A mathematics course for prospective elementary school teachers.

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Abstract

It is possible to get prospective elementary school teachers to learn interesting mathematics well by not only requiring that they learn the material in a mathematics course but also requiring that they teach it as part of an enrichment program at the middle school level. This article discusses the reasons for such a course, and describes a university mathematics course that combines these elements.

Key words: Preparation of elementary teachers, mathematics enrichment, service learning.

Teaching and Learning Mathematics

One of the paradoxes in the mathematics preparation of elementary school teachers is that, though it seems obvious that knowing more mathematics should lead to better teaching, it has been difficult to find empirical evidence to support this belief. In [1] Ball summarizes attempts made through the second half of the 20th century to establish a connection between the learning of elementary school students and characteristics of their teachers. Studies conducted before the mid-1970’s, reviewed in Begle and Geeslin [6] and in
Begle [5], found that no single teacher characteristic proved to be “consistently and significantly correlated with student achievement.” In particular, it seemed that having taken more university mathematics courses was of no benefit. Even for high school teachers the picture is far from simple. In a study published in 1994, Monk [16] concludes that, for the teaching of secondary school mathematics and science, it appears that “gross measures of teacher preparation (such as degree levels, undifferentiated credit counts, or years of teacher experience) offer little useful information for those interested in improving pupil performance”.

Clearly then, simply taking more mathematics courses is not going to help prepare elementary teachers teach better mathematics classes. Yet, it cannot be true that understanding mathematics better is of no benefit to elementary school teachers. One feels that the difficulty finding a strong correlation between mathematics courses taken by teachers and subsequent performance of their students has to do with the kinds of courses taken and the depth at which the course material is understood rather than the number of courses.

In the 1980’s education researchers began to study teachers’ mathematical understanding as it exhibited itself in the context of their classrooms. They hoped that by focusing on teacher practice in the rough and tumble of teaching situations they might discover significant effects of teacher preparation and characteristics on meaningful student outcomes. Lee Shulman [18, 19], wishing to describe the kind of teacher knowledge that leads to good teaching, introduced the notion of “pedagogical content knowledge”. This is knowledge of the subject as it presents itself in the complexity of the classroom. Ball, Lampert and others [1, 3, 2, 13, 12, 10, 7, 8] developed this further and conducted extensive classroom studies to detect and describe situations where subject matter knowledge plays a critical role in the classroom. Their intention was to build a repository of videotaped and transcribed material that could be used to focus teacher education courses as deliberately as possible on the kind of knowledge of mathematics and pedagogy that really matters in the elementary classroom.

Several international studies [21, 15, 4, 14], inspired by the relatively poor performance of U.S. students on TIMSS, confirm the importance of mathematical subject knowledge focused on classroom situations as well as the benefits of a culture of collaborative professional development. By and large, these studies are less focused on the psychology of learning, and more on the depth of teachers’ understanding of mathematics and their familiarity
with strategies for presenting ideas in the classroom. In her widely read study [15, Chapter 5], Ma speaks of “profound understanding of fundamental mathematics.” Her comparison of American elementary school teachers with Chinese counterparts, who have had less formal education than their American colleagues, makes it clear once again that adding more mathematics courses to the requirements of teacher education does not help unless the courses themselves are better tuned to the practicalities faced in the elementary classroom.

Kennedy, Ball, and McDiarmid, in [11], developed instruments for measuring the kind of subject knowledge that matters in the classroom. A recent study reported by Rowan, Chiang, and Miller [17] found some correlation between teachers’ responses to items designed to measure teachers’ mathematical knowledge as displayed in the context of their teaching, and the performance of their students. It seems that we are beginning to identify the kind of knowledge we should be aiming for when we design our mathematics courses for pre-service teachers. A key goal is deep understanding of basic mathematics rather than surface understanding of a large amount of advanced material; and, as much as possible, this understanding of basic mathematics should be qualified and honed by use in the classroom.

