Angular + Spectral distribution

$$\left(\vec{S}\cdot\hat{n}\right)_{ret} = \frac{c}{4\pi}\left(\vec{E}_a \times \vec{B}_a\right)_{ret}$$

$\vec{B} = \hat{n}\times\vec{E}$

so $\vec{E}\times\vec{B} = (\vec{E}\cdot\vec{E})\hat{n} + \cancel{(\vec{E}\cdot\hat{n})}^{0}\vec{E}$

$$= \frac{c}{4\pi}\left|\vec{E}_a\right|^2\hat{n}$$

$$= \frac{e^2}{4\pi c}\left\{\frac{1}{R^2}\left|\frac{\hat{n}\times\left[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{(1-\vec{\beta}\cdot\hat{n})^3}\right|^2\right\}_{ret}$$

$$= \frac{dE}{dA\,dt}$$ energy flux

$$\frac{dE}{dA} = \int_{t_1}^{t_2}\left(\vec{S}\cdot\hat{n}\right)_{ret} dt$$

energy/area radiated betw. abs time t_1 + t_2

in general $t = t' + R(t')/c$ t' ret time

$$\frac{dE}{dA} = \int_{t_1 = T_1 + R(T_1)/c}^{t_2 = T_2 + R(T_2)/c}\left(\vec{S}\cdot\hat{n}\right)_{ret} dt = \int_{T_1}^{T_2}\vec{S}\cdot\hat{n}\,\frac{dt}{dt'}\,dt'$$

energy/area radiated for interval T_1 to T_2 at particle

$$\frac{dP(t')}{d\Omega} = R^2 \frac{dE}{dA\,dt'} = R^2 (\vec{S}\cdot\hat{n}) \frac{dt}{dt'}$$

energy leaving particle per unit particle time per unit solid angle

$$\frac{dt}{dt'} = 1 + \frac{1}{c}\frac{d}{dt'} R(t')$$

recall $\quad R = |\vec{x} - \vec{r}(t')|$ $\qquad \dfrac{dR}{dt'} = \dfrac{-(\vec{x} - \vec{r}(t'))\cdot d\vec{r}/dt}{|\vec{x} - \vec{r}(t')|}$

$$= -\hat{n}\cdot\vec{\beta}$$

$$\frac{dt}{dt'} = (1 - \vec{\beta}\cdot\hat{n})$$

so $$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c} \frac{\left|\hat{n}\times((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}})\right|^2}{(1-\vec{\beta}\cdot\hat{n})^5}$$

Rate at which bullets hit target is higher by factor of $(1-\vec{\beta}\cdot\hat{n})^{-1}$ than rate at which they leave the car

Since most of the radiation is in the forward direction, the effect carries over to total energy as well as angular distribution linear motion $\quad \vec{\beta} \parallel \dot{\vec{\beta}} \qquad \vec{\beta}\cdot\hat{n} \equiv \cos\theta$

θ angle between observatio[n] + motion

$$\hat{n}\times\left((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right) = \hat{n}\times(\hat{n}\times\dot{\vec{\beta}})$$

$$= \hat{n}(\hat{n}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}$$

we've seen this in NR case

$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{\beta}^2}{4\pi c}\,\frac{\sin^2\theta}{(1-\beta\cos\theta)^5} = \frac{dP}{d\phi\, d\cos\theta}$$

want peak in $\cos\theta$

$$\frac{d}{du}\left\{\frac{1-u^2}{(1-\beta u)^5}\right\} = 0 \quad \text{for } \cos\theta_{max} \text{ or } u_{max}$$

i.e. $$-\frac{2u}{(1-\beta u)^5} + \frac{5(1-u^2)\beta}{(1-\beta u)^6} = 0$$

or $$\frac{1}{(1-\beta u)^6}\left\{-2u(1-\beta u)+5(1-u^2)\beta\right\} = 0$$

$$3\beta u^2 + 2u - 5\beta = 0$$

$$u_{max} = \frac{1}{3\beta}\left(-1 + \left(1 + 15\beta^2\right)^{1/2}\right)$$

$\beta \to 0$ $\quad u_{max} \to 0 \quad \theta_{max} \simeq \pi/2$

$\beta \to 1$ write $\beta^2 = 1 - \frac{1}{\gamma^2} \qquad \beta \simeq 1 - \frac{1}{2\gamma^2}$

