Effortful Touch With Minimum Movement: Revisited

Susan J. Lederman
Queen's University

Sapna R. Ganeshan
Bell-Northern Research

Randy E. Ellis
Queen's University

The ecological static moment–torque model proposed by C. Carello, P. Fitzpatrick, I. Domaniewicz, T. C. Chan, and M. T. Turvey (1992) does not uniquely explain the perception of rod length by static holding. Guided by a mechanical analysis of the gravitational forces and torques produced in the hand as it statically holds rods of different lengths and materials at different orientations, we offer 2 additional theoretical explanations, the force–torque and weight–percept models. Experiment 1 demonstrates that all 3 models predict perceived rod length with considerable success. Experiment 2 provides clear experimental support for the force–torque and weight–percept models over the static moment–torque model. Experiment 3 pits the former 2 models against each other. Current results favor the weight–percept model.

Implications for theories of haptic weight perception and design of a new tactile sensor are also considered.

Our article derives from an intriguing program of research by Turvey, Carello, and their colleagues, which is described below. It deals with how humans perform actions on or with objects by holding or wielding. The group has argued that such dynamic or effortful touch has important ramifications for how people perceive the characteristics of the handled object and its position in external space. Although visual and haptic activities are usually coordinated, it is necessary to determine the effectiveness of effortful touch on its own. In addition, we add that there are many situations in which the visual information may be restricted; for example, vision will be limited by viewing perspective, by low levels of illumination, and when the hand or arm obstructs one's sight of the object.

In a recent series of experiments, the Connecticut group has applied Gibson's (1966) ecological or direct perception approach to problems of haptic perception. In general, it is argued that perception is based on the information contained in structured energy array. With respect to the haptic system, Turvey, Solomon, & Burton, 1989) have specifically argued that "perception is based on the information contained in structured arrays of deformations or contacts" (p. 404). Two different structured arrays are considered. The first is the inertia tensor, which quantifies the pattern of moments of inertia that serve to resist rotational acceleration in three-dimensional space during wielding; the second is a corresponding strain tensor, which quantifies the different strains within the biological tissue involved during wielding. Turvey et al. argue that "these arrays contain invariants that are specific to environmental properties, and to which perception, in turn appears to be specific" (p. 404).

The ecological group has applied this theoretical approach in addressing, for example, how people haptically process the length of dynamically wielded rods. The initial experiments (e.g., Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989; Turvey et al. 1989) required participants to judge the extent of visually occluded rods dynamically wielded by movements about the wrist. They concluded that the highly accurate length judgments obtained in their experiments were determined by the rod's resistance to rotational acceleration, that is, the principle maximum moment(s) of inertia of the hand–rod system about the axis of rotation, rather than by rod weight, mass, or center of mass (CM). Perceiving the length of wielded objects becomes a matter of selectively detecting the higher order variable(s) that remain invariant over specific transformations, in this case, presumably variation in the accelerations and/or in the forces and torques that effect changes in acceleration.

Our article deals with a related perceptual problem recently investigated by Carello, Turvey, and their collaborators: Can, and if so how, does an observer haptically perceive the extent of statically held rods? The problem was initially studied by Hoisington (1920) in a series of experiments in which the participants made judgments about the

---

Susan J. Lederman, Department of Psychology, Queens University, Kingston, Ontario, Canada; Sapna R. Ganeshan, Bell-Northern Research Ltd., Ottawa, Ontario, Canada; Randy E. Ellis, Department of Computing and Information Science, Queens University, Kingston, Ontario, Canada.

This research was supported by the following Centers of Excellence: the Manufacturing Research Corporation of Ontario and the Institute for Robotics and Intelligent Systems. Part of this work was based on a master of science degree obtained in computing and information science by Sapna R. Ganeshan. We would like to thank Cheryl O'Neill, Steve Ito, Roberta Klatzky, Andy Bryson, Nilanjun Sarkar, and Jayanth Srinivasan for their contributions.

Correspondence concerning this article should be addressed to Susan J. Lederman, Department of Psychology, Queen's University, Kingston, Ontario, Canada K7L 3N6. Electronic mail may be sent via Internet to lederman@pavlov.psyc.queensu.ca.
lengths of rods that were held loosely at one end and lifted vertically a short distance by a whole-arm movement, while preserving a horizontal orientation. Throughout the trials, no movement of the rods was permitted. Across the experiment, the length, weight, and CM of the rods were varied singly, in pairs, or all together. Participants were required to judge the rod’s length (longer than, equal to, or shorter than) relative to a previously presented rod.

Hoisington (1920) reported that participants failed to discriminate among the rods when only the length varied, but discrimination was very accurate and reliable when the CM varied. It was also good when weight alone, or both weight and length, varied. Hoisington (1920) pointed out that the ratio of the force on the base of the thumb to the force on the forefinger was related to CM, and thus, to length discrimination. In another study, by devising a simulation in which forces could be applied independently to two sides of a short dowel that was hinged in the middle, Hoisington could simulate the haptic feel of rods of a wide range of lengths, although the mass and length of the dowel itself were not varied. According to the ecological approach, Hoisington’s research suggested that for these minimal lifting conditions, perception of rod length is dictated by the gradient or distribution of forces over spatial extent.

We also note, however, that the principles of static-equilibrium analysis suggest that by applying forces at the two ends of a rod gripped at an intermediate location, the reactive force and reactive torque developed at the grip can be varied, thereby simulating systems with different force–torque configurations. Also, this analysis could probably be used to explain the simulation of the haptic feel of a wide range of rod lengths. Hoisington’s (1920) argument that the ratio of the force on the base of the thumb to that on the forefinger affects length discrimination. Furthermore, this points to torque as another possible candidate in the perception of the length of unseen objects held steady in the hand. The contribution of this mechanical variable, however, was not addressed in Hoisington’s study.

In a more recent study by Burton and Turvey (1990), participants were asked to provide length estimates of unseen rods held at one end, under static holding conditions, by moving a board that ran along adjacent tracks. Participants were not permitted to see the hand that was holding the rod, but they were allowed to see the board that was in front of them, whose position they could adjust by moving foot pedals. The distance from the board to the edge of the desk at which the participants sat served as a measure for the length of the rod. Length estimates exhibited an impressive linear dependency on the moment of inertia ($r^2 = .97$) of the rod about the grasp. However, moments of inertia were not considered as a factor affecting the perception of length because they were inaccessible when the object was not undergoing rotation. In keeping with the ecological approach, which emphasizes the role played by invariants, Burton and Turvey (1990) suggested that observers were detecting a different motion-independent variable, namely the static moment (i.e., the first moment of mass distribution), with which the moment of inertia completely covaries. (In mechanics texts, static moments are discussed in the early sections of texts about statics. Discussion of moments of inertia, which do not play a role in the mechanical behavior of systems in static equilibrium, is reserved for the later sections in texts about dynamics, which involves the study of the behavior of bodies subjected to forces that produce changes in their motion.)

Length estimates were obtained by Burton and Turvey (1990) in three experimental conditions: no mass attached, a single mass attached to one side of the grip, and two masses attached at equal distances from the grasp (where the grip was located at an intermediate position on the rod). Rods with no masses attached and those with masses attached on both sides were perceived differently when gripped at the center, thereby refuting roles for either CM or the force gradient, which changes with the position of the CM from the point of application of the downward force on the forefinger. Although static moment was a better predictor of length, the investigators did not entirely dismiss the role played by moments of inertia of the rod; they suggested that participants may in fact have been moving their hands slightly, and that the wrist of the participant would have to be immobilized were the influence of moment of inertia to be eliminated. However, this hypothesis was not experimentally evaluated.

In a subsequent study by Burton and Turvey (1991), which was similar to one conducted on dynamic wielding by Solomon et al. (1989), participants were asked to provide length estimates of the portion of unseen rods forward of the grasp (where the rods were held statically at an intermediate position along the rod’s length). It was reported that perceived extent is a function of the static moment associated with the forward portion of the rod. In all of the experimental conditions in the studies by Burton and Turvey (1990; 1991), the static moment covaries perfectly with the torque acting at the grip. Both variables assume the value zero when the grasp coincides with the CM. In other words, for rods with no masses attached and for rods with two equal masses attached at equal distances from the grasp, the torque and static moment will be equal to zero when the rod is held at its center. Recognizing this fact, the investigators, therefore, calculated the static moment by assuming that the pivot point was located at the wrist. Yet, because a similar technique can be adopted when calculating torque, the arguments presented by Burton and Turvey (1990; 1991) do not explain why static moment was chosen over static torque.