Creating courses for prospective elementary teachers in a department of mathematics

When we try to create a university program that takes these insights seriously we find ourselves facing a number of challenges. In the first place there is the challenge of creating courses that are at a level appropriate for the students who should be taking them, and yet recognized by the institution as university level courses. In the second place, these courses should be designed to ensure that the material is understood by the students at a deeper level than would be the case if they took a more traditional mathematics course. Thirdly, the material should be presented as much as possible in a form that connects to the ways in which the subject comes up in the elementary classroom. Fourthly, the courses should be such that they motivate and engage students who have come to fear mathematics and mistrust their own abilities to understand it at all. Finally, the courses should involve opportunities and requirements for communicating understanding of mathematics.

In many North American post-secondary institutions there is a separa-
tion (unfortunate in view of the findings reported above) between the mathematics courses and the pedagogy courses taken by pre-service teachers. The former are taught in a mathematics department by research mathematicians, while the latter are taught in a school of education. In some cases, the mathematics courses are taken at a university that does not have an education program; students at these schools have to apply for study elsewhere upon their graduation to obtain their teaching qualifications.

Faculty members in a mathematics department have ambitions for their students; becoming an elementary school teacher is not usually high on that list. It is not a given that a course appropriate for pre-service elementary school teachers will be perceived by a mathematics department as a genuine mathematics course rather than a course in pedagogy. To accomplish this it will be necessary to build the course around good mathematics that is challenging and yet accessible to the average pre-service elementary school teacher. To convince colleagues that such a course has a place in the undergraduate program will require a careful explanation of its goals. Fortunately, most colleagues can be persuaded that mathematics does not have to be advanced in order for it to be interesting and worthy of study; and, when pressed, they will acknowledge that we deceive ourselves if we imagine that most of our undergraduates understand most of what we teach them in our regular mathematics courses. Often we content ourselves with the thought that the students who did not understand the material as well as we hoped will learn it better when they see it again in the next course. For intending elementary school teachers there probably will not be a next mathematics course. In a very short time they will be teaching in an environment where (in North America, at least) opportunities for sustained further learning are more limited than they should be, and where a culture of collegial support does not seem to be well developed. In other words, it is essential that the course contain good mathematics that can be learned at a deep level by all the students taking it.

Many of the students in a training program for elementary school teachers, or planning to enter one, have had little or no mathematics since high school, and have found their high school mathematics difficult. These students will approach any university mathematics course with a high degree of apprehension. In questionnaires administered prior to one of my courses, about a third of the students attest to their anxiety. Usually they point to one of the early years of high school as the point at which mathematics became a burden to them. In some cases these students will report that at
that stage their mathematics marks began to slide.

“It got to the point in high school that if I could not grasp a concept on my own there was no one who could teach it to me and that was very frustrating.”

Others will say that they continued to get good marks even though they did not understand what they were doing.

“I really dislike the fact that I feel as though I have just squeaked through math all my life rather than really understanding it. Although I have always gotten good marks in the math courses I have taken, I do not feel comfortable with the subject in any way.”

In nearly all cases these students indicate in their responses that one of the obstacles to their enjoyment of mathematics was their refusal to continue without satisfactory understanding of the ideas behind the formulas.

A second challenge, therefore, is to ensure that the course is accessible to students with a weak mathematics background and a negative attitude to the subject, and to present the course in such a way that these students will feel confident that the material can be understood. There will have to be adequate support for the struggling student, with emphasis on student engagement with the material, and avoidance of evaluation schemes that create non-productive anxiety. At the same time, the course should be such that students are not able to get away with the superficial learning habits that have helped some of them survive mathematics through high school, and put them in this predicament.

The third issue is the matter of motivation. As illustrated by the quotes, many of the students who intend to become elementary school teachers have been “turned off” mathematics. Perhaps they were not naturally inclined toward mathematics in the first place, or came across a teacher who did not have a good sense of the power and beauty of the discipline. Repeated frustration at not getting the necessary time and help to attain the level of understanding the student himself wishes for only adds to the aversion. We should not imagine that for these students a good college instructor and a well-planned course are enough to guarantee interest in the mathematics. If it is rekindled along the way we may be grateful, but there should be other elements in the course to motivate the students, elements more closely related to their professional aspirations.
A related issue is the matter of courage. To instill in the students the confidence, perhaps for the first time, that it is safe to explore mathematics, that they can do it, it is essential that the course give them the opportunity and sufficient time. The best way to achieve this is to choose mathematics that does not rely on material that was learned poorly, but instead begins with topics that the students feel secure about.

