Problem Set 1 PHYS 832

7.18 $$I(t) = \begin{cases} (1-\alpha t)\, I & 0 \le t \le 1/\alpha \\ 0 & t \ge 1/\alpha \\ I & t < 0 \end{cases}$$

outside wire, use Ampère's law $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$

or $$2\pi r B_\phi = \frac{4\pi}{c} I(t) \qquad B_\phi = \frac{2I(t)}{cr}$$

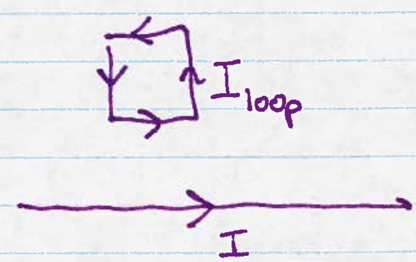
B is out of page!

flux is $$\int_0^a dz \int_s^{s+a} dr\, B_\phi = \frac{2I(t)a}{c}\int_s^{s+a}\frac{dr}{r}$$

$$= \frac{2Ia}{c}\ln\left(\frac{s+a}{s}\right)$$

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{2a}{c^2}\ln\left(\frac{s+a}{s}\right)\frac{dI}{dt}$$

$$= I_{loop} R = \frac{dQ_{loop}}{dt} R$$



$$Q_{loop} = \frac{2a}{c^2 R}\ln\left(\frac{s+a}{s}\right) I$$

so $E_{low} - E_{up} = -d \frac{1}{c} \frac{dB}{dt}$

mpluse $= \int Q E_{low} + (-Q) E_{upp} \, dt$

$= \frac{Q \times d \times B}{c}$ $\int \frac{dB}{dt} dt = -B$

$= \frac{A d E B}{4\pi c}$

8.6 Momentum density $\vec{P} = \frac{1}{4\pi c}\vec{E}\times\vec{B}$

a) Momentum between plates $\vec{P} = P \times A \times d$

$$= \frac{AdEB}{4\pi c}\hat{y}$$

b) $\vec{F} = \frac{q\vec{v}}{c}\times\vec{B}$ force on single charge q traversing wire

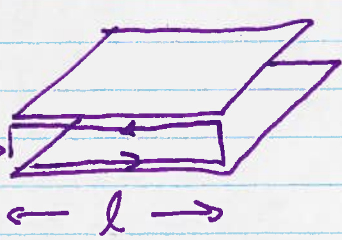
$\vec{I}$ = impulse $= \int \vec{F}\cdot dt$ $\int \vec{v}\,dt = d\hat{z}$

$$= \frac{Q}{c}dB\hat{y}$$ Q = total charge on lower plate $= \frac{AE}{4\pi}$

$$= \frac{AdEB}{4\pi c}\hat{y}$$ ✓

c) $\oint \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{dF}{dt} = -\frac{1}{c}\frac{dB}{dt}\ell d$ ℓ = length (in y) of plates

Faraday loop



$$\oint \vec{E}\cdot d\vec{l} = (E_{lower} - E_{upper})\ell$$ E in y direction

Problem Set 2

11.6 $a = \frac{dv}{dt}$ = ord. accel. in K $= \left(1 - \frac{v^2}{c^2}\right)^{3/2} g$

$a' = g$
$u' = 0$
instantaneous rest frame of rocket

so $\int \frac{dv}{(1 - v^2/c^2)^{3/2}} = \int g\,dt$

$\frac{v}{c} = \tanh y \qquad 1 - \frac{v^2}{c^2} = \frac{1}{\cosh^2 y}$ or $\gamma = \frac{1}{(1 - v^2/c^2)^{1/2}} = \cosh y$

$dv = c\,\frac{dy}{\cosh^2 y}$

$\frac{gt}{c} = \int \cosh y\,dy = \sinh y$ also $dt = \gamma\,d\tau$
$= \frac{c}{g} \cosh y\,dy$

so $\boxed{\frac{g\tau}{c} = y}$ relates proper time on ship to rapidity (or velocity)

