A Note on Second Order Probabilities in the Traditional Deterrence Game

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Abstract

This note focuses on a methodological issue that arises naturally in applications of the traditional deterrence game played under two-sided incomplete information. The problem has potentially interesting implications for the status of the conclusions we draw from various applications of the traditional deterrence game.

KEYWORDS: deterrence game, second order probabilities

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The Traditional Deterrence Game

Here we use a basic version of the traditional deterrence game to represent a large class of decision problems commonly encountered in the analysis of conflict. A more complicated version of the game that encounters the same methodological issue is employed in Carlson and Dacey (2009). The traditional deterrence game is presented in Zagare and Kilgour (1993, 2000) and Morrow (1994). In what follows, we develop a sequential decision analysis of the two-sided incomplete information version of the traditional deterrence game.

There are two actors in the traditional deterrence game: Challenger and Defender. Challenger moves first and can choose to either issue a threat or not issue a threat. If Challenger does not threaten, then the game terminates in the status quo ($SQ$). If Challenger threatens, then Defender can either resist the threat or give in to the threat. If Defender gives in, then the game ends in Defender's acquiescence ($ACQ$). If Defender resists, then Challenger can either escalate or Challenger can back down. If Challenger escalates, then the game ends in war ($WAR$); if Challenger backs down, then the game ends in Challenger's capitulation ($CAP$). The traditional game under complete information is shown in Figure 1.

Figure 1 – The Traditional Deterrence Game Under Complete Information
Challenger and Defender can each be one of two types: hard or soft. The preference orderings for the two types of Challengers and Defenders are as follows:

- **Hard Challenger:** \( ACQ > SQ > WAR > CAP \)
- **Soft Challenger:** \( ACQ > SQ > CAP > WAR \)
- **Hard Defender:** \( CAP > SQ > WAR > ACQ \)
- **Soft Defender:** \( CAP > SQ > ACQ > WAR \)

In a game of two-sided incomplete information, each player is uncertain about the other player’s type. Specifically, Challenger has probabilities \( p \) and \( 1 - p \) that Defender is hard and that Defender is soft, respectively, and Defender has conditional probabilities \( q \) and \( 1 - q \) that Challenger is hard and that Challenger is soft, given that Challenger has threatened, respectively. Furthermore, Challenger holds conditional probabilities that Defender resists and does not resist given that Defender is hard, and that Defender resists and does not resist given that Defender is soft. The tree for the two-sided incomplete information version of the traditional deterrence game is presented in Figure 2.

The analysis of a two-sided incomplete information game played as a sequential decision problem requires counterfactual reasoning within the usual unfolding process. Defender evaluates nodes 1 and 2 of Figure 2 as if Challenger were hard or soft, respectively. Thus, at the nodes marked 1, Defender expects a hard Challenger to escalate, whereas at the nodes marked 2, Defender expects a soft Challenger to back down. Hence, Defender evaluates the nodes marked 1 as WAR, and the nodes marked 2 as CAP. At nodes 3 and 4, Defender resists if and only if \( qV(WAR) + (1 - q)V(CAP) > V(ACQ) \), where \( V \) is Defender’s valuation function.

At node 3, Challenger expects a hard Defender to resist since \( qV(WAR) + (1 - q)V(CAP) > V(ACQ) \) for all values of \( q \), and, therefore, Challenger’s conditional probability that Defender resists, given that Defender is hard, is unity. However, at node 4, Challenger expects a soft Defender to resist if and only if \( qV(WAR) + (1 - q)V(CAP) > V(ACQ) \), which holds only for specific values of \( q \), and, therefore, Challenger’s conditional probability that Defender resists, given that Defender is soft, is Challenger’s probability that \( qV(WAR) + (1 - q)V(CAP) > V(ACQ) \). Let \( P \) denote this probability. Then, by the total probability rule, Challenger’s probability that Defender resists is \( p(1) + (1 - p)(P) = p + (1 - p)P \).
If Defender resists, then a hard Challenger escalates, and the game ends in WAR, whereas if Defender resists, then a soft Challenger backs down, and the game ends in $CAP$. For convenience, we examine capitulation, and therefore we need consider only a soft Challenger. Thus, Challenger faces the decision problem presented in Figure 3.
Figure 3 – The Decision Problem for a Soft Challenger Under Two-Sided Incomplete Information