Finally, the course should have a component in which students are asked to communicate their understanding. Speaking of the ubiquity of superficial, illusory understanding, Shulman writes [20]: “if we can create conditions where they can discuss what they know with others, we significantly raise the likelihood that the problems [of superficial understanding] diminish.” This deeper level of understanding that enables communication is especially important for would-be teachers. Not only will they be required to discuss mathematics with their students, but they will be looked to as resources, and will be required to react quickly and imaginatively to learning opportunities as they present themselves.

Furthermore, if the teacher is to receive maximal benefit from participation in continued professional development activities, knowing how to talk comfortably about mathematics will be very important. Both Ma [15] and Stigler and Hiebert [21] stress the importance of a professional teaching culture that emphasizes discussion of teaching strategies that focus on subject content and presentation. To the extent that opportunities for professional development are available in North America (apparently less than in China or Japan), it is important that teachers feel as comfortable talking about the content of their mathematics courses as they do discussing other pedagogical concerns.

**A course that seems to work.**

Over the past five years I have developed a course in the Department of Mathematics and Statistics at Queen’s University that attempts to achieve these goals. The course was developed together with, and is now organized around and essential to, *StepAhead*, a mathematics enrichment program taught by the students in the course, for the benefit of grade 7 and 8 students in local schools. The university students each visit a local school, in pairs, for an hour per week over ten weeks. Most of the mathematics discussed in the university classes serves as preparation for these school visits; and as instructor, I tell the students that as much as possible I will attempt to model ways in
which the mathematics can be presented to young teenagers, and conduct
the classes as if I were teaching that group. The textbook [9] used for the
course is a two-volume enrichment manual written for the purpose, and in-
formed by many years’ personal experience teaching enrichment mathematics
to students in grades 7 and 8. The syllabus for the enrichment program is
outlined at the end of this paper.

By using mathematics that is accessible to good middle school students I
try to ensure that the course does not, at the outset, involve material that is
threatening to the university students taking it. Since the enrichment pro-
gram is fairly ambitious for students at the middle school level (I ask the
classroom teachers to identify the students who would benefit from the pro-
gram, using 25% of the class as a guideline), the mathematics is interesting
for me to teach and challenging for the pre-service teachers to learn. Fur-
thermore, the enrichment material is largely independent of the high school
curriculum, so university students with a stronger mathematics background
do not feel they are under-challenged by the course, and weaker students are
not tempted to fall back on formulaic understanding remembered from their
own high school years.

It is significant that, since the university students have to present enrich-
ment classes based on the material they are learning in class, and since many
of the students they will face are very quick, it is not possible to get away
with superficial understanding. From the first of their ten weekly enrichment
visits my students realize that they have to be ready for unexpected questions
and developments during their classroom visits. The university classroom is
significantly transformed by the fact that the students have to deliver the
material they are learning. They constantly ask themselves (and the instruc-
tor) how they will explain the mathematics to their students. Every week
we set a block of time aside to review the school visits together.

To maximize the communication component in the class, the university
students are required to prepare their lessons in pairs and to visit the schools
together. This also helps ensure that the program delivered to the schools is
of high quality, and minimizes the potential for problems caused by sending
out inexperienced teachers on their own.

The format of the course takes care of the problem of motivation. When
university students are offered the opportunity to teach a group of young
teens, they have much less trouble focusing on the course than they would
without that incentive. The format of the course also works well in providing
feedback to the students. The pressure of that weekly school visit is much
greater than looming assignment deadlines or exams could ever be. The students gain confidence as they find they can understand the material. They gain respect for the real challenges of the classroom, and for the extent to which good students are able to learn difficult material.

As an added bonus, the local schools are enormously grateful for what they consider to be an excellent enrichment program. At the end of the course I routinely write to the principals of the schools, asking them to use an enclosed questionnaire to provide feedback on the school visits. One of the questions I put is “Did the grade 7/8 students appear to enjoy the classes? I have yet to see a negative answer. The following are typical:

“The students were always eager to get to class and told me they loved the challenging/difficult math they were working on.”

“From the participants: ‘We should have this for a longer time!’”