We began our current study by observing that the effects of the forces and torques felt in the hand (or the wrist) were not considered in the studies by Burton and Turvey (1991) and Solomon et al. (1989). When a hand-held rod is wielded, stresses and strains develop in the tissues of the hand and wrist, as a result of the forces and torques acting on them. (The rest of the arm is not considered, because the stress developed here does not affect the hand or wrist in any significant way.) The stresses developed in a body are dictated by the nature of the forces acting on them. In other words, the stresses in the hand and the wrist bear a direct and proportional relation to the forces and torques acting on
them due to the rod and also due to the effects of holding it steady.

Because of the complexities involved with the material and structural properties of the hand and wrist, it is difficult to perform a stress analysis for the problem under consideration. Although the nature of the stresses and strains cannot be fully understood, the nature of the forces acting on the hand and wrist can be studied with relative ease. To do so, an object held stationary in the hand without any rotary or translatory movement is treated as a system at rest. The system can then be effectively investigated by performing a detailed mechanical analysis, on the basis of the principles of static equilibrium.

This type of an analysis was not provided by Burton and Turvey (1990; 1991). Therefore, they have not convincingly proved that static moment uniquely determines the perceived length of unseen rods statically held in the hand. In the current article, we will experimentally show that the static moment is not in fact used (Experiment 2).

Carello et al.'s Static Moment–Torque (SM–T) Model: An Ecological Approach

A recent set of experiments by Carello et al. (1992) serves as the primary focus for our study, and hence is described in detail. Carello et al. began by providing a mechanical analysis that was used to address a prediction derived from Hoisington's (1920) claim that the force gradient impressed on the hand when a rod is held is critical to perceiving extent. We present their mechanical analysis here, as it is also crucial to Carello et al.’s calculations of torque, and therefore, to their conclusions regarding a role for this parameter in judging the length of statically held rods.

Recall that Hoisington (1920) hypothesized that haptic perception of extent under conditions of effortful touch with minimal movement was understandable through a force gradient given by the ratio of downward to upward forces impressed on the hand while it held a rod, where the downward force, \( D \), and the upward force, \( U \), are provided at the middle phalanges of the index finger and the heel of the hand when a rod is grasped at the end. To test this prediction experimentally, Carello et al. (1992) derived the following equation that describes the ratio of the downward force and the upward force (their corresponding free-body diagram is shown in Figure 1a):

\[
\frac{D}{U} = \left( \frac{a}{a + b} \right) \tag{1}
\]

where \( D \) is the downward force, \( U \) is the upward force, \( b \) is the distance separating the two forces, and \( a \) is the distance of the upward force from the CM of the rod.

On the basis of Equation 1, Carello et al. (1992) predicted that the perception of a rod’s length should decrease with increasing distance between \( D \) and \( U \), and conducted a series of experiments derived from this prediction. Carello et al. reported that simple regression of the ratio of the two forces accounted for only 39% of the variance in perceived length \( (r^2 = .39) \). Consequently, Carello et al. suggested that torque could be a possible candidate affecting perceived length.

The length estimates provided by participants for rods held at two supports, where the supports were provided by the two hands with the palms facing each other, were plotted against the torque calculated about the point of application of \( U \). Carello et al. (1992) concluded in their Experiments 2–4 that torque is a significant factor affecting length. In fact, torque accounted for 94% of the variance in mean perceived length (Carello et al.’s Experiment 2), and the relation between torques and perceived length approximated closely a single-valued function.

In Experiment 5 of Carello et al.’s (1992) study, visually occluded rods were presented in three different orientations: vertical, horizontal, and at 45° to the ground plane. This particular design was used by the investigators to separate the effect of torque from static moment, with which it is highly correlated. Perceived extent decreased as the orientation became more vertical and the effect tended to be more pronounced with longer rods. Carello et al. reported that when static moment and torque varied orthogonally, per-
ceived length was affected by both parameters, although not to the same extent—static moment dominated. Carello et al. reported a multiple regression of mean perceived length on static moment and torque that yielded an $R^2$ value of .96, and $r^2$ values for simple correlations between perceived length and each of the two predictor variables were .77 and .49, respectively. Henceforth, we shall refer to this as the static moment–torque (SM–T) model.

One question that the ecological approach needs to clarify is: What sort of entity is torque? In Carello et al.’s (1992) introductory remarks (p. 291), they noted that under static holding conditions, the static moment about an arbitrary point was invariant over static torques. Yet, on the basis of results from their subsequent experiments, they concluded that gravitational torque also contributes to participants’ perceived length estimates. Is torque to be treated as an invariant? If so, over what specific transformations? If it is not invariant, then what are the implications for an ecological analysis in which perceived length is specified by a combination of variant and invariant variables?

A Static-Equilibrium Analysis of the Hand–Rod System Gripping an Object at One End

As the perception of statically held rod length necessarily involves this kind of physical system, we began our own research on the topic as well by presenting a formal static analysis of the rod-gripper system, performed with the help of force diagrams (free-body diagrams). Definitions of the mechanical parameters are included in the Appendix. The term gripper could refer either to the hand (for purposes of the current problem) or to a mechanical device capable of providing force–torque measurements (see Ellis, Ganeshan, & Lederman, 1994; Ganeshan, Ellis, & Lederman, 1994), thus rendering our analysis applicable to both human and machine domains.

Our analysis of the gripped rod will resolve the net forces and torques, produced by the rod and grip, about a single point; one should examine Figure 1B to compare our free-body diagram to that of Carello et al. (1992). The current mechanical analysis considers the general case in which an object is gripped at one extreme and held statically in different orientations with respect to the ground plane. This analysis applies to the single-support condition in Carello et al.’s (1992) Experiment 5 (as opposed to the two-support conditions in the preceding experiments in that study), and to our current experiments. We demonstrate here that the length of the object becomes indeterminate when the object is held perpendicular to the ground plane.

Assumptions

The assumptions made in the analysis of the system are as follows:
1. The object is uniform.
2. The angle or orientation of the object with respect to the ground is assumed to be a known quantity.
3. The rod-gripper system is at rest and in a state of static equilibrium.

Uniformity applies if the body has the same geometric properties throughout its length, for example, a rod having a constant cross section throughout its length.

The rod-gripper system is considered to be in static equilibrium when there is no macroscopic motion—in this situation, the forces and the torques are each maintained in balance.

Nomenclature for Static Analysis

We consider a uniform, isotropic object whose mass and total length are given by $M$ and $L$, respectively. Because the object is rigid and in static equilibrium, the total mass of the object is taken to be a point mass acting at the CM of the object. This convenient assumption is valid in static analysis, because it does not affect the computation of the forces and torques. In the case of a uniform object, the CM is located at the midsection of the object, that is, it is located at a distance of $L/2$ from either end.

The definitions of terms used in the mechanical analysis with vector quantities symbolized in boldface type are as follows:

<table>
<thead>
<tr>
<th>Term</th>
<th>Subscript</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of mass of object</td>
<td>CM</td>
</tr>
<tr>
<td>Force due to the weight of the object</td>
<td>F</td>
</tr>
<tr>
<td>Reactive force provided by the gripper</td>
<td>$F_R$</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>$g$</td>
</tr>
<tr>
<td>Total length of object</td>
<td>$L$</td>
</tr>
<tr>
<td>Torque subtended at the gripper by object</td>
<td>$T$</td>
</tr>
<tr>
<td>Reactive torque provided by the gripper</td>
<td>$T_R$</td>
</tr>
<tr>
<td>Mass of object</td>
<td>$M$</td>
</tr>
<tr>
<td>Orientation of object with respect to the ground plane</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

Coordinate systems are constructed so that downward forces and anticlockwise torques are considered negative, whereas upward forces and clockwise torques are considered positive, according to the left-hand rule.

Analysis

Let the object be gripped at one of its ends, and select a point in the grip about which forces and torques will be resolved (the wrist point). The object may be held perpendicular to the ground plane, parallel to the ground, or at some angle to it. The nature of the forces and torques acting on the gripper depend on the orientation of the object with respect to the ground.

The free-body diagram of the object vertical to the ground (with the object being above the gripper in a inverted-pendulum state, as in the vertical condition of our Experiment 1) is shown in Figure 2a. Even if the object is held below the grip in a pendulum state (our Experiment 2), the analysis remains essentially the same. The force, $F$, on the basis of the weight of the object acting from its CM causes pure translatory motion of the object in the direction perpendicular to the ground plane. To maintain static equilibrium, the gripper will have to produce a reactive force, $F_R$.
When the object is held at an angle \( \theta \) to the ground plane (Figure 2b), the reactive force \( F_R \), offered by the gripper to counteract the effect of force \( F \) remains the same as in the previous situation. However, a reactive torque, \( T_R \) is developed at the grip due to the twisting effect produced by the vertical component of the force \( F \). The direction of \( T_R \) is opposite to that of torque \( T \) produced by the object, whereas its magnitude is the same as that of \( T \), that is,

\[
-T_R = T, \quad \text{and} \quad T = F \cos(\theta) \left[ \frac{L}{2} \right] = -\frac{1}{2} Mg L \cos(\theta). \tag{3}
\]

If \( \theta \) is zero, as in the case where the object is held parallel to the ground plane, the magnitude of the reactive torque will have a maximum value. As \( \theta \) increases, the magnitude of the reactive torque decreases, and when the object is perpendicular to the ground plane (\( \theta = 90^\circ \)), then the total length (and the mass) of the object can be determined using Equations 2 and 3.