The payoff table for a soft Challenger’s decision problem is:

<table>
<thead>
<tr>
<th>Threaten</th>
<th>Not Threaten</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defender Resists</td>
<td>CAP</td>
<td>$SQ$</td>
</tr>
<tr>
<td>Defender does not Resist</td>
<td>$ACQ$</td>
<td>$SQ$</td>
</tr>
</tbody>
</table>

Challenger will threaten if and only if

$$(p + (1 - p)P)v(CAP) + (1 - p)(1 - P)v(ACQ) > v(SQ)$$  \hspace{1cm} (1)$$

where $v$ is Challenger’s valuation function. This inequality rearranges to the following:

$$\frac{p + (1 - p)P}{(1 - p)(1 - P)} < \frac{v(ACQ) - v(SQ)}{v(SQ) - v(CAP)}$$  \hspace{1cm} (2)$$

We refer to the ratio $\frac{p + (1 - p)P}{(1 - p)(1 - P)}$ as the probability surface and to the ratio $\frac{v(ACQ) - v(SQ)}{v(SQ) - v(CAP)}$ as the probability surface.
\[
\frac{v(ACQ) - v(SQ)}{v(SQ) - v(CAP)}
\]
as the threshold. The former is an everywhere increasing function of both \( p \) and \( P \), and the latter is a positive number. The graph of the probability surface and the threshold is presented in Figure 4.

**Figure 4 – The Probability Surface and the Threshold**
The probability surface and the threshold intersect in an arc. Figure 5 presents the arc when the threshold is equal to 2. Challenger chooses threaten if and only if the foregoing inequality holds, i.e., if and only if the pair $\langle p, P \rangle$ falls inside the arc shown in Figure 5.

**Second Order Probabilities**

The two-sided incomplete information version of the traditional deterrence game raises the issue of second order probabilities. As noted earlier, Challenger holds the probability $P$ that a soft Defender’s expected valuation of resisting is greater than Defender’s valuation of giving in. That is, $P$ is Challenger’s probability that $qV(WAR) + (1-q)V(CAP) > V(ACQ)$. Thereby, $P$ is a second order probability, i.e., a probability over a probability, here $q$. Second order probabilities cause serious difficulties for the interpretation of probability theory that is relevant to the analysis of the traditional deterrence game.

There are three (primary) interpretations of probability theory: the objective interpretation, wherein probability is a frequency (Bernoulli 1713); the...
logical interpretation, wherein probability is a logical relation, like deducibility, between propositions (Keynes 1921, Carnap 1950); and the subjective interpretation, wherein probability is a personal degree of belief and is measured as a decision maker’s betting odds (Ramsey 1926, de Finetti 1937, Savage 1954). Only the subjective interpretation of probability is applicable to the decision-making settings of the kind we encounter in the foregoing analysis of the traditional deterrence game.

The standard account of the subjective interpretation of probability is based upon the Dutch book argument (Ramsey 1926, de Finetti 1937). A Dutch book is a set of bets that the decision maker accepts but is a sure-thing loss for the decision maker. The Dutch book argument is straightforward: if a decision maker’s betting odds do not obey the laws of probability theory, then there exists a Dutch book against the decision maker. Jeffrey (2004, 5-6) presents an example of a Dutch book. In order for second order probabilities to satisfy the subjective interpretation of probability theory, we must be able to establish that there are no Dutch books against the decision maker. However, second order probabilities are such that no events exist against which the bets can be settled. Thus, second order probabilities encounter a difficulty – the events needed to support the subjective interpretation of probability theory are not available. Smets (1997, 243) puts the point as follows:

