

Magnetic Monopoles

Maxwell's equations cry out for monopoles

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho_e$$

$$\vec{\nabla} \cdot \vec{B} = 4\pi\rho_m$$

$$\vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = -\frac{4\pi}{c} \vec{J}_m$$

$$\vec{\nabla} \times \vec{E} - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = \frac{4\pi}{c} \vec{J}_e$$

equations are invariant under

$$\vec{E} = E' \cos \xi + \vec{B}' \sin \xi$$

$$\vec{B} = -\vec{E}' \sin \xi + \vec{B}' \cos \xi$$

$$\rho_e = \rho_e' \cos \xi + \rho_m' \sin \xi$$

$$\rho_m = -\rho_e' \sin \xi + \rho_m' \cos \xi$$

ξ real parameter.

Thus, what we call electric charge + magnetic charge is somewhat arbitrary.

- 1) Dirac 1931 argues that if monopoles exist they imply charge quantization (from ang. mom. quantization)
- 2) Monopoles are predicted to exist in some GUT models (but may be diluted by inflation)

By convention, assume $g_{m, \text{electron}} = 0$

can we place a limit on $g_{m, \text{nucleon}}$?

$$B_{\text{Earth's surface}} = \frac{g_m N_N}{R_{\oplus}^2}$$

$$= \frac{g_m M_{\oplus} / m_p}{R_{\oplus}^2}$$

< 1 Gauss to avoid detection

Now $\frac{GM_{\oplus}}{R_{\oplus}^2} = g = 10^3 \text{ cm/s}^2$

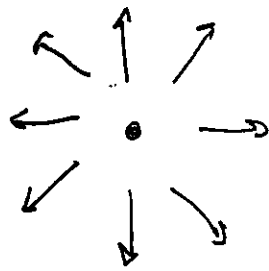
$$\therefore \frac{B}{g} = \frac{g_m}{G m_p}$$

$$\text{or } g_m < \frac{1 \text{ Gauss} \times G m_p}{10^3 \text{ cm/s}^2}$$

$$< 10^{-34} \text{ e.s.u.}$$

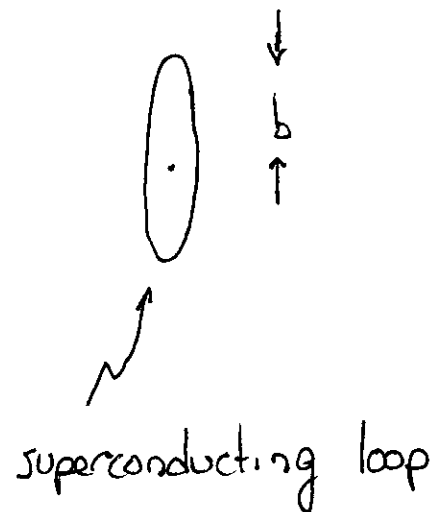
$$\text{or } \frac{g_m}{|e|} < 2 \times 10^{-23}$$

Detection



$$\vec{B} = g \frac{\hat{r}}{r^2}$$

magnetic charge



As monopole passes thru loop, \vec{B} in loop changes
+ there is EMF which drives current.

$$\int \vec{E} \cdot d\vec{l} = -\frac{1}{c} \frac{dF}{dt}$$

consider charge in loop of mass m , charge q

$$\oint \vec{E} \cdot d\vec{l} = 2\pi b E = 2\pi b \cdot \frac{m}{e} \frac{dv}{dt}$$

v speed of charge in loop

$$= -\frac{1}{c} \frac{dF}{dt}$$

$$2\pi b \frac{m}{e} (v_f - v_i) = -\frac{1}{c} (F_f - F_i)$$

let $v_i = 0$

$F_i = 0$ monopole far from loop

$$F_f = \oint_S da \hat{n} \cdot \vec{B}$$

$$= \oint R^2 d\Omega \cdot \frac{q}{R^2} = 4\pi q$$



$$\frac{m\dot{b}r}{e} = - \frac{2g}{c}$$

$$m\dot{b}r = L = \text{ang. momentum of charge in loop}$$

$$= - \frac{2ge}{c}$$

$$= n\hbar \quad \Rightarrow \quad |e| = \frac{n\hbar c}{2g}$$



Limit on magnetic monopole density in galaxy

Galactic field 3×10^{-6} G Age 3×10^{17} s

Monopoles would short-circuit this field!