In the following section, two alternative theoretical approaches are offered. The first one is based directly on the mechanical force–torque analysis presented immediately above. The second one suggests that length perception maps to a perceptual variable, the corresponding weight percept, which in turn is influenced primarily by the forces and torques.

### The Gravitational Force–Torque (F–T) Model: A Traditional Psychophysical Approach

When a rod is held in the hand in a gravitational field, stresses and strains are developed there because of the effects of the force and the torque acting at the grip (caused by the weight of the rod). The magnitude and direction of these forces acting on the hand depend on the location at which the rod is grasped and on the orientation of the rod with respect to the ground plane. In the mechanical analysis above, we showed that the length of a uniform rod can be derived from the reactive forces and torques developed at the grip, using the principles of static analysis. On the basis of this analysis, we propose the gravitational force–torque (F–T) model: The perceived length of unseen rods held stationary in the hand depends on both the force applied by the rod (i.e., its weight) and the torque acting at the grip.

For a physical system subjected to the action of forces, metric information regarding the forces and torques is first transduced from the stresses developed in the body. The model proposed here takes into consideration the physical inputs that directly produce the deformation patterns sensed by mechanoreceptors embedded in the relevant haptic tissues.
The Weight–Percept Model: A Constructive Approach

A more interpretive account of the perceived length of statically held rods might argue for the importance of some intervening percept. For example, in his theory of unconscious inference, Helmholtz (1962) proposed that the perceptual constancies (e.g., size, brightness) were the result of an unconscious, computational trade-off between the proximally sensed retinal image and some intermediate percept (perceived distance and perceived illumination, respectively). We will use the term percept models to describe this general class of perceptual theorizing (see reviews of contemporary versions of such models by Epstein, 1982, & Sedgewick, 1986). Gogel (1980), for example, demonstrated the importance of perceived distance in the apparent motion of objects.

In reading Hoisington’s (1920) original article, we observed that when individuals were asked to provide a full psychological report at the end of the session, some individuals reported that they had initially estimated weight and subsequently used these estimates to judge rod length. Individuals reported assigning greater length estimates to heavier rods. Hoisington commented: “It is much more probable that weight was the first item of perception, immediately translated into length terms” (p. 120). And indeed, we observed the same tendencies in our own pilot participants. Because individuals may estimate the rod’s weight in every situation, irrespective of the grasp location or orientation of the rod with respect to the ground plane, it raises the possibility that perceived length maps to estimated weight. We refer to this as the weight–percept model. On the basis of the mechanical analysis, we further suggest that the weight estimates derive primarily from the gravitational forces and torques, as set out in our mechanical analysis.

The Current Experiments

The following experiments attempted to experimentally evaluate the relative validity of the three models outlined above. In Experiment 1, participants judged the length of homogeneous rods varying in mass (and thus, weight) and in the orientation at which they were held relative to the ground plane. All three models proved highly successful in predicting length estimates; however, with such high simple correlations among the predictor variables, it was not possible to assess their relative merits. Experiment 2 was, therefore, designed to pit the SM–T model experimentally against the F–T and weight–percept models: Static moment was varied over a considerable range while holding the force (i.e., weight) constant, and as shall be demonstrated, the perceived weight constant; torque was eliminated by requiring participants to hold the rod vertically, relative to the ground plane. Having demonstrated strong support for the F–T and weight–percept models, Experiment 3 then pitted these two against each other by using the haptic size–weight illusion (Ellis & Lederman, 1993) to induce a change in perceived weight with no variation in the gravitational forces and torques. Collectively, the results of our study favor the weight–percept model.


Participants were presented with visually occluded rods of widely varying masses to obtain a broad range of values for force and torque, as compared to the studies previously described. The values for force F and torque T vary from 1.22 N to 5.25 N and 0 to 1.575 N m, respectively (with handle taken into account). In contrast, the values for F and T used by Burton and Turvey (1990) varied from 0.36 to 0.96 N and from 0.0828 to 0.5856 N m, respectively.

Rods of three different lengths and three different materials were used. This design allowed us to study the effect of the mass (and thus, weight) of the rods on perceived length, when actual length remains unchanged; thus, we could determine if different rods having the same physical length but different masses (produced by using different density materials) were perceived to be of equal length.

To decorrelate the force (rod weight) from the torque acting at the grip, each rod was presented in three orientations: vertical, horizontal, or 45° to the ground plane. Coincidentally, Carello et al., 1992, independently devised the same manipulation of rod orientation to decouple static moment and gravitational torque.) This manipulation increased the range of torque values further than is possible when the rod is always held parallel to the ground. It is important to note that for a given rod, the mass, weight, length, and static moment of the rod remained constant across all three orientations, whereas the torque varied from a minimum value, zero, in the vertical condition, to a maximum value in the horizontal condition (see static analysis above). In addition to length estimates, estimates of weight were also obtained to test for the contribution of the weight percept to perceived length.

Method

Participants. A total of 16 experimentally naive graduate students at Queen’s University were paid to participate in the experiment, their ages ranging from 18 to 35 years. All wrote with their right hand. Six were men and 9 were women.

Stimuli and apparatus. A set of nine rods were prepared. Rod diameter was 0.75 in. (1.91 cm). Three rod lengths were used: 12 in. (30.5 cm), 18 in. (45.7 cm), and 24 in. (61.0 cm). Each rod length was constructed out of three different materials, that is, wood, plastic, and aluminum. Corresponding masses (including the handle) for the short, medium, and long lengths were 124, 144, and 169 g, respectively, for wood; 171, 224, and 274 g, respectively, for plastic; and 304, 418, and 535 g, respectively, for aluminum. Thus, the ratio of the shortest to heaviest masses was 1:1.2, whereas the ratio of the lightest to heaviest masses was 1:4.3. Each rod was screwed into a handle with a thin plywood end plate at the proximal end (its total mass, 71.8 g, was constant across all fixtures); the handle was used to prevent the participants from
Knowing the material out of which the rod was constructed. The length of the handle was 4.5 in (11.0 cm), which comfortably accommodated an average-sized hand (see Figure 3). In calculating the weight of the rod, the weight of the handle, the verticle plate, and the screws were also taken into account. Because the weight of the vertical plate and the screws (extending beyond the grasp) and their distances from the grasp were negligible relative to those of the rod, these were not used in calculating the torque felt at the grip.

Procedure. Participants sat on a chair facing a curtain, beyond which a table was placed at a comfortable height. They were required to rest their right forearm on the table (which helped to minimize the effect of torque on the arm); their hand protruded beyond the distal edge. Participants extended their palm out straight in a pronated position; on initial contact with the stimulus rod (all extended outward to the participant’s left), they rotated their right hand quickly to grasp each rod supported by the experimenter at one of three different orientations: 0° (horizontal), 45° upward to the left (slanted), and 90° (vertical) in the frontoparallel plane. The targeted orientation was achieved as follows: A white sheet was mounted on the wall immediately behind the experimenter, with a light source positioned in front so as to cast a shadow of the rod on the wall. On each trial, the experimenter aligned the shadow with appropriate orientation guide marks to achieve the desired orientation. Participants grasped the rod by wrapping their palm around the handle; the thumb and index finger lay flush against the handle’s perpendicularly aligned end plate. On a signal from the participant, the experimenter released her own grasp on the rod; participants were instructed to maintain the rod in the given static orientation until responding.

To obtain an estimate of error indicating the deviation of the rod from the desired hand position, the angular deviation of the rod from the nominal orientation was measured for the first 4 participants. This was achieved by measuring the deviation of the shadow of the hand-held rod. This measurement was made at the end of each trial during the practice block. The mean angular deviation was 4.74°, 4.13°, and 5.83° (3.2, 2.97, and 5.83) for the vertical, slanted, and horizontal conditions, respectively. These data indicate that the actual rod positions were very close to the nominal positions required; the small deviations measured should, therefore, have little effect on the calculations of torque. Also, the data show little difference in error across the different orientations. In fact, because the error measurements were obtained during the initial practice block, it is possible that during the succeeding test trials error may still have been lower.