$v = c \tanh y = \frac{dx}{dt}$ or $\frac{dx}{d\tau} = c \tanh y \cosh y = c \sinh y$
$= c \sinh g\tau/c$

so $x = \frac{c^2}{g}\left[\cosh g\tau/c - 1\right]$

early time $t = \tau$

* $x = \frac{c^2}{g}\left[1 + \frac{1}{2}\left(\frac{g\tau}{c}\right)^2 - 1\right]$

$\simeq \frac{1}{2} g t^2$ ✓

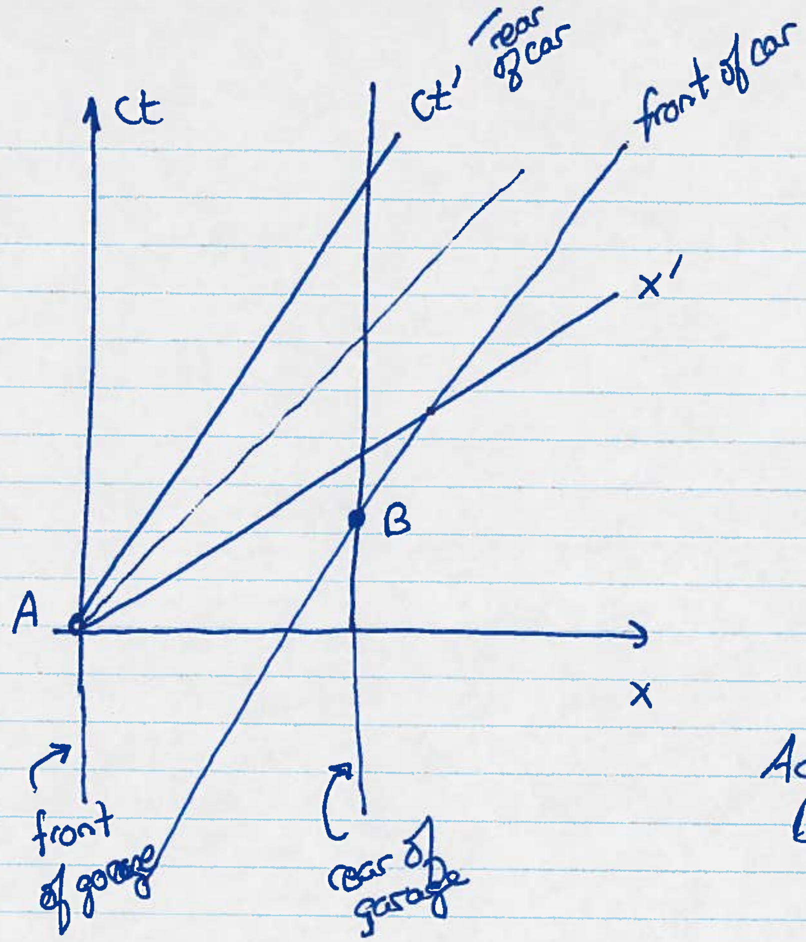
At 5 years $\frac{g\tau}{c} = 5.15$

$\cosh \frac{g\tau}{c} = 86.2$ and $x = 83$ ltyrs

total distance is 166 ltyrs

total time elapsed on earth for RT is 332 yrs

1)



