Summary

$$\frac{dP(t)}{d\Omega} = |\vec{A}(t)|^2$$

$$\vec{A}(t) = \left(\frac{c}{4\pi}\right)^{1/2} \left(R\vec{E}_a\right)_{ret}$$

$$= \left(\frac{e^2}{4\pi c}\right)^{1/2} \left(\frac{\hat{n} \times \left[(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}\right]}{(1 - \vec{\beta}\cdot\hat{n})^3}\right)_{ret}$$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |\vec{A}(t)|^2 dt = \int_{-\infty}^{\infty} |\vec{A}(\omega)|^2 d\omega$$

$$= 2\int_0^{\infty} |\vec{A}(\omega)|^2 d\omega$$

so $$\frac{d^2 I}{d\omega\, d\Omega} = 2|\vec{A}(\omega)|^2$$

Now integrate over angles

$$2\pi \int \sin\theta \, d\theta \times \left\{ \frac{1}{\left(1/\gamma^2 + \theta^2\right)^{5/2}} + \frac{5}{7} \frac{\theta^2}{\left(1/\gamma^2 + \theta^2\right)^{7/2}} \right\}$$

$$\sin\theta d\theta \cong \theta d\theta = \frac{1}{2} d\theta^2 \qquad \text{so let} \quad x = \theta^2$$

$$= \frac{1}{2} \int_0^\infty dx \left(\frac{1}{\left(1/\gamma^2 + x\right)^{5/2}} + \frac{5}{7} \frac{x}{\left(1/\gamma^2 + x\right)^{7/2}} \right)$$

$$= \frac{\gamma^3}{3} + \frac{2}{15} \frac{5}{7} \gamma^3 = \frac{\gamma^3}{3} \left(1 + \frac{2}{7} \right)$$

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2}+\theta^2\right)^2\left[\underbrace{K^2_{2/3}(\xi)}_{\hat{\epsilon}_{\parallel}} + \underbrace{\frac{\theta^2}{1/\gamma^2+\theta^2}K^2_{1/3}(\xi)}_{\hat{\epsilon}_{\perp}}\right]$$

$$\frac{dI}{d\Omega} = \int_0^\infty \frac{d^2I}{d\omega\, d\Omega}\, d\omega$$

$$\frac{\omega\rho}{c} = 3\xi\left(\theta^2+1/\gamma^2\right)^{-3/2}$$

so $$\left(\frac{\omega\rho}{c}\right)^2 d\omega = 27\xi^2\left(\theta^2+1/\gamma^2\right)^{-9/2} d\xi\, \frac{c}{\rho}$$

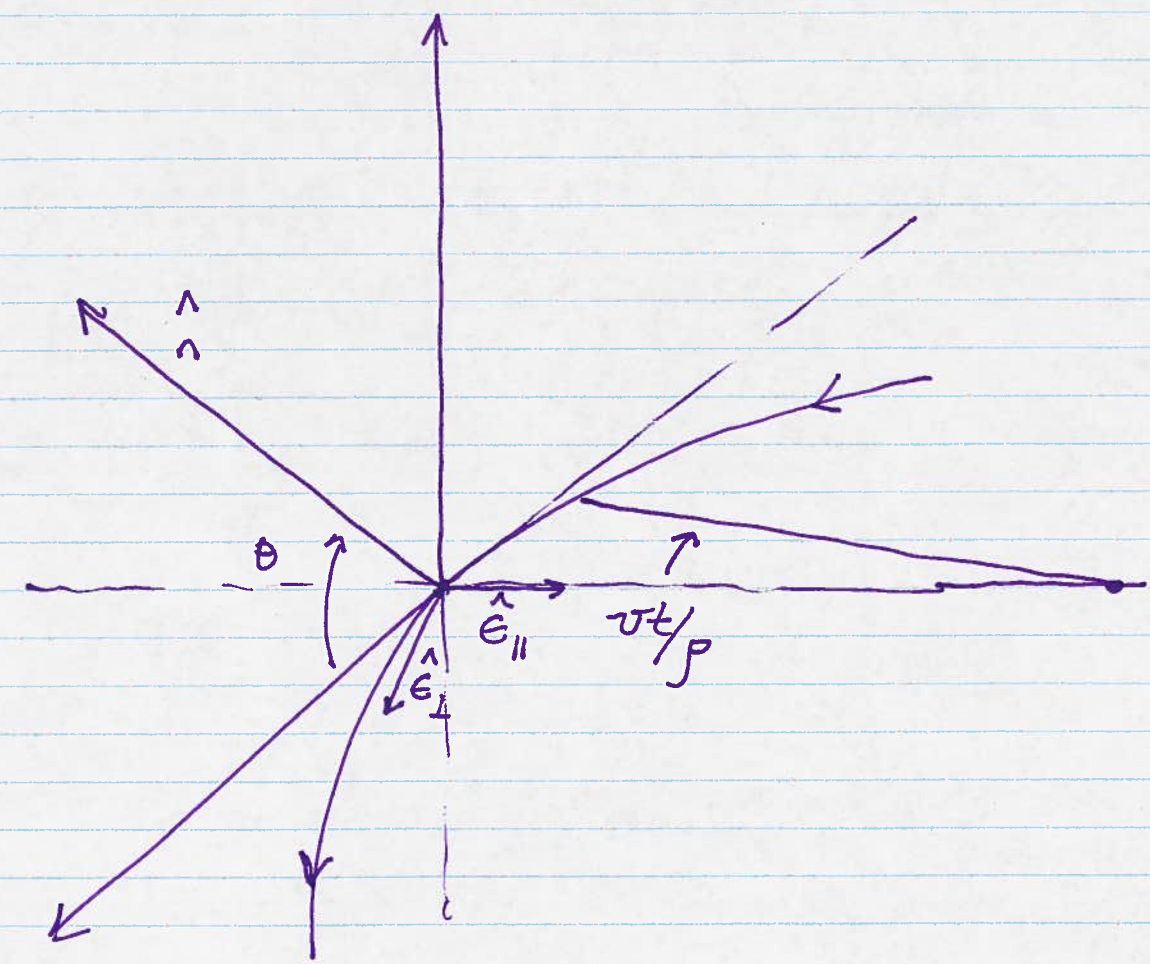
$$\frac{dI}{d\Omega} = \frac{9e^2}{\pi^2\rho}\,\frac{1}{(1/\gamma^2+\theta^2)^{5/2}}\left\{\int_0^\infty \xi^2 d\xi\left[K^2_{2/3}(\xi) + \frac{\theta^2}{1/\gamma^2+\theta^2}K^2_{1/3}(\xi)\right]\right\}$$