An absolute magnitude estimation procedure (Zwislocki & Goodman, 1980) was used to obtain estimates of rod length and, subsequently, of weight. The participant was instructed to assign the positive number (decimal, fraction, or whole number) that best matched the rod’s length (from the outer edge of the end plate to the rod’s far tip) or weight. No standard or modulus was used. The length estimates were obtained before the weight estimates to ensure that participants were not biased toward using weight estimates in their length judgments.

Experimental design. A completely within-subject design was used, with three completely crossed levels: length (3), material (3), and orientation (3). Each participant was initially given one block of 27 practice trials, followed by two additional blocks of 27 test trials in which rod length was estimated. An additional three blocks (one practice, two test) of 27 trials were subsequently performed during which rod weight was estimated. The 27 conditions within each block were completely randomized.

Results

Length estimates. An analysis of variance (ANOVA) was performed on the participant’s length estimates using a repeated-measures design with three levels, that is length (3 levels), material (3 levels), and orientation (3 levels). The length estimates were initially normalized across participants to adjust for differences in the numerical scales used; they were subsequently logarithmically transformed to achieve homogeneity of variance because the means of magnitude-estimate scores typically covary in a fairly linear manner with associated variances (e.g., see discussion by Marks, 1982). However, we note that similar patterns of results were obtained in all experiments without the logarithmic transformation.

Main effects for length and material were highly statistically significant, $F$s(2, 30) = 98.5 and 96.5, respectively; both $ps < .0000$. Length estimates increased with increasing length, ensuring that we had chosen lengths that were perceptually different from one another. The estimates also varied consistently with material, increasing in the following order: wood, plastic, and aluminum. In other words, for each length, the wooden rod was perceived to be the shortest and the aluminum rod the longest. Post hoc tests on the main effects indicated that all pairwise comparisons in both groups were highly significant, with no $p$ value greater than .005.

Because the main effect for orientation was not statistically significant, two-way interactions with both length and material were significant at the .01 level or better: $F$s(4, 60) = 3.26 and 4.24 for Length × Orientation and Material × Orientation, respectively. In general, these interaction effects indicate that length estimates tended to increase with decreasing orientation (90°–0°) most consistently as density.

Figure 3. Experiment 1: the experimental set-up. A participant poses holding a rod horizontally relative to the ground plane. Overall, rods varied in length and material and were held in different static orientations.
and rod length increased (the latter interaction was similar to that obtained previously by Carello et al. (1992); the orientation effect was not apparent for the short, plastic rods or for the wooden rods.

The geometric means of the normalized length estimates are plotted in Figure 4a on a log scale as a function of orientation and length; the data are plotted in separate panels for wood, plastic, and aluminum.

Although a magnitude estimation procedure does not permit us to assess absolute accuracy of length estimation, we can compare physical length ratios to perceived length ratios using geometric mean estimates. Averaging materials, because the Length × Material interaction was not significant, participants' estimates increased 2.34-fold for a 2-fold increase in actual rod length (12 vs. 24-in., 30.5 vs. 61.0-cm rods); they increased 1.69-fold and 1.38-fold for 1.5-fold and 1.33-fold increases in actual length (12 vs. 18-in., 30.5 vs. 46.0-cm, and 18 vs. 24-in., 46.0 vs. 61.0-cm rods), respectively. Thus, participants' estimates reflected a consistent tendency to overestimate the ratios of rod-length pairs.

Weight estimates. In keeping with the length-estimate data, the corresponding weight estimates were initially subjected to normalization, and subsequently, to logarithmic transformation. An ANOVA was performed on the resulting weight estimates using the same repeated-measures design. Again, only the main effects based on length and material were statistically significant. All main effects and their interactions were highly statistically significant at the .005 level or less, with the exception of the Material × Length interaction ($p = .11$). The weight estimates varied in strikingly similar fashion to the length estimates, as apparent in Figure 4b. Weight estimates increased with variations in material, in the order of wood, plastic, and aluminum, and with increasing length. The two-way interactions, Orientation × Length and Orientation × Material, were both sta-

![Figure 4](image_url)

*Figure 4.* Experiment 1: geometric means of normalized length (a) and weight (b) estimates as a function of orientation and material.
PERCEIVING STATICALLY HELD ROD LENGTHS

859

estimated significant, $F(2, 30) = 12.88$ and $9.07$, $p < .0000$. Like the length estimates, weight estimates increased with decreasing orientation, with highest consistency for the densest and longest rods.

For each of the three pairs of rod lengths made of the same material (18 vs. 12, 24 vs. 18, 24 vs. 12 in.; 46.0 vs. 30.5, 61.0 vs. 46.0, 61.0 vs. 30.5 cm), the physical versus perceived weight ratios were as follows: for wood, 1.16 versus 1.55, 1.17 versus 1.45, and 1.36 versus 2.24, respectively; for plastic, 1.31 versus 1.85, 1.22 versus 1.73, and 1.60 versus 3.20; for aluminum, 1.38 versus 1.87, 1.28 versus 1.47, and 1.76 versus 2.76. Thus, participants' estimates tended to reflect an overestimation of the actual rod weight ratios.

**Correlational analyses.** A simple correlation matrix is presented in Table 1, which includes log$_{10}$ perceived length and log$_{10}$ perceived weight estimates, as well as the mechanical parameters highlighted by our mechanical analysis and by Carello et al. (1992). Both normalized and subsequently logarithmically transformed scores of the mechanical variables (including torque) were initially considered; however, the logarithmic set produced higher correlations with other variables than did the raw-data set. In all subsequent analyses, therefore, logarithmically transformed values were used. We note that the length estimates are very highly correlated with weight and static moment, less so with torque, and still less with length; perceived length and perceived weight correlated most strongly ($r = .983$), as evident in Figure 5.

Given that the force based on rod weight is similarly present in all three conditions, whereas the torque is theoretically unavailable in the vertical condition, two additional sets of simple correlations were obtained, again using the logarithmically transformed data. One set of simple correlations was based only on the vertical condition results, where torque $= 0$; the other set of simple correlations was based on an aggregate of the oblique and horizontal conditions. In the vertical condition, physical weight gave even slightly higher values for $r$ when it was correlated with both perceived length and perceived weight ($r = .963$ and .985, respectively), whereas the $r$ values between perceived weight and perceived length increased very slightly from .983 to .987. With the vertical condition, results excluded, torque was very strongly correlated with both perceived length and perceived weight ($r = .959$ and .976, respectively). The $r$ value obtained for perceived weight and perceived length was .983, which is the same as that obtained when all three orientation conditions are considered.

The parameter selection for the following multiple regression analyses was guided by the results obtained from the simple correlational analyses. A standard multiple regression analysis was performed to evaluate the three linear models of perceived length. Carello et al.'s (1992) SM-T model of the perceived length of statically held rods (from their Experiment 5) included static moment (compare gravitational force in our F-T model) and gravitational torque as predictor variables. To evaluate this model using the current data, perceived length was regressed on static moment and torque. Logarithmically transformed scores were used for reasons explained above. An $R^2$ value of .972 was obtained, which was very similar to the value obtained by Carello et al.:

$log_{10}$ perceived length $= 0.69 log_{10}$ static moment

$+ 0.10 log_{10}$ torque $- 0.27$. (4)

On the basis of our mechanical analysis, the psychophysical F-T model predicts $log_{10}$ perceived length from $log_{10}$ force (weight) and $log_{10}$ torque. The linear equation obtained for

Table 1

**Experiment 1: A Simple Correlation Coefficient Matrix of Relevant Physical and Perceptual Variables**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Log perceived length</td>
<td>.580</td>
<td>.948</td>
<td>.678</td>
<td>.980</td>
<td>.983</td>
</tr>
<tr>
<td>2</td>
<td>Log length</td>
<td></td>
<td>.402</td>
<td>.374</td>
<td>.618</td>
<td>.552</td>
</tr>
<tr>
<td>3</td>
<td>Log force (weight)</td>
<td></td>
<td></td>
<td>.578</td>
<td>.964</td>
<td>.970</td>
</tr>
<tr>
<td>4</td>
<td>Log torque</td>
<td></td>
<td></td>
<td></td>
<td>.600</td>
<td>.670</td>
</tr>
<tr>
<td>5</td>
<td>Log static moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.985</td>
</tr>
<tr>
<td>6</td>
<td>Log perceived weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. Experiment 1: geometric means of normalized length estimates as a function of the geometric means of the normalized weight estimates.
the F–T model, with an $R^2$ value of .924, was

$$\log_{10} \text{perceived length} = 1.10 \log_{10} \text{force}$$

$$+ 0.13 \log_{10} \text{torque} + 0.42. \quad (5)$$

To evaluate the weight-percept model, we could only examine the simple correlation between perceived length and perceived weight, which as previously noted, was exceptionally high (.983). The phenomenological reports of the participants also suggested a dependence of length estimates on perceived weight. Twelve out of 16 participants volunteered that they had based their judgments of length on the estimated weight of the object. The other 4 participants suggested that they were influenced by the twisting effect (torque) felt at the grip.