K rest frame of garage

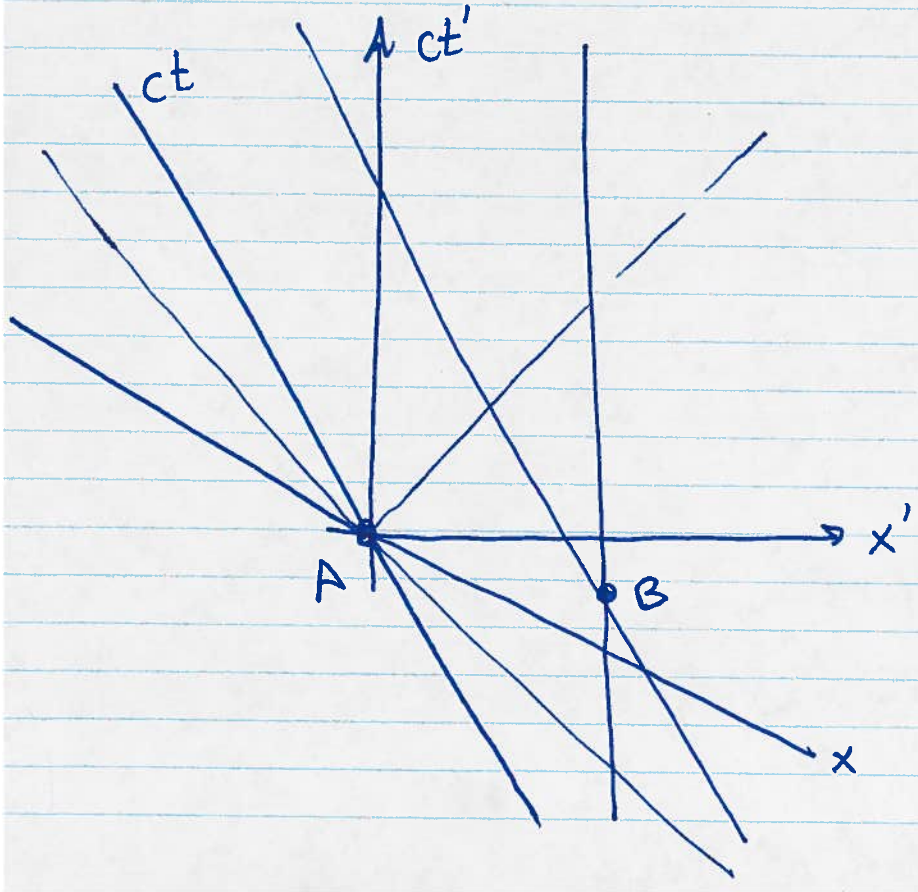
K' rest frame of car

A door slams as rear of car passes

B front of car crashes thru rear of garage

According to K observers, B occurs after A (car fits in garage)

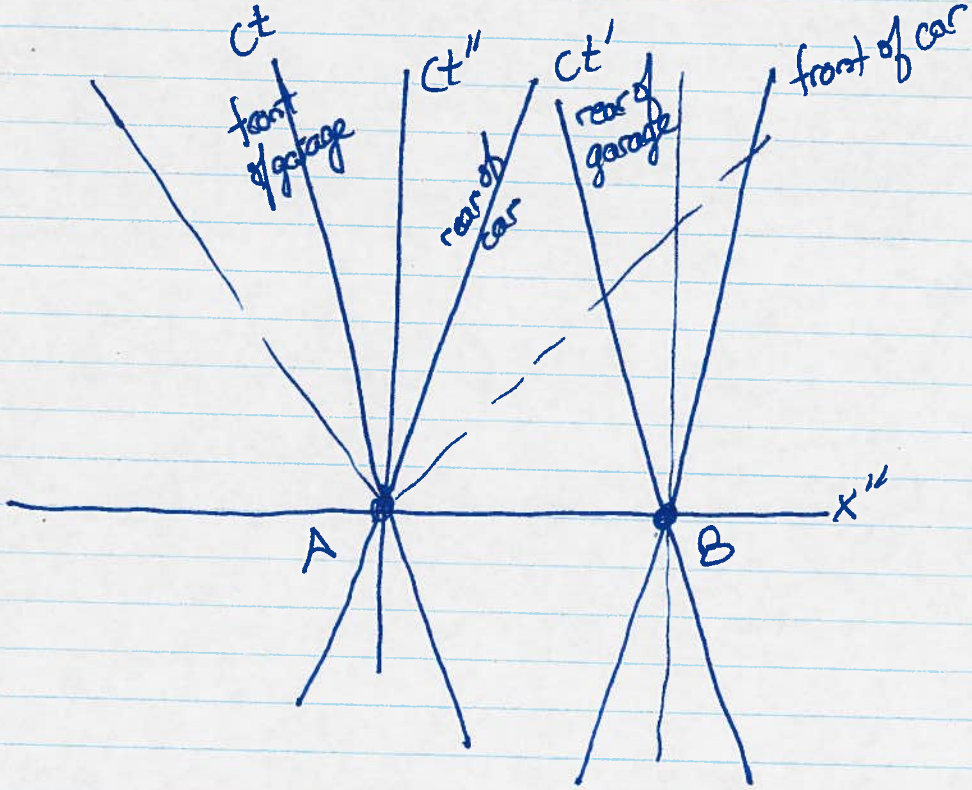
According to K' observers B occurs before A (car doesn't fit)



again $t_A' > t_B'$

$t_A < t_B$

Want intermediate frame - car + garage moves in opposite directions at same speed