$$u_{max} = \frac{1}{3}\left(1 + \frac{1}{2\gamma^2}\right)\left(-1 + \left(1 + 15 - \frac{15}{\gamma^2}\right)^{1/2}\right)$$

$$= \frac{1}{3}\left(1 + \frac{1}{2\gamma^2}\right)\left(-1 + 4\left(1 - \frac{15}{32\gamma^2}\right)\right)$$

$$= \left(1 + \frac{1}{2\gamma^2}\right)\left(1 - \frac{5}{8\gamma^2}\right) = 1 - \frac{1}{8\gamma^2} = \cos\theta_{max}$$

$$= 1 - \frac{\theta_{max}^2}{2}$$

$$\Longrightarrow \quad \theta_{max} = \frac{1}{2\gamma}$$

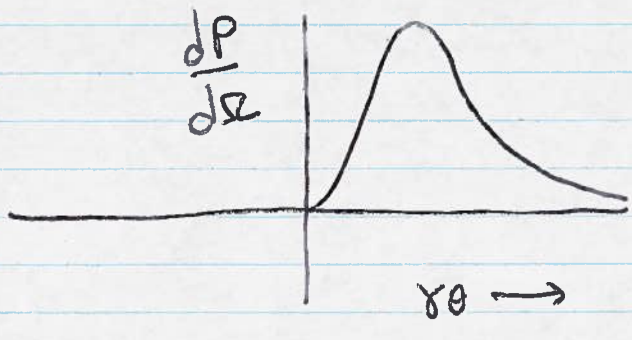
$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{\beta}^2}{4\pi c}\,\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

at $\beta \to 1$ most of the radiation will be in the forward direction $\sin\theta \simeq \theta$ $\cos\theta \simeq 1-\frac{\theta^2}{2}$

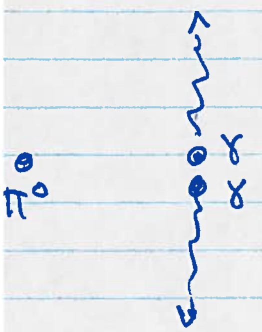
$\beta \simeq 1 - \frac{1}{2\gamma^2}$

so
$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{\beta}^2}{4\pi c}\,\frac{\theta^2}{\left(1-\left(1-\frac{1}{2\gamma^2}\right)\left(1-\frac{1}{2\theta^2}\right)\right)^5}$$

$$= \frac{e^2\dot{\beta}^2}{4\pi c}\,32\gamma^{10}\,\frac{\theta^2}{(1+\gamma^2\theta^2)^5} = \frac{8e^2\dot{\beta}^2}{\pi c}\gamma^8\,\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^5}$$



Consider $\pi^0 \rightarrow \gamma + \gamma$



rest frame of π^0

$$E_\pi = 2E_\gamma$$

$$p_\pi = 2p_\gamma \cos\theta$$

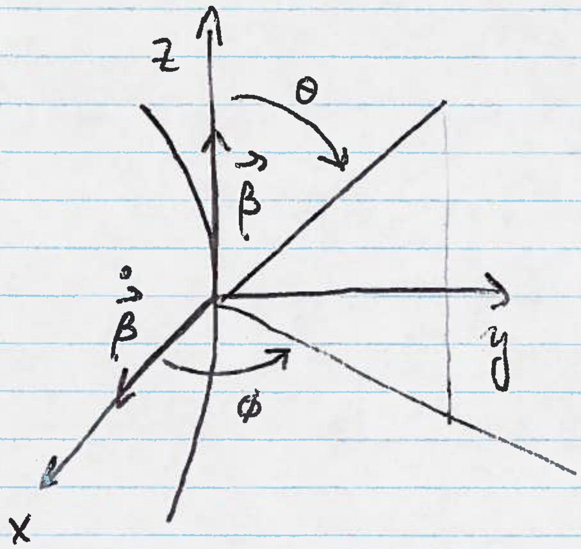
$$\frac{cp_\pi}{E_\pi} = \beta_\pi = \frac{2cp_\gamma \cos\theta}{E_\gamma} = \cos\theta$$