To the question “Did the classes seem useful for their program?” one teacher who sat in on all the classes wrote:

“The classes have become an important part of our fall mathematics enrichment programming for the Challenge Program students.”

Comments about the university students are also invariably positive. One vice principal wrote: “The two students who took part in your program/class this year were nothing short of amazing.” It appears that while creating an important opportunity for the university students, we have also stumbled upon a deeply felt need.

Course content and design

Course content is constrained by two simple criteria: The material should be accessible and yet challenging to students at the grade 7/8 level; and the material should have the potential for engaging the university students. We try to avoid giving middle school students accelerated access to high school mathematics. In particular, we avoid using algebra other than at a very basic level, not only because students in grades 7 or 8 are only just beginning to learn algebra, but also because with the use of algebraic methods there
is a temptation for students to suspend reflection in favour of calculation. This applies to both the university students and the young teenagers they teach, but it is particularly important for the university students, who will remember some of the algebra they took in high school but may have a very weak understanding of the mathematics underlying the mechanics. It is fortuitous that you can tell these university students that they must find solutions that do not use algebra because the middle school students have not had the necessary techniques.

Because some of the enrichment classes are at small schools where one grade alone would not provide a sufficiently large group of students for an enrichment program, we allow schools to combine grade 7 and 8 students in one class. To ensure that these students do not run into the same material on two successive years, we have two distinct programs, used in alternate years. One program focuses on numbers and number patterns, while the other focuses on geometry.

To provide a picture of the course’s content we will describe the topics covered in a recent year when the focus was on numbers and number patterns. While the material changes somewhat from year to year, the core remains fairly constant. You will notice that there is a high degree of coherence between the concepts included in the course. As much as possible without giving up on that coherence, we center the lessons on problems, and encourage a habit of student exploration and discovery.

We began the course with a short exploration of patterned sequences, such as $1, 3, 5, 7, \ldots$, $1, 4, 9, 16, \ldots$, and $1, 3, 6, 10, \ldots$. This allowed us to explore the idea of infinity and students’ understanding of pattern rules and functions. We used cases where different pattern rules produce the same infinite sequence, to motivate and explore the role of a proof.

We then switched to a discussion of prime factors. Assuming the uniqueness of prime factorization without questioning it, we explored the role of these prime factors in giving each number its unique characteristics. As part of this discussion we reviewed the “tricks” for determining divisibility by 2, 3, 4, 5, 6, 8 and 9, focusing on the reasons these techniques work. In most cases both the method and the explanation had been seen before, but not for 3 and 9. We promised to explain these later. We continued the divisibility unit with a discussion of large primes. After drawing students’ attention to the website for the “Great Mersenne Prime Search” (www.mersenne.org/prime.htm) we discussed the largest prime known to date and challenged students to estimate the number of digits in $2^n - 1$. We also discussed the relation between
prime factors and greatest common divisors and least common multiples. In the schools these topics are typically taught by writing down the lists of factors [respectively multiples] of the two numbers and then identifying the greatest [resp. least] common entry by inspection. The connection to prime factors was new to the students.

Following this discussion we focused on rational and irrational numbers. We began by reviewing the conversion from fractions to decimal expressions. We considered why a fraction of integers always results in a terminating or repeating decimal. We studied which fractions produce terminating expressions. We discussed the relationship between the length of the repeating decimal pattern and the denominator of the fraction. We then discussed examples of numbers that are not rational. This included a proof that $\sqrt{2}$ is irrational. We included a probabilistic argument to show that (essentially in terms of the Lebesgue measures of the sets) there are more irrational numbers than rational numbers.

The unit on rational and irrational numbers was followed by a short and simple discussion of modular arithmetic. We did this partly because it was fun, and partly because it allowed us to give the promised explanation of the tricks for divisibility by 3 and 9.

At the end of the course we had some time to discuss counting techniques (essentially a very basic introduction to permutations and combinations) followed by a simple discussion of probability.

Mathematics and imagination

There is a wonderful richness in this material, even if approached in very simple form. The discussion of infinity is a case in point. It is easy to find good geometric examples to stimulate students’ imaginations. Because they are not visual, we do not easily associate numbers with imagination. Nevertheless, there were two places in the course where the idea of infinity played a particularly important role in igniting students’ imaginations.

