

# The Refined Humbert Invariant for Abelian Product Surfaces with Complex Multiplication

## Abstract:

The *refined Humbert invariant* is a positive quadratic form  $q_{(A,\theta)}$  which is intrinsically attached to a principally polarized abelian surface  $(A, \theta) \in \mathcal{A}_2(K)$ , where  $K$  is an algebraically closed field, and reflects many of the geometric properties of  $(A, \theta)$ . Its values are closely related to the classical Humbert invariant which was introduced by Humbert in 1900.

In the case that  $A$  is isogenous to  $E \times E$ , where  $E$  is an elliptic curve with complex multiplication,  $q_{(A,\theta)}$  determines the equivalence class of a positive ternary integral quadratic form. The purpose of this article is to give a classification of those primitive ternary forms that can occur in this case.

The proof of this classification is constructive and be made into an algorithm which is presented here. This algorithm decides whether or not a given ternary form  $f$  can be equivalent to some refined Humbert invariant  $q_{(A,\theta)}$  and if so, constructs the abelian surface  $A$  and the principal polarization  $\theta$ .

**Keywords:** Abelian surfaces, elliptic curves with complex multiplication, Humbert invariant, integral quadratic forms.

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