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First Results from a Superconductive Detector for Moving Magnetic Monopoles

Bias Cabrera

Physics Department, Stanford University, Stanford, California 94305

(Received 5 April 1982)

A velocity- and mass-independent search for moving magnetic monopoles is being performed by continuously monitoring the current in a 20-cm²-area superconducting loop. A single candidate event, consistent with one Dirac unit of magnetic charge, has been detected during five runs totaling 151 days. These data set an upper limit of 6.1×10^{-10} cm⁻² sec⁻¹ sr⁻¹ for magnetically charged particles moving through the earth's surface.

PACS numbers: 14.80.Hv

The detection of a moving magnetic charge with a superconducting ring is based solely on the long-range electromagnetic interactions between the magnetic charge and the macroscopic quantum state of the ring.¹ Such a detector measures a moving particle's magnetic charge regardless of its velocity, mass, electric charge, or magnetic dipole moment. In this paper, the first experimental results from use of this scheme are presented.

Superconductors make natural magnetic charge detectors, as suggested by comparing the flux quantum of superconductivity $\varphi_0 = hc/2e$ with the flux emanating from a single Dirac charge $4\pi g = hc/e$. Dirac² was led to his value for the elementary magnetic charge by postulating that the wave function of a single electron in the field of a pole should be single valued. In superconductivity, the postulate of a single-valued macroscopic wave function leads to flux quantization. The factor of 2 arises from the electric charge, $2e$, of the Cooper pairs.

Consider a magnetic charge g moving at velocity v along the axis of a superconducting wire ring of radius b . Integrating Maxwell's generalized equation for the monopole current, $\text{curl}(\vec{E}) + (1/c)d\vec{B}/dt = -(4\pi/c)\vec{j}_m$, over the area S_Γ in the plane of the ring, we obtain

$$\oint_{\Gamma} \vec{E} \cdot d\vec{l} + c^{-1} (d\varphi/dt) = -(4\pi g/c)\delta(t),$$

where S_Γ is bounded by a path Γ that is everywhere inside the wire. If we neglect the finite response time of the superelectrons, \vec{E} will vanish along Γ and $\varphi(t) = -4\pi g\theta(t)$, where we set $\varphi = 0$ for $t = -\infty$. The total flux through S_Γ is $\varphi = \varphi_g + \varphi_s$, φ_g from the monopole and φ_s from the induced supercurrent. We find

$$\varphi_g(t) = 2\pi g \left[1 - 2\theta(t) + \frac{\gamma vt}{[(\gamma vt)^2 + b^2]^{1/2}} \right],$$

and $\varphi_s = -I(t)L$, where $I(t)$ is the induced supercurrent and L the self-inductance of the ring. Thus, substituting $4\pi g = 2\varphi_0$, we obtain

$$I(t) = \frac{\varphi_0}{L} \left[1 + \frac{\gamma vt}{[(\gamma vt)^2 + b^2]^{1/2}} \right].$$

This result is independent of the choice of surface S_Γ bounded by the path Γ and corresponds to a change of $2\varphi_0$ through the ring (Fig. 1). The change in current will occur with a characteristic time given by $b/\gamma v$.

In the general case, any trajectory of a magnetic charge g which passes through the ring will result in a flux-quantum change of 2, while one which misses the ring will produce no flux change. In the less likely event that a magnetic charge passes through the ring wire itself, it will leave a trapped doubly quantized vortex, and some intermediate total current will persist. Any electric charge or magnetic dipole moment of the particle

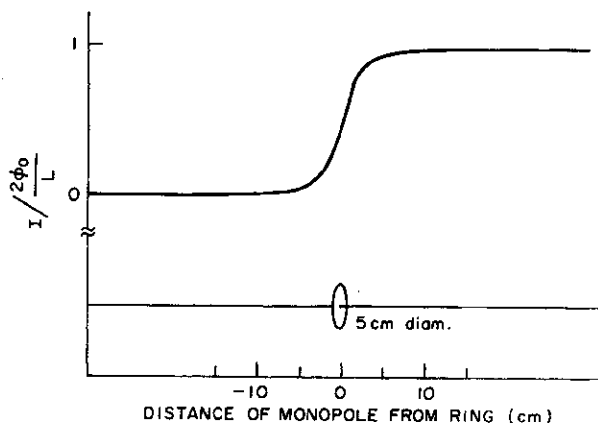


FIG. 1. Induced current in a superconducting ring for an axial monopole trajectory.

will cause only small transient fluctuations and no dc shifts. Thus, moving magnetically charged particles can be detected by monitoring the current in the superconducting ring.