Use velocity addition

u' = speed of car rel to garage

u'' = speed of car rel to K''

v = speed of K'' to garage

$$= \frac{u'' + v}{1 + \frac{u''v}{c^2}}$$

$u'' = v$

$$= \frac{2v}{1 + v^2/c^2}$$

solve for v

$\left(1 + \frac{v^2}{c^2}\right)\frac{u'}{c} = \frac{2v}{c}$ $\quad \frac{u'}{c} = \frac{4}{5}$ so $1 + \beta^2 - \frac{5\beta}{2} = 0$

$\beta = \frac{5}{4} \pm \left(\frac{25}{16} - 1\right)^{1/2} = \frac{5}{4} \pm \frac{3}{4} = \frac{1}{2}$ take $\beta < 1$ sol'n

Problem set 3

11.14 a) $F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix}$ $F_{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix}$

$$F^{\alpha\beta} F_{\alpha\beta} = F^{00} F_{00} + F^{01} F_{01} + \dots$$

$$= 0 + (-E_x^2) + (-E_y^2) \dots$$

$$= 2(B^2 - E^2)$$

$\mathcal{F}^{\alpha\beta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ & 0 & E_z & -E_y \\ & & 0 & E_x \\ & & & 0 \end{pmatrix}$ $\mathcal{F}_{\alpha\beta} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ & \cancel{E_z} + E_z & -E_y \\ & & 0 & +E_x \\ & & & 0 \end{pmatrix}$

$$\mathcal{F}^{\alpha\beta} F_{\alpha\beta} = -E_x B_x + E_y B_y + \dots \quad \dots \quad -4\vec{E}\cdot\vec{B}$$

These are the only invariants $\left(\mathcal{F}^{\alpha\beta} \mathcal{F}_{\alpha\beta} = 2(E^2 - B^2) = -F^{\alpha\beta} F_{\alpha\beta} \right)$

b) If there exists a frame where $\vec{E} = 0$ then

$$\vec{E} \cdot \vec{B} = 0 \quad + \quad B^2 - E^2 > 0$$

i.e. $\vec{E} \perp \vec{B}$ in all frames and $|\vec{B}| > |\vec{E}|$

c) $G^{\alpha\beta} = \begin{pmatrix} 0 & -D_x & -D_y & -D_z \\ & 0 & -H_z & H_y \\ & & 0 & -H_x \\ & & & 0 \end{pmatrix}$

$$G^{\alpha\beta} G_{\alpha\beta} = 2\left(H^2 - D^2\right)$$

$$G^{\alpha\beta} \mathcal{G}_{\alpha\beta} = -4\vec{D}\cdot\vec{H}$$

$$G^{\alpha\beta} F_{\alpha\beta} = 2\left(\vec{H}\cdot\vec{B} - \vec{E}\cdot\vec{D}\right)$$

$$G^{\alpha\beta} \mathcal{F}_{\alpha\beta} = -2\vec{D}\cdot\vec{B} + 2\vec{E}\cdot\vec{H} = -2\left(\vec{B}\cdot\vec{B} + \vec{E}\cdot\vec{H}\right)$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} = \frac{\gamma^2 - 1}{\gamma^2}$$

$$\frac{\gamma\beta^2}{\gamma+1} = 1 = \frac{\gamma(\gamma^2-1)}{\gamma+1} = \gamma(\gamma-1) = 1$$

11.16 a) Want expression that reduces to $\vec{J}' = \sigma \vec{E}'$ in rest frame of fluid (NR limit)

$$J^\alpha = \frac{\sigma}{c} F^{\alpha\beta} U_\beta \quad \text{would seem to work.}$$

however $$U_\alpha J^\alpha = \frac{\sigma}{c} F^{\alpha\beta} U_\alpha U_\beta$$

but RHS is zero $\left(F^{\alpha\beta} = -F^{\beta\alpha}\right)$ but LHS is not necessarily zero.