for ultrarelativistic π's $\quad \beta \simeq 1 - \frac{1}{2\gamma^2} = 1 - \frac{\theta^2}{2}$

$$\Rightarrow \quad \theta \simeq \frac{1}{\gamma}$$

Circular motion

- imagine orbit in xz plane

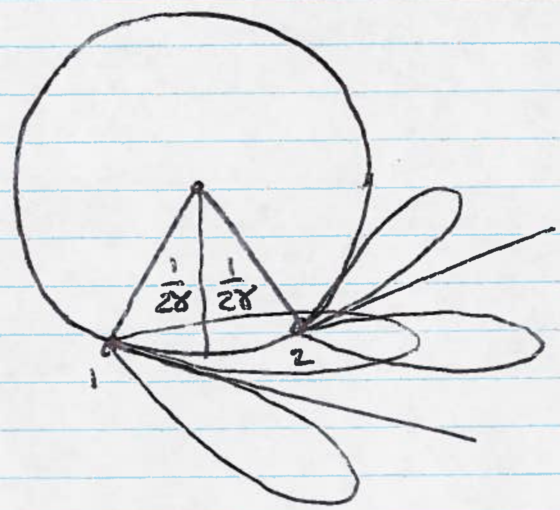


$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c} \frac{\dot{\beta}^2}{(1-\beta\cos\theta)^3}\left(1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right)$$

again forward peaked (factors of $1-\beta\cos\theta$)

Particle in arbitrary, extreme relativistic motion

radiation dominated by acceleration $\perp$ to velocity



Time between 1 + 2

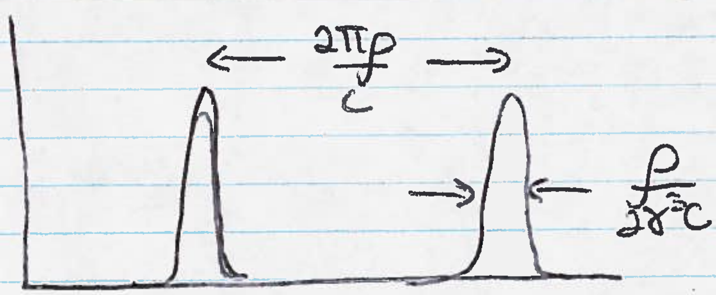
$$T_2 - T_1 = \frac{\rho}{\gamma v}$$

Path diff. between light from 1+2

$$\Delta L = \frac{\rho}{\gamma}$$

so $\quad t_2 - t_1$ at observer $= \frac{\rho}{\gamma v} - \frac{\rho}{\gamma c}$

$$= \frac{\rho}{c\gamma}\left(\frac{c}{v} - 1\right) = \frac{\rho}{2\gamma^3 c}$$



$$\omega_c = \frac{3}{2}\gamma^3 \frac{c}{\rho}$$

characteristic frequency for spectral distribution

Angular power distribution in term of frequency

$$\frac{dP(t)}{d\Omega} = |\vec{A}(t)|^2$$

P(t) power in terms of observer clock (which we use for obs. - freq measurement)

$$\vec{A}(t) = \left(\frac{c}{4\pi}\right)^{1/2} (R\vec{E})_{ret}$$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |\vec{A}(t)|^2 dt$$

total energy Integrating over all time.

$$\vec{A}(t) = \left(\frac{1}{2\pi}\right)^{1/2} \int_{-\infty}^{\infty} \vec{A}(\omega) e^{-i\omega t} d\omega$$

$$\tilde{\vec{A}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{A}(t) e^{i\omega t} dt$$

Parseval's theorem $$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |\tilde{\vec{A}}(\omega)|^2 d\omega$$

$$= \int_{-\infty}^{\infty} \tilde{\vec{A}}(\omega) \cdot \tilde{\vec{A}}^*(\omega) d\omega$$ + use $\delta(t-t') \circ \int$