The first appearance of the idea of infinity came in the context of the exploration of number sequences. It is one thing to think of a number sequence in terms of its pattern rule (the rule that generates the sequence); it is quite another to think of the outcome, the infinite sequence, as an object in its own right. We have found that Hilbert’s (well-known) Infinite Hotel is an excellent way to get students’ imaginations going around the idea of infinity. We started the discussion with the infinite hotel and one infinite
bus. The seats in the bus are numbered, as are the hotel rooms. To effect
orderly transfer of the bus passengers to the hotel, the hotel manager had
to decide on a single instruction to the passengers: “find the room whose
number corresponds to the number on your seat”. Once the importance of
a single rule that can be applied to all the passengers had become clear, we
changed the problem by bringing an additional single-occupant car on the
scene, followed later by additional infinite buses. In each case the single, or
at least finite, instruction came in the nature of a formula, and so constituted
a natural instance of simple algebra. The other virtue of the infinite hotel
story is that it brought out very naturally some of the non-intuitive features
of infinite cardinals. How many infinite buses will fill the hotel? Two? Five?
Infinitely many?

Infinity appeared a second time when we discussed non-terminating dec-
imals and irrational numbers. We spent at least a week exploring the rela-
tionship between (real) numbers and points on the number line. We located
points corresponding to infinite decimal expressions. We identified particular
irrational numbers, such as $\pi$.

Our discussion of the relationship between $0.999\ldots$ and 1 was especially
interesting. After several lessons on the relationship between fractions and
repeating decimals and the difference between rational and irrational num-
bers, students were asked whether they thought the equation

$$0.999\ldots = 1$$

was correct. One of the students, who had clearly paid close attention when
we discussed conversions of numbers, and who had acquired some mathemat-
ical sophistication, got up to write on the board:

$$\begin{align*}
  x &= 0.999\ldots \\
  10 \times x &= 9.999\ldots \\
  9 \times x &= 9 \\
  x &= 1
\end{align*}$$

The discussion that followed this was fascinating. Without commenting on
the solution, I asked the rest of the class to say whether the student was
right. Almost immediately there were dissenting voices, and within minutes,
the student who had written out that very convincing argument announced
that he took it back: “The numbers are not the same”. I participated freely
in the discussion, but made sure to participate by asking questions rather
than speaking definitively to the issue.
At the end of about 20 minutes, the students were each given a sheet of paper and asked to summarize and react to what they had learned over the past week about rational and irrational numbers. The results were revealing in that they showed how difficult it was for students to think of an infinite sequence as a completed entity as opposed to a process. One student wrote: “I’m still a little confused how $0.999\ldots = 1$ because $0.999\ldots$ shrinks smaller and smaller, getting infinitely close to one but should theoretically never actually get there or become one.” Another student added: “I don’t understand $\pi$ as a number either. It’s an expression.”

In mathematics teaching we make lots of demands on student imagination. Though all of these demands have a potential for giving delight, they are not all of the same order. The richest challenges to the imagination are the ones that bring together quite different ideas and invite us to see them connected. Completed infinities such as inherent in discussions of countable sets or representations of the continuum are wonderful for students who know little mathematics precisely because of the way they challenge students to extend their thinking. One student commented at the end of the class discussion “I had no idea mathematics could be so much like philosophy”. In a class of students who have been accustomed to thinking of mathematics as a large but finite set of difficult abstract facts and formulas, that is not a bad point to get to.

**Summary**

By inviting intending elementary school teachers to teach enrichment mathematics classes for students in grades 7 and 8 it is possible to create a university mathematics course that is effective for learning good mathematics, for acquiring the beginnings of pedagogical content knowledge, and for providing a mathematics enrichment programme that is appreciated by local schools.
References


Biographical sketch

Leo Jonker did his doctoral work at the University of Toronto. He is currently a professor in the Department of Mathematics and Statistics at Queen’s University, and holds a Queen’s Chair in Teaching and Learning. His mathematical interest is in the area of dynamical systems. In recent years he has been interested in the teaching of mathematics in elementary and high schools, in teacher preparation, especially for the elementary school level, and in teaching and learning in university courses.