Can include term $A\, U_\beta J^\beta U^\alpha$ as 4-vector

since $U_\alpha U^\alpha = c^2$, set $A = -\frac{1}{c^2}$ + we have desired expression

$$J^\alpha - \frac{1}{c^2} U_\beta J^\beta U^\alpha = \frac{\sigma}{c} F^{\alpha\beta} U_\beta$$

b) 0^{th} comp.

$$① \quad c\rho - \frac{1}{c^2}\gamma^2\left(c\rho - \vec{v}\cdot\vec{J}\right)c = \frac{\sigma}{c}\vec{E}\cdot\vec{v}\,\gamma$$

vector part $$② \quad \vec{J} - \frac{1}{c^2}\gamma^2\left(c\rho - \vec{v}\cdot\vec{J}\right)\vec{v} = \frac{\sigma}{c}\vec{v}\times\vec{B}\,\gamma + \frac{\sigma}{c}\gamma\vec{E}c$$

Take $\vec{v}\times ① - c\times ②$

$$c\vec{v}\rho - c\vec{J} = \frac{\sigma}{c}\gamma\left[\left(\vec{v}\cdot\vec{E}\right)\vec{v} - c\vec{v}\times\vec{B} - c^2\vec{E}\right]$$

or $\quad \vec{J} = \rho\vec{v} + \gamma\sigma\left[\vec{E} + \vec{\beta}\times\vec{B} - (\vec{\beta}\cdot\vec{E})\vec{\beta}\right]$

c) $\quad \rho' = 0$

But J^α is a 4-vector so

$$\rho' = \gamma\left(\rho - \frac{\vec{\beta}}{c}\cdot\vec{J}\right) \quad \text{so} \quad \rho = \frac{1}{c}\vec{\beta}\cdot\vec{J}$$

$$\vec{J} = \gamma\sigma\left(\vec{E} + \vec{\beta}\times\vec{B} - \vec{\beta}(\vec{\beta}\cdot\vec{E})\right) + \frac{1}{c}(\vec{\beta}\cdot\vec{J})\vec{v}$$

Can simplify

$$\vec{\beta}\cdot\vec{J} = \gamma\sigma\left(\vec{\beta}\cdot\vec{E} - \beta^2(\vec{\beta}\cdot\vec{E})\right) + \beta^2(\vec{\beta}\cdot\vec{J})$$

$$\Rightarrow \quad \vec{\beta}\cdot\vec{J} = \gamma\sigma(\vec{\beta}\cdot\vec{E})$$

$$\Rightarrow \quad \boxed{\vec{J} = \gamma\sigma\left(\vec{E} + \vec{\beta}\times\vec{B}\right)}$$

Problem Set 5

12.15 $(\nabla^2 - \mu^2)\vec{A} = -\frac{4\pi}{c}\vec{J}$ $(\nabla^2-\mu^2)\Phi = -4\pi\rho$

for a point charge $\rho = q\delta^3(r)$ and solution is

$$\Phi = q\frac{e^{-\mu r}}{r}$$

essentially a Green's fn solution for $(\nabla^2-\mu^2)G = -4\pi\delta(\vec{r})$

so $$\vec{A}(\vec{x}) = \frac{1}{c}\int d^3x'\,\vec{J}(\vec{x}')\,\frac{e^{-\mu|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

$$\vec{J}(\vec{x}') = c\,\vec{\nabla}'\times\vec{m}f(\vec{x}') = -\vec{m}\times\vec{\nabla}'f(\vec{x}')$$

$$\vec{A}(\vec{x}) = -\vec{m}\times\int d^3x'\,\frac{e^{-\mu|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\,\vec{\nabla}'f(\vec{x}')$$

