E.g. $\vec{\beta} = \begin{cases} 0 & t < 0 \\ \beta\hat{z} & t > 0 \end{cases}$

Integral $\hat{n}\times(\hat{n}\times\vec{\beta}) \int_0^\infty e^{i\omega t(1-\vec{\beta}\cdot\hat{n})}\, dt$

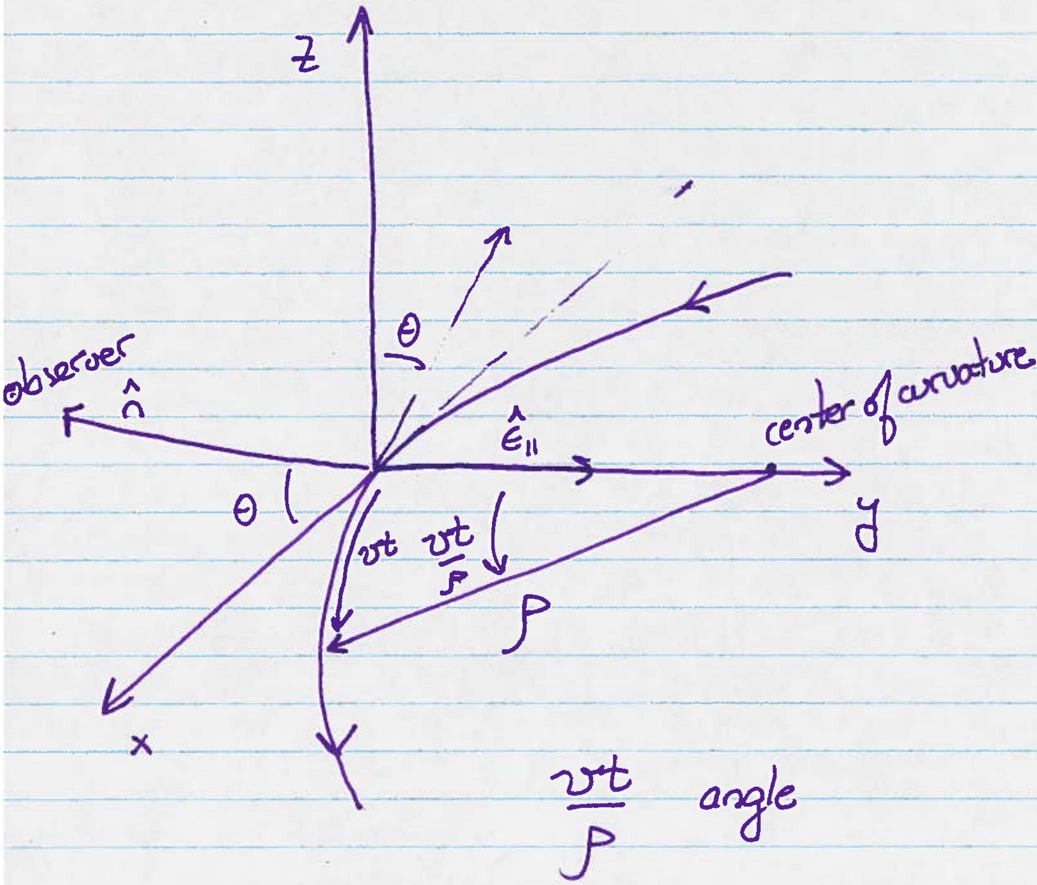
ϵ prescription. Multiply integrand by $e^{-\epsilon t}$

let ϵ go to zero at <u>end</u> of calculation

$$\hat{n}\times(\hat{n}\times\vec{\beta})\,\frac{1}{i\omega}\,\frac{e^{i\omega t(1-\vec{\beta}\cdot\hat{n})}}{1-\vec{\beta}\cdot\hat{n}}\Bigg|_0^\infty = \frac{i}{\omega}\,\frac{\hat{n}\times(\hat{n}\times\vec{\beta})}{1-\vec{\beta}\cdot\hat{n}}$$

so $$\frac{d^2I}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2 c}\,\beta^2\,\frac{\sin^2\theta}{(1-\beta\cos\theta)^2}$$

another problem arises - spectrum is flat - hence infinite energy!



ρ is radius of curvature of circular arc

vt = distance from origin travelled by particle

observer is in xz-plane

$$\hat{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{z}$$

polarization vectors

$$\hat{\epsilon}_{\parallel} = \hat{y} \qquad \hat{\epsilon}_{\perp} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{z}$$

$$\vec{\beta} = \beta\left[\cos\left(\frac{vt}{\rho}\right)\hat{x} + \sin\left(\frac{vt}{\rho}\right)\hat{y}\right]$$

$$\vec{r}(t) = \rho\left[\sin\frac{vt}{\rho}\,\hat{x} - \cos\left(\frac{vt}{\rho}\right)\hat{y}\right]$$