To further determine the extent to which the weight estimates were predicted by the gravitational forces and torques, we performed the following multiple regression, which yielded an $R^2$ value of .959:

$$\log_{10} \text{perceived weight} = 1.37 \log_{10} \text{force}$$

$$+ 0.13 \log_{10} \text{torque} + 1.82. \quad (6)$$

**Discussion**

Experiment 1 was designed to evaluate the predictive power of three different models of the perception of the lengths of statically held rods. These include Carello et al.'s (1992) ecological SM–F model and two newly proposed models derived from a formal mechanical analysis—the psychophysical F–T model and the weight–percept model that is based on the participants' weight percepts. To fully explore the role of the gravitational forces and torques induced during static holding at one end of the rod, the experiment varied weight (through changes in density) and orientation relative to the ground plane, in addition to rod length. The orientation manipulation was performed to break the covariation between the forces and torques, to assess their separate contributions.

The results of the correlational analyses indicate high predictive value for all three models. We note, however, that the simple correlation between log (static moment) and log (force), that is weight, also yield a very high correlation, .972. Because the static moment is a function of the mass of the rod, an equally good model will result if static moment is considered in place of the physical weight. It is clear that as the mechanical variables involved are strongly correlated based on the inherent nature of their definitions, within either the current experimental conditions or those of Carello et al. (1992), one cannot conclusively assess the relative efficacy of the SM–T, F–T, and weight–percept models.

**Experiment 2: Pitting the F–T and Weight–Percept Models Against The SM–T Model**

Carello et al. (1992) conclude that in a static grasp, "the primary mechanical invariant . . . is the first (or static) moment" (p. 301) and that perceiving rod length "is tied to the deformation consequences of the first moment even when contact with the rod is restricted to one narrow anatomical location, wherever it might be" (p. 301). An elementary mechanical analysis indicates that static moment must be mediated by forces and torques in producing the tissue deformations their model assumes, and that the effects of varying static moment (yet maintaining constant mass) must vanish when a rod is held vertically. However, Carello et al. argue that the static moment is directly perceived, in some unspecified way: the haptic perceptual instrument is said to "read" each first moment under conditions of holding" (p. 301). Thus, we can pit the SM–T model against the F–T model by varying static moment while eliminating torque, a parameter common to both models.

We used a custom-built apparatus that maintains rods in a vertical position, with no micromotion or nonvertical tremor. As previously explained, when a rod is grasped vertically (i.e., 90° perpendicular to the ground plane), the torque is 0. In this orientation, a weight fixed at different points along a statically grasped rod will alter the static moment without varying rod weight. In Experiment 2, a weight was added close to the grasp end (proximal-weight condition) versus at the far end (distal-weight condition) of the rod. The static moment is, therefore, larger in the latter than in the former condition, whereas the force (weight) of a given rod is unchanged.

Carello et al.’s (1992) SM–T model predicts that for each physical length, individuals should perceive the rod to be longer in the distal-weight condition. In contrast, our F–T and weight–percept models (to the extent that the gravitational forces and torques strongly determine perceived weight) predict that perceived length will remain the same for both proximal-weight and distal-weight conditions.

**Method**

**Participants.** Fifteen 1st-year undergraduates (5 men; 10 women) volunteered from the Introductory Psychology subject pool for credit. Their ages ranged from 18 to 27 years. All participants wrote with their right hand.

**Apparatus and stimuli.** A set of 4 rod lengths, each with its own additional weight, was machined from .25 in. (.635 cm) diameter of hardened steel. The lengths were selected to sample an even greater range than used in Experiment 1. The additional steel weights were selected so as to reproduce the same weight range, and thus, the same weight ratio (1:4.3) as used in Experiment 1, where we first concluded that weight played an important role in the length perception of statically held rods. In the distal-weight condition, the distal end of the weight was lined up with the far edge of the rod; in the proximal condition, the weight was positioned as close to the grip edge as possible, allowing 1 in. (2.54 cm) for the width of the finger grip. The corresponding ratio of largest to smallest static moments was 30:1, thus, potentially biasing the results toward the SM–T model. The length, mass, and static moments for each rod in the proximal- and distal-weight conditions are presented in Table 2.

A metal apparatus (Figure 6) was constructed to force participants to hold the rods in the true vertical position with respect to
the ground plane, thus, eliminating torque. It consisted of a vertical support stand, to which was attached a square frame that could be rotated about an axis which was parallel to the ground. A circular hole was drilled through the side of the vertical frame further from the support stand; the diameter of the hole was equal to that of the stimulus rods. Linear antifriction ball bearings were inserted into the side walls of the opening. With this arrangement, the experimenter could rotate the square frame on each trial to permit her to insert each stimulus rod easily through the opening. She then locked the frame in place on a vertical position, so that the stimulus rod could, at that point, only be moved in a vertical direction. An additional horizontal guide arm was attached to the top of the vertical stand, above the frame. The guide arm could be rotated temporarily into place above the frame, after the latter was locked into a vertical position, so as to align the near end of the stimulus rod correctly, and to guide the participants’ hand to the proper grip location.

In Experiment 2 participants grasped and maintained the rod in a vertical pendulum position at the top, rather than in the inverted-pendulum position used in Experiment 1, because the former position was easier for the participant to achieve: any torques are equal in magnitude to those of the inverted pendulum but opposite in sign. This sign reversal is of little consequence for testing the validity of the current hypothesis; more to the point, the torques are near zero when the rod is held in a vertical position (see our mechanical analysis of the hand-rod system gripping an object at one end).

Procedure. Participants were blindfolded and wore head-phones through which they listened to white masking noise. They were instructed to judge the lengths of a set of rods as follows. Their right forearm and wrist rested on a counter top, with their hand projecting beyond. At the start of each trial, participants positioned the top side of their hand up against the horizontal guide arm, moving along the horizontal guide arm until the hand contacted the proximal end of the stimulus rod, which was also positioned by the experimenter up against the guide arm. Each rod was grasped with the thumb in opposition to the index and middle fingers. On a signal the experimenter released her own grip and swung the sidearm away while participants supported the full weight of the rod. They were instructed not to make any movements, and to hold the rod in its original position without slip. Participants were not told about the additional weight, nor could they feel it in either proximal or distal condition.

Participants were initially given practice on hand positioning and forming the proper rod grip. They were subsequently given a block of trials to practice making magnitude estimations of rod length (as in Experiment 1). Finally, they performed three formal replications of the set of eight randomly presented stimulus conditions (four rods, two weight positions). A brief break was given after the length judgments. Participants then estimated the rod weight of three additional blocks as they had done previously for length estimations.

### Table 2

<table>
<thead>
<tr>
<th>Rod length (cm)</th>
<th>Total rod mass (including added mass)</th>
<th>Total static moment (proximal)</th>
<th>Total static moment (distal)</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.5</td>
<td>136.44</td>
<td>522.27</td>
<td>1,100.24</td>
<td>0</td>
</tr>
<tr>
<td>30.5</td>
<td>214.11</td>
<td>675.51</td>
<td>1,956.77</td>
<td>0</td>
</tr>
<tr>
<td>61.0</td>
<td>348.57</td>
<td>2,084.19</td>
<td>6,327.59</td>
<td>0</td>
</tr>
<tr>
<td>91.5</td>
<td>539.14</td>
<td>4,515.60</td>
<td>14,979.16</td>
<td>0</td>
</tr>
</tbody>
</table>

### Experimental design.

A completely within-subject design was used. Participants estimated both the lengths and weights of four stimulus lengths, each presented with a weight added to the rod at the proximal or distal end. For each set of judgments, these eight conditions were repeated in three separate blocks, with stimuli randomized across the full set of 24 trials.

### Results

#### Length estimates.

A three-factor, within-subjects ANOVA was performed on the log normalized length estimates, the factors being rod length (4 levels), weight position (2 levels), and replications (3 levels). Supporting our selection of perceptually different lengths, the main effect of rod length was highly statistically significant—estimated rod length increased with increasing rod length, $F(3, 42) = 233.45, p = .000$. Most important to the current experiment, there was no effect of weight position, $F(1, 14) = 77.86, p = .40$. The two-way interaction between length and position only approached significance, $F(3, 42) = 2.93, p = .05$, and was not consistently in the predicted direction: near length estimates were actually larger than far length esti-
mates for the two shortest rods (i.e., by .02 and .06 log units, respectively); they were shorter by .02 log units for both of the longer rods. Moreover, regardless of position, the effect accounted for a negligible fraction of 1% of the variance attributable to treatment effects. None of the other effects reached statistical significance (i.e., all ps > .300).