*Second-order probabilities, i.e. probabilities over probabilities, do not enjoy the same support as subjective probabilities. Indeed, there seems to be no compelling reason to conceive a second-order probability in terms of betting and avoiding Dutch books. So the major justification for the subjective probability modeling is lost. Further, introducing second-order probabilities directly leads to a proposal for third-order probabilities that quantifies our uncertainty about the value of the second-order probabilities. Such iteration leads to an infinite regress of meta-probabilities that cannot be easily avoided.*

The question then becomes: What interpretation of probability theory is satisfied by the second order probabilities encountered in the analysis of the traditional deterrence game? Marshak (1975, 1121) frames the question in terms of the meaning of second order probabilities, as follows:

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1 The paper referred to as Marschak, et al. (1975) is composed of an introductory note by Jacob Marschak, the outline of a presentation by Morris H. DeGroot, a brief paper by Jacob Marschak (entitled “Do Personal Probabilities of Probabilities Have an Operational Meaning?”) and comments by Karl Borch, Herman Chernoff, Morris H. DeGroot, Robert Dorfman, Ward Edwards, T. S. Ferguson, Koichi Miyasawa, Paul Randolph, L. J. Savage, Robert Schlaifer, and
Suppose all probabilities are defined as ‘subjective’, ‘personal’ – i.e., as being revealed by a ‘rational’, ‘consistent’ decision-maker’s choices under uncertainty: if he prefers to bet on one rather than on another event, the former is said to be the subjectively more probable one. Moreover, a few rather plausible quasi-logical postulates of ‘rationality’ of choices imply that such ‘subjective probabilities’ have indeed the properties of ‘mathematical measure’. Our question is: what meaning, if any, can be assigned to the probability of a hypothesis, law, theory that is itself a probability distribution so that its falsity or truth is not, in general, an observable event on which bets can be made and paid-off.

Here, the “hypothesis … that is itself a probability distribution” is the proposition \( qV(WAR) + (1 - q)V(CAP) > V(ACQ) \). Challenger does not know the truth-status of this proposition and assigns to it the probability \( P \). More importantly, and to Marschak’s point, the proposition is such “that its falsity or truth is not, in general, an observable event on which bets can be made and paid-off”. Thus, the foregoing inequality is the kind of proposition Marschak finds troublesome.

Marschak addresses the issues at hand via the question, “Do probabilities of probabilities have an operational meaning?” (Marschak 1975, 127-133). The term ‘operational’ has (at least) two uses. As used by Marschak, ‘operational’ refers to the existence of a formal system whereby second order probabilities have a coherent interpretation as subjective probabilities. The foregoing discussion shows that second order probabilities cannot be interpreted as subjective probabilities. As used in the empirical testing of theories and models, ‘operational’ refers to the existence of a practical system whereby values can be assigned to second order probabilities on the basis of available data. Various analyses show that practical systems do exist whereby second order probabilities can thus be interpreted as probabilities. For example, Carlson (1995, 525) characterizes a second order probability of escalation in terms of military capabilities in order to test hypotheses drawn from an analysis of escalation based on the traditional deterrence game.

We are left with an interesting two-part conclusion. Given that the probabilities we employ in the traditional deterrence game make sense only as subjective probabilities, the second order probabilities we encounter in the
foregoing analysis do not have an operational meaning. That is, we do not have a foundation for interpreting second order probabilities, as encountered in the traditional deterrence game, as betting odds and thereby as subjective probabilities. Thus, under Marschak’s use of ‘operational’, second order probabilities do not have an operational meaning. Contrariwise, under the second use of ‘operational’, when assessed in terms of measurable factors such as military capabilities, second order probabilities do have an operational meaning.

It is interesting to note that the purely formal (i.e., axiomatic) aspects of second-order probabilities have been addressed. Gaifman (1988) provides a formal structure for higher order probabilities, and Nau (2006) applies second-order probabilities in an analysis similar to the analysis of the traditional deterrence game employed here. Nonetheless, second-order probabilities, as subjective probabilities, remain problematic for the reasons discussed in Marschak (1975) and highlighted in Smets (1997).

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