$$\hat{n}\times(\hat{n}\times\vec{\beta}) = (\hat{n}\cdot\vec{\beta})\hat{n} - \vec{\beta}$$

$$= \beta\left\{\cos\theta\cos\left(\frac{vt}{\rho}\right)(\cos\theta\,\hat{x} + \sin\theta\,\hat{z}) - \cos\left(\frac{vt}{\rho}\right)\hat{x} - \sin\left(\frac{vt}{\rho}\right)\hat{y}\right\}$$

$$= \beta\left\{-\sin^2\theta\cos\left(\frac{vt}{\rho}\right)\hat{x} + \cos\theta\sin\theta\cos\left(\frac{vt}{\rho}\right)\hat{z} - \sin\left(\frac{vt}{\rho}\right)\hat{y}\right\}$$

$$= \beta\left\{\hat{\epsilon}_\perp\sin\theta\cos\left(\frac{vt}{\rho}\right) - \sin\left(\frac{vt}{\rho}\right)\hat{\epsilon}_\parallel\right\}$$

$$\omega\left(t - \frac{\hat{n}\cdot\vec{r}(t)}{c}\right) = \omega\left[t - \frac{\rho}{c}\cos\theta\sin\left(\frac{vt}{\rho}\right)\right]$$

Now make small angle/time/$1-\beta$ approximations

$$\langle t\rangle \sim \langle \rho/c\gamma\rangle \text{ and } \langle\theta\rangle \sim \frac{1}{\gamma}$$

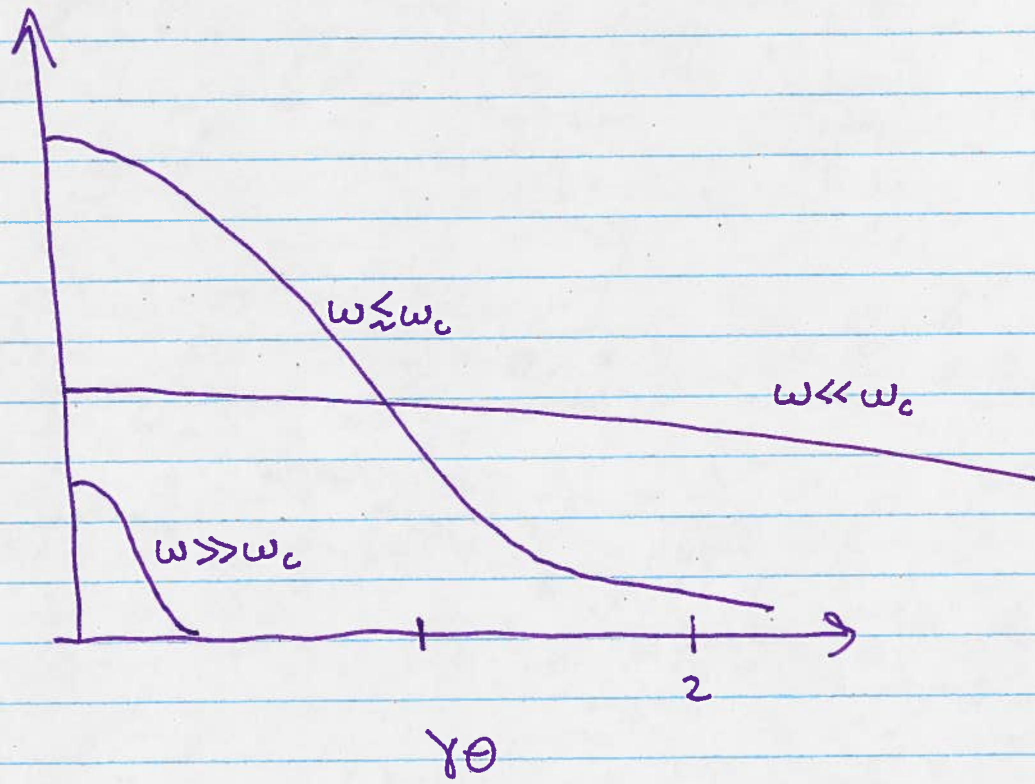
$$t - \frac{\rho}{c}\cos\theta\sin\frac{vt}{\rho} \simeq t - \frac{\rho}{c}\left(1-\frac{\theta^2}{2}\right)\left(\frac{vt}{\rho} - \frac{1}{6}\left(\frac{vt}{\rho}\right)^3\right)$$

$$= t\left\{1 - \left(1-\frac{\theta^2}{2}\right)\left(\frac{v}{c} - \frac{1}{6}\frac{v^3}{c}\frac{t^2}{\rho^2}\right)\right\}$$

$$= t\left\{1 - \left(1-\frac{\theta^2}{2}\right)\left(1 - \frac{1}{2\gamma^2} - \frac{c^2t^2}{6\rho^2}\right)\right\}$$

$$= t\left\{\cancel{1} - \cancel{1} + \frac{\theta^2}{2} + \frac{1}{2\gamma^2} + \frac{c^2t^2}{6\rho^2}\right\}$$