integrate by parts. then use $\vec{\nabla}'\frac{e^{-\mu|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} = -\vec{\nabla}\frac{e^{-\mu|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$

then $\vec{\nabla}$ comes outside integral

$$\vec{A}(\vec{x}) = -\vec{m}\times\vec{\nabla}\int d^3x'\,f(\vec{x}')\,\frac{e^{-\mu|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

$$f(\vec{x}\,') = \delta(\vec{x}\,')$$

$$\vec{A}(\vec{x}) = -\vec{m} \times \vec{\nabla} \frac{e^{-\mu|\vec{x}|}}{|\vec{x}|}$$

$$= -\hat{m} \times \hat{r}\left[\frac{\partial}{\partial r}\left(\frac{e^{-\mu r}}{r}\right)\right] = +\hat{m} \times \hat{r} f(r)$$

$$f(r) = \left(\frac{\mu}{r} + \frac{1}{r^2}\right) e^{-\mu r}$$

$$\vec{\nabla} \times \vec{A} = \vec{B} = \vec{\nabla} \times \left(\vec{m} \times \hat{r} f(r)\right)$$

$$= \hat{m}\left(\vec{\nabla} \cdot \hat{r} f(r)\right) - \left(\hat{m} \cdot \vec{\nabla}\right) \hat{r} f(r)$$

$$= \hat{m}\left(\left(\vec{\nabla} \cdot \hat{r}\right) f + \hat{r} \cdot \vec{\nabla} f\right) - \frac{f}{r}\left[\hat{m} - \hat{r}(\hat{m} \cdot \hat{r})\right]$$

$$- \hat{r}(\hat{m} \cdot \hat{r}) \frac{\partial f}{\partial r}$$

$$\vec{\nabla} \cdot \hat{r} = \vec{\nabla} \cdot \frac{\vec{r}}{r} = \frac{3}{r} - \frac{1}{r} = \frac{2}{r} \qquad (\hat{r} \cdot \vec{\nabla}) f = \frac{\partial f}{\partial r}$$

$$\vec{B} = \hat{m}\left(\frac{2f}{r} + \frac{\partial f}{\partial r}\right) - \frac{f}{r}\left(\hat{m} - \hat{r}(\hat{m} \cdot \hat{r})\right) + \hat{r}(\hat{m} \cdot \hat{r}) \frac{\partial f}{\partial r}$$

$$\frac{\partial f}{\partial r} = \left(-\frac{\mu}{r^2} - \frac{2}{r^3} - \frac{\mu^2 r}{r}\right) e^{-\mu r}$$

$$\vec{B} = \hat{m}\left(\frac{f}{r} + \frac{\partial f}{\partial r}\right) + \hat{r}(\hat{r}\cdot\hat{m})\left(\frac{f}{r} - \frac{\partial f}{\partial r}\right)$$

$$\frac{\partial f}{\partial r} = -\frac{\mu}{r^2} - \frac{2}{r^3} - \frac{\mu^2}{r} - \frac{\mu}{r^2} = -\frac{1}{r^3}\left(2\mu r + 2 + \mu^2 r^2\right) e^{-\mu r}$$

$$\frac{f}{r} = \frac{1}{r^3}\left(\mu r + 1\right) e^{-\mu r}$$

$$B = \frac{\vec{m}}{r^3} e^{-\mu r}\left(-\mu r - 1 - \mu^2 r^2\right) + \hat{r}(\hat{r}\cdot\vec{m})\frac{e^{-\mu r}}{r^3}\left(3\mu r + 3 + \mu^2 r^2\right)$$

$$= \left(3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}\right)\left(1 + \mu r + \frac{\mu^2 r^2}{3}\right) - \frac{2}{3}\mu^2 \vec{m}\frac{e^{-\mu r}}{r}$$

$$u = r_{min}/r \qquad dr = -r_{min}\,du/u^2$$

$$\Delta W = \frac{4}{3}\frac{z^2e^2}{m^2c^3}\left(\frac{m}{2}\right)^{1/2}\frac{z^2Z^2e^4}{(zZe^2)^{1/2}}\frac{1}{r_{min}^{5/2}}\int_0^1 \frac{u^2\,du}{(1-u)^{1/2}}$$