In this experiment a four-turn, 5-cm-diam loop, positioned with its axis vertical, is connected to the superconducting input coil of a SQUID (superconducting quantum interference device) magnetometer.³ The passage of a single Dirac charge through the loop would result in an $8\phi_0$ change in the flux through the superconducting circuit, comprised of the detection loop and the SQUID input coil (a factor of 2 from $4\pi g = 2\phi_0$ and of 4 from the turns in the pickup loop). The SQUID and loop are inside a 20-cm-diam, 1-m-long cylindrical superconducting shield closed at the bottom, and these are mounted inside a single Mumetal cylinder. The combined shielding provides 180-dB isolation from external magnetic field changes and an ambient field of 5×10^{-8} G.⁴

The voltage output of the SQUID electronics, which is directly proportional to the supercurrent in the detection loop, is continuously recorded through a 0.1-Hz low-pass filter onto a strip-chart recorder. In addition, several times per day digital voltmeter readings are taken to guard against recorder failures.

The detector sensitivity has been calibrated in three independent ways: (a) by measuring the SQUID response to a known current in calibration Helmholtz coils and calculating their mutual inductance to the superconducting loop ($\pm 4\%$); (b) by estimating the self-inductance of the superconducting circuit ($\pm 30\%$); and (c) by directly observing flux quantization within the superconducting circuit ($\pm 10\%$). All three methods agree with

in their independent uncertainties.

Two additional effects influence the exact detector response. First, a magnetic monopole whose trajectory intersects either the transformer loop in the SQUID, the twisted leads from the SQUID to the loop, or the loop wire itself would produce a shift of nonintegral magnitude. Computation of the average area ratio of the loop to the remainder of the transformer circuit indicates that such events will be suppressed by a factor of 25 compared to loop events. Second, a particle traversing the superconducting shield will leave doubly quantized vortices wherever the trajectory intersects a wall. The effect is a magnetic field change inside the shield and an applied flux change across the loop. The total induced current change in the loop is $\Delta I = (8\phi_0/L)[\eta - \xi(A_1/A_s)]$, where $A_1/A_s = 0.06$ is the ratio of the loop to shield cross-sectional areas. For a trajectory that intersects the loop, $\eta = 1$, and for one that misses, $\eta = 0$. The geometric factor ξ depends on the trajectory impact parameter and inclination angles, and has maximum value of 1 for axial trajectories through the shield and a minimum of 0 for transverse ones. Current changes of $(0.06)8\phi_0/L$ or less will be observed for trajectories that pass through the shield but not the loop. The probability for such events with $\Delta I > (0.02)8\phi_0/L$ is about 10 times larger than for the loop.

As of 11 March 1982 data have been recorded for a total of 151 days. Several intervals throughout a continuous one-month time period are shown in Fig. 2(a), where no adjustment of the dc level has been made. Typical disturbances caused by daily liquid-nitrogen and weekly liquid-helium transfers are evident. A single large event was recorded [Fig. 2(b)]. It is consistent with the passage of a single Dirac charge within a combined uncertainty of $\pm 5\%$ (resulting from the calibration uncertainty and the distribution of geometric factor ξ). It is the largest event of any kind in the record. In Fig. 3 are plotted the 27 events exceeding a threshold of $0.2\phi_0$, which remain after exclusion of known disturbances such as transfers of liquid helium and nitrogen.⁵ An event is defined as a sharp offset with well-defined stable levels for at least 1 h before and after. Only six events were recorded during the 70% of the running time when the laboratory was unoccupied.