Physical-length ratios versus perceived-length ratios of the six possible pairs drawn from the stimulus set described in Table 3 were 1.0 versus 1.37, 2.0 versus 2.32, 2.0 versus 1.70, 3.0 versus 3.33, 3.0 versus 2.44, and 1.5 versus 1.43, indicating some variability in the pattern of results. Ratios of the perceived estimates reflected both over- and under-estimation of the physical length ratios.

**Weight estimates.** A similar three-factor within-subjects ANOVA was performed on the logarithmically transformed normalized weight estimates. A main effect of rod length was obtained, $F(3, 42) = 137.31$, $p = 0.00$. Weight estimates also increased with increasing length. None of the other main effects of their interactions was statistically significant.

Corresponding physical-weight ratios versus perceived-weight ratios for the six possible pairs discussed in the length-estimate analysis above were 1.57 versus 1.46, 2.56 versus 2.41, 1.63 versus 1.64, 3.95 versus 3.50, 2.52 versus 2.39, and 1.55 versus 1.45, reflecting a tendency to underestimate physical-weight ratios. The reason for the differences in the pattern of results obtained in Experiments 1 and 2 are not immediately obvious, but do not affect our predictions or conclusions.

**Correlational analyses.** Simple correlations between variables (log values) of interest from Experiment 1 are presented in Table 3. Correlations with torque are not shown as torque was eliminated in this experiment using the vertical holding position. The only difference, therefore, between the ecological SM–T and psychophysical F–T models lies in the role of the highly correlated static moment versus force. Under the current testing conditions, the obtained results of experimentally decoupling these two variables clearly favor the F–T model. Whereas this conclusion is based on accepting the null hypothesis, it seems appropriate for the following reason. Participants in Carello et al.'s (1992) Experiment 5 were able to use static moment to judge the length of statically held rods when that variable varied for only a 2-fold range (we argue that they could not use differences in force or perceived weight, because the actual forces were very similar: 62, 63, and 98 g). In the current experiment, we varied static moment over a 30-fold range, and force over a 4.3-fold range. Thus, participants in this experiment were actually biased to use static moments; yet, they behaved as predicted by the F–T model. However, because the length estimates correlated almost perfectly with the perceived-weight estimates, which in turn correlated as highly with force, the data further support the weight–percept model.

**Experiment 3: Pitting the F–T Model Against the Weight–Percept Model**

The results of Experiment 2 allow us to conclude that F–T and weight–percept models better account for the length estimates than does the ecological SM–T model. However, they do not allow us to differentiate between their relative effectiveness. Either one accounts for almost all of the variance in the length estimates.

The current experiment attempts to pit these two models against each other, by inducing a change in perceived weight without a concomitant alteration in the gravitational force and torque parameters. The technique we used involved inducing a static version of the haptic size–weight illusion in the participant (Ellis & Lederman, 1993). More specifically, the participant was asked to statically grasp rods whose diameters (size) vary, whereas the physical length and weight of such rods remain unchanged. The weight–percept model predicted that participants should estimate the narrower diameter rod to weigh more than the larger diameter rod, and consequently, should further estimate its length to be longer. The F–T model predicted no such differences as the force and torque at the grip remain unchanged.

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experiment 2: A Simple Correlation Coefficient Matrix of Relevant Physical and Perceptual Variables</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>1. Log perceived length</td>
</tr>
<tr>
<td>2. Log length</td>
</tr>
<tr>
<td>3. Log force (weight)</td>
</tr>
<tr>
<td>4. Log static moment</td>
</tr>
<tr>
<td>5. Log perceived weight</td>
</tr>
</tbody>
</table>
Method

Participants. A total of 28 participants took part. They were drawn from the subject pool used in Experiment 2. One group made rod length estimates and the other made only weight estimates.

Stimulus rods. Three pairs of rods were machined, with pair lengths of 8.0 in. (20.3 cm) 11.0 in. (27.9 cm), and 14.0 in. (35.6 cm). We could not select lengths from the range used in previous experiments, because participants would have been unable to statistically support their weights comfortably. The outer diameters of the small and large rod of each pair were 11/16 in. (1.8 cm) and 3.0 in. (7.6 cm), respectively. The narrow diameter rods were made of solid, cold-finished steel. The large diameter rods were aluminum cylindrical tubes, with a thickness that produced a rod mass that was equivalent to the corresponding narrow-diameter rod mass (and therefore, weights). The masses for each pair of rod lengths were 384, 525, and 666 g, respectively. On the basis of informal pilot work, we ascertained that the aluminum and steel rods were not differentiable in terms of their perceived thermal characteristics.

Procedure. The general procedure was similar to that used in Experiment 2, with the exception of the holding technique (see Figure 7). Once again, blindfolded participants rested their right forearm and wrist on the countertop at their side, with the hand extending forward beyond the front edge. The hand was pronated, with the thumb rotated and abducted. The experimenter placed one end of the rod within the palm, so that the thumb could grasp the rod end along the bottom. The four fingers were extended and held tightly together; they were offset from the edge of the rod by the distance of the thumb, and wrapped around the top of the rod. The experimenter ascertained that exactly the same grip was used for all rods, to maintain the same physical mechanical parameters. An average grip width of 2.5 in. (6.4 cm) was assumed in calculating the mechanical variables of interest. The rod was presented and held in a steady, horizontal position relative to the ground plane throughout the trial. Participants initially practice producing the proper grip on a wooden dowel with a diameter of 1.4 in. (3.5 cm).

As before, an absolute magnitude estimation procedure was used to judge rod length or rod weight. Participants were also given a block of practice trials to learn how to make magnitude estimates. The experiment lasted about 15 min.

Experimental design. Participants in one group judged the lengths of four blocks (replications) of six rods (three lengths, two diameters), for a total of 24 trials. The stimulus order was randomized across the full set of trials. Participants in the second group judged the weight of the rods in the same 24 conditions.

Results

Weight estimates. To test our prediction concerning perceived length, it was first necessary to confirm that a haptic size-weight effect had been induced in the static hefting task. We performed a within-subject, three-factor (rod length, rod diameter, and replications) ANOVA on the log weight estimates after normalization across participants. Not surprisingly, a highly significant main effect for length was obtained, $F(2, 26) = 62.79, p = 0.0000$. Of specific interest is the effect of rod diameter, which was highly statistically significant, $F(1, 13) = 23.07, p = 0.0000$. Participants judged the narrower diameter rod to weigh more than the wider diameter rod, there being a 31% increase in weight estimates for a 4.4-fold decrease in rod diameter. None of the other main effects or interactions reached significance.

The physical-weight ratios versus perceived-weight ratios for 11.0-in. versus 8.0-in., 14.0-in. versus 11.0-in., and 14.0-in. versus 8.0-in. pairs were as follows: 1.375 versus 1.69, 1.27 versus 1.45, and 1.75 versus 2.45, respectively. As in Experiment 1, participants' estimates reflected a consistent overestimation of the actual weight ratios.

Length estimates. An ANOVA was next performed on the log normalized magnitude estimates for length, using the same three-factor (rod length, rod diameter, replications), within-subjects design. The main effect for length, as expected, was highly statistically significant, $F(2, 26) = 103.75, p = 0.0000$. In addition, the effect of rod diameter was also highly significant, $F(1, 13) = 39.72, p = 0.0000$. Participants judged the narrower rod to be longer than the wider rod, with a 26% increase in length estimates for a 4.4-fold decrease in rod diameter. As in the weight estimate analysis, no other main effects or interactions were statistically significant.

The physical-length ratios versus perceived-length ratios for 11.0-in. versus 8.0-in., 14.0-in. versus 11.0-in., and 14.0-in. versus 8.0-in. pairs were, respectively, 1.37 versus 2.03, 1.27 versus 1.51, and 1.73 versus 3.06. Participants' magnitude estimates of length consistently reflected an overestimation of the actual length ratios, as previously found in Experiment 1.

Correlational analyses. A simple-correlation matrix is shown in Table 4. All physical variables were initially logarithmically transformed, as in previous analyses. The correlation between log normalized perceived length and log normalized perceived weight estimates was slightly higher (.984) than with force, torque, and static moment, which were .940, .945, and .945, respectively.