$$= \frac{7}{16}\,\frac{e^2}{\rho}\,\frac{1}{(1/\gamma^2+\theta^2)^{5/2}}\left[\overset{\hat{\epsilon}_{\parallel}}{1} + \overset{\hat{\epsilon}_{\perp}}{\frac{5}{7}\,\frac{\theta^2}{1/\gamma^2+\theta^2}}\right]$$

at $\theta = 0$ it's all $\hat{\epsilon}_{\parallel}$

$$\vec{A}(\omega) = \left(\frac{e^2}{8\pi^2 c}\right)^{1/2} \int_{-\infty}^{\infty} e^{i\omega t} \left.\frac{\hat{n} \times \left((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}\right)}{\left(1 - \vec{\beta}\cdot\hat{n}\right)^3}\right)_{t = t' + \frac{R(t')}{c}} dt$$

$$= \left(\frac{e^2}{8\pi^2 c}\right)^{1/2} \omega \int_{-\infty}^{\infty} \left(\hat{n} \times (\hat{n} \times \vec{\beta})\right) e^{i\omega\left(t - R(t)/c\right)} dt$$



$$\omega\left(t - \frac{\hat{n}\cdot\vec{r}}{c}\right) = \frac{\omega}{2}\left(\left(\frac{1}{\gamma^2} + \theta^2\right)t + \frac{c^2}{3\rho^2}t^3\right)$$

Now we had found

$$\hat{n}\times(\hat{n}\times\vec{\beta}) = \beta\left(-\hat{\epsilon}_{\parallel}\sin\left(\frac{vt}{\rho}\right) + \hat{\epsilon}_{\perp}\cos\left(\frac{vt}{\rho}\right)\sin\theta\right)$$

$$\cong \beta\left(-\hat{\epsilon}_{\parallel}\frac{vt}{\rho} + \hat{\epsilon}_{\perp}\theta\right)$$ to leading order

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{n}\times(\hat{n}\times\vec{\beta})\, e^{i\omega(t - \hat{n}\cdot\vec{r}(t)/c)}\, dt\right|^2$$

$$= \frac{e^2\omega^2}{4\pi^2 c}\left|-\hat{\epsilon}_{\parallel}A_{\parallel}(\omega) + \hat{\epsilon}_{\perp}A_{\perp}(\omega)\right|^2$$

$$A_{\parallel}(\omega) = \frac{c}{\rho}\int_{-\infty}^{\infty} t\, e^{\frac{i\omega}{2}\left[\left(\frac{1}{\gamma^2}+\theta^2\right)t + \frac{c^2 t^3}{3\rho^2}\right]} dt$$

$$A_{\perp}(\omega) = \theta\int_{-\infty}^{\infty} e^{[\quad]} dt$$

A bit messy:

t - integration variable

ω - frequency

γ - Lorentz factor

θ - angle of obs. to xy plane

ρ - radius of curvature of particle

Change variables: $x = \frac{ct}{\rho}\frac{1}{(1/\gamma^2+\theta^2)^{1/2}}$ $\quad t = \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2};$

$$\left(\frac{1}{\gamma^2}+\theta^2\right)t + \frac{c^2t^3}{3\rho^2} = \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}x + \frac{\cancel{c^2}}{3\cancel{\rho^2}}\frac{\cancel{\rho^3}}{c^3}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}x^3$$

$$= \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}\left(x+\frac{x^3}{3}\right)$$

$$\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}$$ dimensionless frequency

$$A_{\parallel}(\omega) = \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)\int_{-\infty}^{\infty} x\, e^{i\frac{3}{2}\xi\left(x+\frac{x^3}{3}\right)}\,dx$$

Airy integrals

also written as

Modified Besselfunctions

found in optics!

x is odd about $x=0$ so $e^{i\{\}} \rightarrow i\sin\{\}$

$$A_{\parallel}(\omega) = \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)\underbrace{\int_{-\infty}^{\infty} x\sin\left(\frac{3\xi}{2}\left(x+x^3/3\right)\right)dx}_{\frac{1}{\sqrt{3}}K_{2/3}(\xi)}$$ MBF

$$A_{\perp}(\omega) = \frac{\rho}{c}\,\theta\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}\underbrace{\int_{-\infty}^{\infty} \cos\left(\frac{3\xi}{2}\left(x+x^3/3\right)\right)dx}_{\frac{1}{\sqrt{3}}K_{1/3}(\xi)}$$