The following statements about spurious detector response can be made:

(a) *Line voltage fluctuations* caused by two power outages and their accompanying transients

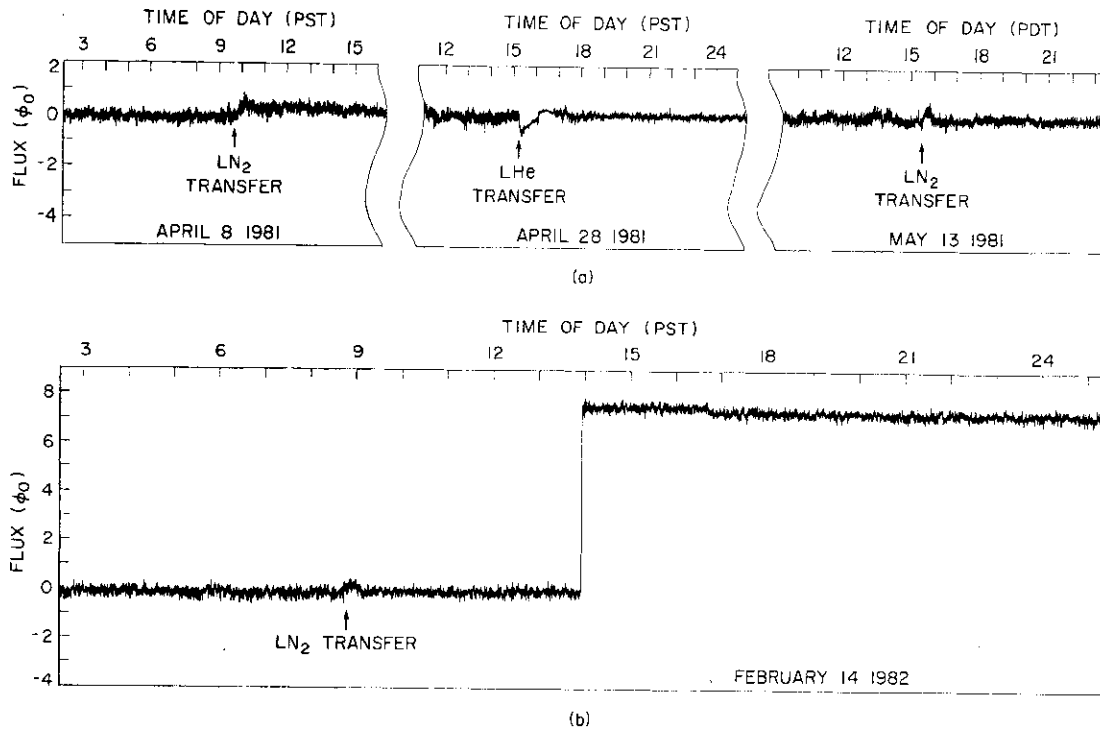


FIG. 2. Data records showing (a) typical stability and (b) the candidate monopole event.

failed to cause detectable offsets.

(b) *rf interference* from the motor brushes of a heat gun failed to produce any offsets when operated in close proximity to the detector.

(c) *External magnetic field* changes are attenuated by 180 dB, primarily from an exponential factor of $e^{-1.83z/a}$, where $z = 72$ cm is the distance in from the open top of the superconducting shield and $a = 10$ cm is the shield radius.

(d) *Ferromagnetic contamination* is minimized using clean-bench assembly techniques and

checked with magnetometer measurements within the shield.

(e) *The critical current* of the loop is not reached for currents a thousand times greater than $8\phi_0/L$ and is typically 10^8 times greater.

(f) *Mechanically induced offsets* have been intentionally generated and are probably caused by shifts of the four-turn loop-wire geometry which produce inductance changes. Sharp raps with a screwdriver handle against the detector assembly cause such offsets. On two occasions out of 25 attempts these have exceeded $6\phi_0$ (75% of the shift expected from one Dirac charge); however, drifts in the level were seen during the next hour.

(g) *No seismic disturbance* occurred on 14 February 1982.

(h) *Energetic cosmic rays* depositing ≈ 1 GeV/cm in traversing the wire would raise the local wire temperature by only ≈ 0.01 K, but a 5-K change is needed to reach the critical temperature.