The multiple regression obtained for the linear F-T
Table 4

<table>
<thead>
<tr>
<th></th>
<th>Log perceived length</th>
<th>Log length</th>
<th>Log force (weight)</th>
<th>Log torque</th>
<th>Log static moment</th>
<th>Log perceived weight</th>
<th>Log diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log perceived length</td>
<td>-</td>
<td>.948</td>
<td>.941</td>
<td>.945</td>
<td>.945</td>
<td>.984</td>
<td>-.293</td>
</tr>
<tr>
<td>Log length</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>.944</td>
<td>.000</td>
</tr>
<tr>
<td>Log force (weight)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>1.000</td>
<td>.937</td>
<td>.026</td>
</tr>
<tr>
<td>Log torque</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>.941</td>
<td>.012</td>
</tr>
<tr>
<td>Log static moment</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.942</td>
<td>.011</td>
</tr>
<tr>
<td>Log perceived weight</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Log diameter</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

model, with an $R^2$ of .927, was

$$\log_{10} \text{perceived length} = -15.01 \log_{10} \text{force} + 8.1 \log_{10} \text{torque} + 14.0. \quad (7)$$

To further assess the extent to which the weight-percept model was instantiated by perceived weight being determined by the forces and torques, we regressed these mechanical variables against the weight estimates. The regression for the weight-percept model, with an $R^2$ value of 0.922, was

$$\log_{10} \text{perceived weight} = -19.5 \log_{10} \text{force} + 10.5 \log_{10} \text{torque} + 18.0. \quad (8)$$

Clearly, the F–T model did not account for all of the variability in the weight estimates in this experiment. The additional contribution of rod diameter was assessed by adding it as a third predictor variable in a separate multiple regression analysis. The $R^2$ value of this three-factor model of weight perception increased from .922 to .970.

Discussion

Experiment 3 attempted to assess the role of a perceptual variable, the weight percept, over and above the forces and torques in judging rod length. The perceived weight of pairs of rods that varied only in diameter was altered by inducing a static version of the haptic size–weight illusion: The narrower rod was perceived to be heavier than a wider rod of equal length, weight, and torque. The participants' estimates of length are almost perfectly correlated with the weight estimates. Furthermore, given the current experimental conditions, perceived weight makes a residual contribution to the estimation of the length of statically held rods, over and above those of the forces and torques proposed in the F–T model. We conclude that the weight-percept model best explains the estimation of the length of statically held rods.

General Discussion

In keeping with Gibson's ecological theory of perception, Turvey, Carello and their colleagues have argued that perception is specific to information, which in turn is specific to the environment. According to this theoretical perspec-

tive, information corresponds to properties of the structured energy distribution that remain invariant with exploration. Perception results if and only if a particular information structure is detected, thus, suggesting a unique mapping from perception to invariant. The approach has come to be called the direct theory of perception, because its advocates claim that sensory systems directly extract higher order information about the permanent properties of objects and layouts that is available in the flux of stimulation received by an active perceiver from the world.

In support of the direct perception theory, the Connecticut group has published the results of an extensive research program focusing on how the haptic system processes the length of dynamically and statically held rods. Studies of dynamic wielding (e.g., Solomon & Turvey, 1988) argued that perceived reachable distance is a function of the rod's resistance to rotational acceleration (i.e., the moment of inertia of the hand–rod system about the axis of rotation). Studies of static holding have argued for the critical role of the static moment (e.g., Burton & Turvey, 1990), and more recently, for a linear model of perceived rod extent that includes both the static moment and torque at the wrist (Carello et al., 1992). This last set of studies has guided the focus of our article.

The ecological SM–T model proposed by Carello et al. (1992) predicts length estimates remarkably well. However, the study does not clearly demonstrate that perception maps uniquely to static moment or torque (about the wrist), which are themselves highly correlated. As demonstrated here by the formal mechanical analysis, there are also other candidate parameters, such as force. But rod weight could not have served as a determinant of rod length in the Carello et al. study because the stimulus weights were very similar. We also proposed perceived weight as a candidate, although the ecological theory explicitly denies any role for intervening perceptual variables.

The various ecological models proposed and the F–T model share a common approach to the problem of haptic length perception in that they all treat an object that is wielded or statically held in the hand as a physical system that can be subjected to mechanical analysis. Furthermore, they both assume that our sensory systems respond to the stimulus information present in the environment without being mediated by intermediate perceptions pertaining to other independent variables. The mechanical parameters chosen by the ecological models are derived from the theory
of invariants, which emphasizes motion-independent variables (e.g., static moment). However, as we mentioned earlier, the status of torque in an ecological interpretation needs to be clarified: Is it an invariant, and if so, over what specific transformations? If it is not an invariant, then it does not seem to satisfy one of the essential tenets of the theory. The parameters in our psychophysical F-T model are guided by the formal static analysis of the hand–object system.

We began our work by assuming that the only available sources of information that can be sensed in this situation are the gravitational forces and torques. A mechanical analysis of the gripper-object system was presented for conditions in which objects of different materials are held stationary at different orientations with respect to the ground plane. The analysis was rendered general so that it could be applied to studies of haptic length perception by both humans and machines, as we discuss in the final section.

The results of the mechanical analysis of the forces and torques were used to propose two models of length perception in addition to the SM-T model. The psychophysical F–T model uses the gravitational forces and torques acting on the hand to predict the length estimates. The weight–percept model proposes that perceived length maps to perceived weight. Carello et al.'s (1992) model argues for a strong contribution of static moment and a weaker contribution from the torque produced in the hand.

Experiment 1 was similar to Experiment 5 in the study by Carello et al. (1992), inasmuch as participants were required to judge rod lengths by statically holding each rod at three different orientations. In addition, we constructed each rod length out of three different materials. Thus, unlike the Carello et al. experiment, the rods varied quite considerably in mass, which in turn affects the force (weight), first moments of mass, torques, and so on. Under the current experimental conditions, torque accounted for 46% of the variance in the perceived length estimates in contrast to about 25% in Carello et al.'s Experiment 5. Moreover, all three models predicted perceived length remarkably well (R² values from the multiple regression analyses were all >.90).

In calculating the simple overall correlation between perceived length and torque, however, Carello et al. (1992) did not consider the fact that torque was unavailable in the vertical condition and hence could not contribute to the perception of length in this orientation condition. To assess the relative contributions of the proposed predictor variables for conditions where changing torque was or was not available, we therefore performed two additional multiple regressions. One was based on the aggregated data from both horizontal and 45° slant conditions, and the other was based on the data from the vertical condition. As expected, in the vertical condition, correlations of the relevant perceptual estimates with weight were a little higher than when all three orientations were included. In the aggregated horizontal and slanted orientation conditions, both percepts were considerably more strongly correlated with torque than in the corresponding three-orientation analyses, and slightly more correlated with force.

These results seem to suggest different relative contributions by the candidate parameters, the forces, and the torques, depending on the orientation of the rod. But we did not find this surprising. In certain situations, as in the case where the rod is held perpendicular to the ground or gripped at its CM, torque is unavailable; in certain other situations, as in the case where the same rod is held at different orientations with respect to the ground plane, the weight may remain constant whereas the torque varies. We propose, therefore, that the haptic system will capitalize on whatever information is available (e.g., by selectively attending to weight cues when torque is unavailable, or by emphasizing the effect of torque when weight remains constant). Thus, it is hypothesized that the force (rod weight) and the torque applied at the grip will both contribute to the perception of the length of unseen objects, their strengths depending on the extent to which each source of information is available in a given circumstance. Furthermore, we emphasize that in this approach, no unique invariant is considered to be the governing factor affecting the perception of length; rather, we suggest that perceived length is systematically affected by the combined effect produced by both forces and torques.

At this stage in our investigation, the F–T model applies to symmetric rods statically held within a gravitational field; the force is equivalent to the weight of the object and the torque is the twisting force about the Ixx axis. We are currently attempting to extend our mechanical analysis and empirical tests to other axes and to provide a unified F–T theory of the perceived extent of rods, whether statically held or dynamically wielded.

Because the mechanical variables and the perceived weight estimates that were proposed as predictor variables were so highly intercorrelated, Experiments 2 and 3 were designed to experimentally unconfound their individual contributions. Experiment 2 deliberately pitted the SM–T model against both the F–T and weight–percept models, which themselves could not be further dissociated using this paradigm. The results strongly favored predictions made by the latter two models. A 30-fold change in static moment did not produce any effect on estimated length; as predicted, under conditions of constant gravitational force (weight), perceived length remained unchanged. In keeping with the weight–percept model, the perceived weight remained unchanged as well.