These are "leading" order terms


$\omega \lesssim \omega_c$
$\omega \ll \omega_c$
$\omega \gg \omega_c$
2
$\gamma\theta$

$$K_\nu(\xi) \cong \begin{cases} \frac{\Gamma(\nu)}{2}\left(\frac{2}{\xi}\right)^{\nu} & \xi \ll 1 \\ \left(\frac{\pi}{2\xi}\right)^{1/2} e^{-\xi} & \xi \gg 1 \end{cases}$$

$$\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2} + \theta^2\right)^{3/2}$$

So radiation is negligible for $\xi \gg 1$

as ω is increased, the critical angle decreases
i.e. low frequency radiation is spread out over wider range in angle

for $\theta = 0$ critical frequency goes as γ^3

$$\omega_c = \omega\left(\begin{matrix}\xi = 1/2 \\ \theta = 0\end{matrix}\right)$$

$$= \frac{3}{2}\frac{c}{\rho}\gamma^3 = \frac{3c}{2\rho}\left(\frac{E}{Mc^2}\right)^3$$

we obtained this
from hand waving
arguments

low frequency limit $\xi \ll 1$ for $\theta = 0$

critical angle found at $\xi = 1$ ($\theta > \theta_c$ + radiation is exponentially truncated)

$$\theta_c = \left(\frac{3c}{\omega\rho}\right)^{1/3} \quad \text{and since} \quad \omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

$$= \gamma\left(\frac{2\omega_c}{\omega}\right)^{1/3}$$

at low frequencies, radiation comes out at angles greater than $\frac{1}{\gamma}$

high frequency regime, $\xi \gg 1$

$$\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2} + \theta^2\right)^{3/2} \simeq \frac{\omega\rho}{3c}\frac{1}{\gamma^3}\left(1 + \frac{3\theta^2\gamma^2}{2}\right)$$

$$= \omega_c\left(1 + \frac{3\gamma^2\theta^2}{2}\right)$$

so $$\frac{d^2 I}{d\omega d\Omega} = \left.\frac{d^2 I}{d\omega d\Omega}\right|_{\theta=0} e^{-3\omega_c\gamma^2\theta^2/2}$$

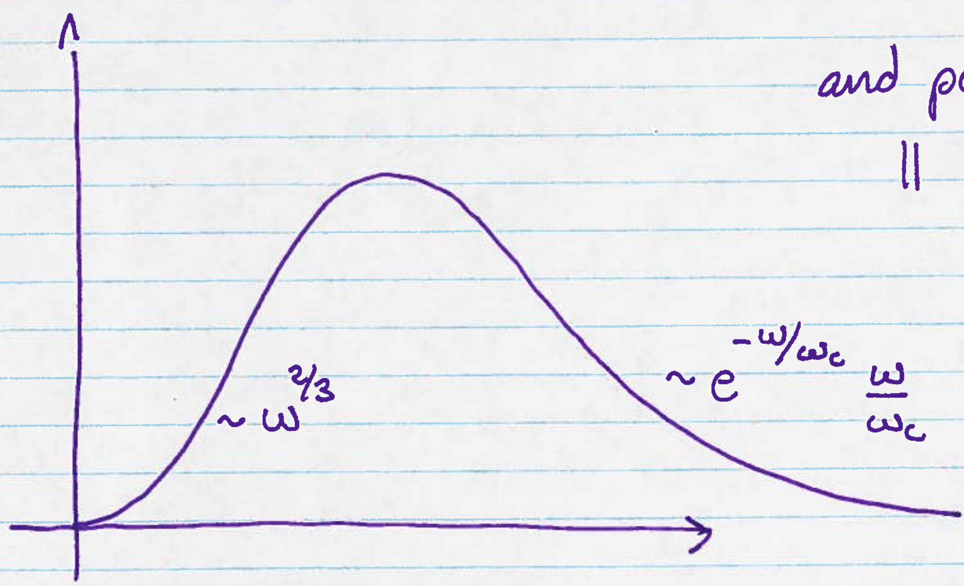
radiation confined to angular range narrower that $1/\gamma$

Since most of radiation comes out near $\theta = 0$

evaluate

$$\left.\frac{d^2 I}{d\omega\, d\Omega}\right|_{\theta=0} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2 \frac{1}{\gamma^4} K^2_{2/3}(\xi)$$

$$= \begin{cases} \dfrac{e^2}{c}\left(\dfrac{\Gamma(2/3)}{\pi}\right)^2 \left(\dfrac{3}{4}\right)^{1/3} \left(\dfrac{\omega\rho}{c}\right)^{2/3} & \omega \ll \omega_c \\[2ex] \dfrac{3}{4\pi}\dfrac{e^2}{c}\gamma^2 \dfrac{\omega}{\omega_c} e^{-\omega/\omega_c} & \omega \gg \omega_c \end{cases}$$



and polarized
$\parallel$ to orbital plane!