A spontaneous and large external mechanical impulse is not seen as a possible cause for the event; however, the evidence presented by this single event does not preclude the possibility of a spontaneous internal stress release mechanism. Regardless, to date the experiment has set an

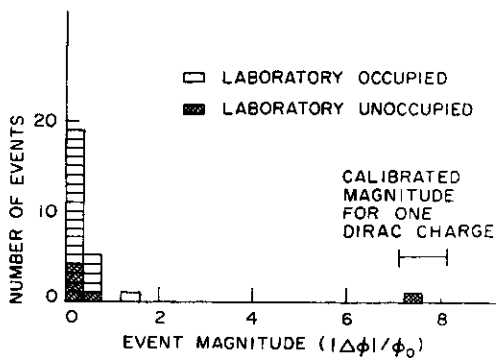


FIG. 3. Histogram of all event magnitudes.

upper limit of $6.1 \times 10^{-10} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$ for the isotropic distribution of any moving particles with magnetic charge greater than 0.06 g.

An observational upper bound on the mass density of monopoles is given by limits on the local "missing mass."⁶ Visible matter has a measured local density of $0.09 M_{\odot}/\text{pc}^3$ (solar masses per cubic parsec), whereas the mass density calculated from the velocity distribution out of the galactic plane is $0.14 M_{\odot}/\text{pc}^3$. This local "missing mass" density estimate of $0.05 M_{\odot}/\text{pc}^3$ is in good agreement with the halo mass estimates extrapolated back to our local galactic radius, which give $0.03 M_{\odot}/\text{pc}^3$. If we assume this entire "hidden mass" to be made up of monopoles of mass $10^{16} \text{ GeV}/c^2$ with isotropic velocities of order 300 km/sec, as suggested from grand unification theories,⁷ the number passing through the earth's surface would be $4 \times 10^{-10} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$.⁸ This would result in 1.5 events per year through the detector loop.

The search with the present detector is being continued, and two new systems of larger sensing area are being built.

The author wishes to thank F. Bloch, R. Bond, W. M. Fairbank, S. Felch, R. Hofstadter, M. Tamber, P. Trower, and R. V. Wagoner for many stimulating and helpful discussions. This work has been funded in part by National Science Foundation Grant No. DMR 80-26007 and by NASA Grant No. NAS8-32355.

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⁸Recently J. D. Ullman, *Phys. Rev. Lett.* **47**, 289 (1981), has reported on the use of a proportional counter array to set an upper flux limit of $3 \times 10^{-11} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$. However, differing estimates for slow monopole ionization rates [S. Geer and W. G. Scott, *CERN pp Note* 69, April 1981 (unpublished); K. Hayashi, to be published; J. S. Trefil, to be published] indicate the complexities of such calculations. These calculations are crucial to predicting whether slow monopoles are observable in a given ionization detector.

Consider a single monopole in uniform field B

$$\frac{dp}{dt} = gB$$

$$\approx mc \quad \text{on timescale} \quad \tau = \frac{mc}{gB}$$

$$= \frac{mc}{eB} \times \frac{e}{g}$$

$$= \frac{9 \times 10^{-27} \text{ g} \cdot 3 \times 10^{10} \text{ cm/s}}{4.8 \times 10^{-10} \text{ esu} \cdot 3 \times 10^{-6} \text{ G}} \frac{e}{g}$$

$$= 0.25 \times \frac{e}{g}$$

$$\frac{dE}{dt} \approx gBc$$

Suppose there are N_g monopoles/cm³

$$\frac{d\epsilon}{dt} = \text{change in energy density} \approx gBcN_g = - \frac{d}{dt} \frac{B^2}{8\pi}$$

$$\frac{1}{B^2} \frac{dB^2}{dt} \approx \frac{8\pi g c N_g}{B}$$

$$\tau_{B, \text{decay}} \approx \frac{B}{8\pi g c N_g} > 3 \times 10^{17} \text{ s}$$

$$N_g < \frac{B}{8\pi e c 3 \times 10^{17} \text{ s}} \frac{e}{g}$$

$$< 3 \times 10^{-26} \text{ cm}^{-3} \frac{e}{g}$$

$$\text{ISM } N_N \approx 1 \text{ cm}^{-3}$$