Because the ecological approach advocates the directness of perception with no need to postulate intervening interpretive processes, the possible contribution of additional perceptual factors (e.g., estimated rod weight) has never been formally considered. Neither Experiment 1 nor Experiment 2 could effectively evaluate an independent contribution of the weight percept to the perceived extent of statically supported rods, although there were some hints. For example, in Experiment 1 the r² value for the simple correlation between perceived weight and perceived length was higher (.983) than the R² value for the F–T model of perceived length (.924). Clearly, the forces and torques can account for most of the variance in the perceived weight estimates; but, there was a small residual variation (6%) in
perceived weight that could not be explained in this way. In Experiment 2, in accord with the F–T model, the simple correlation between perceived length and actual weight was very high (.991; as torque is zero, this variable was not included), whereas the correlation between perceived length and perceived weight was marginally higher (.995). Ceiling effects prevent us from differentiating further among these two values. As actual and perceived weight were themselves almost perfectly correlated (.999), perceived weight could just as well have accounted for all of the variance in the perceived length estimates.

The direct assessment of perceived weight was achieved in Experiment 3 by successfully inducing a static version of the haptic size–weight illusion, which has previously been documented only for conditions of dynamic hefting (e.g., Ellis & Lederman, 1993). This technique allowed us to maintain the values of the forces and torques constant, and altering the perceived weight of rods of equal length and different diameters. As predicted, the manipulation produced a corresponding change in the length estimates: the narrow rod felt both heavier and longer than the correspondingly wider one.

This final result suggests that the weight–percept model does the most effective job of predicting the perceived length of statically grasped rods. The current data further support the possibility that participants may be using a two-stage process to instantiate the weight–percept model. Stage 1 involves the estimation of rod weight, on the basis of primarily, although not exclusively, the gravitational forces and torques at the grip. Selection of these predictor variables was guided by the results of our static equilibrium analysis. Stage 2 involves mapping perceived length to perceived weight.

Is perceived weight used because of a generally privileged relation to length perception by static holding? Alternatively, has it merely been selected from a number of percepts, any one of which could have served equally well as a perceptual measurement tool in these experiments? We would suggest the former explanation has greater ecological validity inasmuch as the weight and size of solid objects tend to strongly covary in nature. Whereas there are other percepts one might consider (perceived orientation, shape), these were not mentioned in participant reports obtained by Hoisington (1920) and ourselves.

In his review of this article, Turvey suggested that our results are theoretically ambiguous. Turvey proposed that participants may have made small uncontrollable movements of the rod about both the longitudinal axis of the rod and about the axis parallel to the ground plane. To this extent, additional information about rod length is provided by the resistances offered to such rotations, known as moments of inertia ($I_{zz}$ and $I_{xx}$ respectively, e.g., Pagano & Turvey, 1993). As applied to Experiment 2, Turvey argues that as the value of $I_{zz}$ was constant, then perceived length would be expected to remain unchanged as well. Here, we would point out that roboticists typically ignore $I_{xx}$ in their calculations when the rod diameter is very small, as is the case here. Moreover, the much larger and changing $I_{xx}$ cannot be important either inasmuch as it covaries perfectly with static moment, which our study has shown does not influence length judgments. With respect to Experiment 3, Turvey argues further that decreases in $I_{zz}$ with decreasing diameter also predict increasing length estimates, without the need to postulate the weight–percept model. We are planning to resolve this ambiguity with additional research.

**Haptic Weight Perception**

The current findings also permit us to draw conclusions concerning the major determinants of haptic weight perception by static holding. In Experiment 1, perceived weight estimates were predicted extremely well by the combined sensed forces and torques; however, when considered across all orientation conditions, they were also predicted equally well by the static moment. However, the first moment of mass distribution cannot account for the obtained rod orientation effects, because its values remain invariant as a function of orientation. The results of Experiment 2 further experimentally confirm that participants were not using the static moment, despite large variation in the value of these variables. In contrast, the weight estimates were virtually perfectly predicted by rod weight (as torque was zero). Experiment 3 demonstrates that the participants' weight estimates can be further altered without changing the gravitational forces and torques, by inducing a size–weight illusion: objects of different diameters, but the same length, weight, and torque are perceived to vary in weight—the narrower one appears to be lighter than the wider one.

**Application of the Mechanical Analysis to Robotic Perception of Rod Lengths by Static Holding**

Questions posed concerning human perception typically address the inverse problem, namely, what environmental properties determine or contribute to a given perception? In contrast, questions posed concerning machine perception commonly address the forward problem, namely how can a machine perceive a particular environmental property? Whereas the transduction techniques are strikingly different for the two systems, the environmental parameters acting on those systems are in fact the same. In other words, both biological and machine systems are affected by the same environmental properties, and are subject to the same physical laws about the external world.

We believe the current task also poses interesting questions for a robot that must manipulate objects in the environment, whether directly or through the use of a tool. Perceiving the length may be important in situations where the object is manipulated in space-constrained environments: if the dimensions of the object are known, then the object can be manipulated (reoriented, translated, etc.) without causing damage to other objects in the immediate workspace or to the robot itself. Perceiving length may also be required in situations where it is important to know the location at which an object is grasped, or in tasks that require further manipulation on the basis of knowing the...
length (e.g., automated manufacturing tasks such as sorting rods into different piles according to length).

Therefore, we extended the mechanical analysis here to show that it is possible to derive both the lengths and the masses of a statically gripped rod by solving two simultaneous equations. On the basis of this analysis, a new tactile sensor was designed and tested with highly promising results (Ellis et al., 1994; Ganeshan et al., 1994). To date, rod mass can be estimated to within 2% accuracy. Rod length can be estimated to within 6% accuracy, but by downsizing the sensor's strain gauges, we expect that measurement error can be further reduced to about 1%.

What links these seemingly diverse fields, experimental psychology and robotics, is the general goal of having a human operator control the activities of a robot in a remote workspace, that is, telerobotics (or more generally, teleoperation). Rods may be viewed as one of the simplest tools a human operator may be asked to control remotely. The results from these two parallel projects further suggest that the highly accurate forces and torques sensed by the robot may need to be appropriately altered when delivered as inputs to the operator's hand, to counteract the biases documented in human length perception. Otherwise, the operator may not achieve length constancy under such circumstances: He or she may perceive the rod length to change when the robot gripper alternates between tools of different materials, and when it alters tool orientation.

References


(Appendix follows on next page)
Appendix

Definitions

Equilibrium conditions: For any system to be in equilibrium, there should be a balance of all the forces acting on the system. On the basis of the kinds of forces that are acting on the system, there could be several equilibrium conditions. However, the most common equilibrium conditions that are encountered are force equilibrium, \( \sum F = 0 \); and torque equilibrium, \( \sum T = 0 \).

Force: Any action that alters or tends to alter a body's state of rest or uniform motion in a straight line.

Free-body diagram: A diagram depicting all the forces and the torques acting on the body under consideration, along with the reactions provided by the supports, is called a free-body diagram or a force diagram.

Mass: The property that measures inertia. In the SI system, it is measured in grams, kilograms, and so forth.

Moment of inertia: Also known as the second moment of mass distribution, and is often defined as the resistance of a body to angular acceleration. It is given by \( \int m r^2 \, dm \). The concept of a moment of inertia can be thought of as the sum of the product of the mass of each particle in a rigid body and the square of its distance from the axis.

Static analysis: Solves the relationships among the internal forces and the torques of a mechanical system, in the presence of applied forces and torques. Usually, the bodies in the system are rigid. The analysis is also called equilibrium analysis, or static-equilibrium analysis.

Static moment: Also called the first moment of the mass distribution, for a continuous body it is \( \int r \, dm \). For a rigid body of mass \( M \), the static moment at a distance \( r \) from the center of mass is just \( Mr \).

Torque: If the line of action of a force does not coincide with the axis under consideration, it produces a twisting effect about the axis. This twisting effect is called torque. Its magnitude is given by the cross product of the force and the perpendicular distance of its line of action from the given axis.

Weight: The force exerted on matter by the gravitational pull of the earth. In the case of a body of mass \( M \), the weight is equal to \( Mg \), where \( g \) is the acceleration of free fall. Weight is measured, in the SI system, in newtons.

Received September 1, 1993
Revision received April 3, 1995
Accepted May 22, 1995

Call for Nominations

The Publications and Communications Board has opened nominations for the editorship of Developmental Psychology for the years 1999–2004. Carolyn Zahn-Waxler, PhD, is the incumbent editor.

Candidates should be members of APA and should be available to start receiving manuscripts in early 1998 to prepare for issues published in 1999. Please note that the P&C Board encourages participation by members of underrepresented groups in the publication process and would particularly welcome such nominees. Self nominations are also encouraged.

To nominate candidates, prepare a statement of one page or less in support of each candidate and send to

Janet Shibley Hyde, PhD, Search Committee Chair
c/o Lee Cron, P&C Board Search Liaison
American Psychological Association
750 First Street, NE, Room 2004
Washington, DC 20002-4242

Members of the search committee are Bennett Bertenthal, PhD; Susan Crockenberg, PhD; Margaret Spencer, PhD; and Esther Thelen, PhD.

First review of nominations will begin December 9, 1996.