now $$\frac{zZe^2}{r_{min}} = \frac{1}{2}mv_0^2 \qquad r_{min} = \frac{2zZe^2}{mv_0^2}$$

$$\Delta W = \frac{4}{3}\times\frac{\cancel{16}^{\,8}}{15}\times\frac{1}{\cancel{2^{1/2}}}\frac{1}{\cancel{2^{5/2}}}\frac{z^2\cancel{e^2}}{\cancel{m^2}c^3}m^{1/2}\frac{\cancel{z^2Z^2}e^4}{(zZ\cancel{e^2})^{1/2}}\frac{m^{5/2}v_0^5}{(zZ\cancel{e^2})^{5/2}}$$

$$= \frac{8}{45}\frac{zmv_0^5}{Zc^3}$$

14.5 a) Larmor formula

$$P = \frac{2}{3}\frac{z^2e^2}{c^3}|\dot{\vec{v}}|^2$$

$$\dot{\vec{v}} = \frac{1}{m}\frac{dV}{dr}\hat{r} \quad \text{so} \quad P = \frac{2}{3}\frac{e^2}{m^2c^3}\left|\frac{dV}{dr}\right|^2$$

$$\Delta W = 2\int_{r_{min}}^{\infty} dr\,\frac{dt}{dr}\frac{dW}{dt} \quad \text{where} \quad P = \frac{dW}{dt} \quad \text{and}$$

$$\frac{m}{2}\left(\frac{dr}{dt}\right)^2 = V(r_{min}) - V(r)$$

or $$\frac{dr}{dt} = \left(\frac{2}{m}\right)^{1/2}\left(V(r_{min}) - V(r)\right)^{1/2}$$

so $$\Delta W = \frac{4}{3}\frac{z^2e^2}{m^2c^3}\left(\frac{m}{2}\right)^{1/2}\int_{r_{min}}^{\infty}\left|\frac{dV}{dr}\right|^2\frac{dr}{\left(V(r_{min})-V(r)\right)^{1/2}}$$

for $$V(r) = \frac{zZe^2}{r} \qquad V' = -\frac{zZe^2}{r^2}$$

$$\Delta W = \frac{4}{3}\frac{z^2e^2}{m^2c^3}\left(\frac{m}{2}\right)^{1/2}\frac{z^2Z^2e^4}{(zZe^2)^{1/2}}\int_{r_{min}}^{\infty}\frac{dr}{r^4\left(\frac{1}{r_{min}} - \frac{1}{r}\right)^{1/2}}$$

$$\mathcal{L}_1 - \mathcal{L}_2 = -\frac{1}{8\pi}\partial_\alpha A_\beta \partial^\beta A^\alpha$$

sine $\mathcal{L}_2 \equiv -\frac{1}{16\pi}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{c}J_\alpha A^\alpha$

$$= -\frac{1}{8\pi}\left(\partial_\alpha A_\beta \partial^\alpha A^\beta - \partial_\alpha A_\beta \partial^\beta A^\alpha\right) - \frac{1}{c}J_\alpha A^\alpha$$

and $\mathcal{L}_1 = -\frac{1}{8\pi}\partial_\alpha A_\beta \partial^\alpha A^\beta - \frac{1}{c}J_\alpha A^\alpha$

$$\partial_\alpha A_\beta \partial^\beta A^\alpha = \partial_\alpha\left(A_\beta \partial^\beta A^\alpha\right) - A_\beta \partial^\beta \partial_\alpha A^\alpha$$

$$= \partial_\alpha K^\alpha \quad \text{in Lorentz gauge}$$

Since $A = \int \mathcal{L}\, d^4x$

we can use divergence theorem

$$A' = \int\left(\mathcal{L} + \partial_\alpha K^\alpha\right)d^4x = \int \mathcal{L}\, d^4x + \cancelto{0}{\oint K^\alpha n_\alpha d\Sigma}$$

$$\mathcal{L} = -\frac{1}{8\pi} \partial_\alpha A_\beta \partial^\alpha A^\beta - \frac{1}{c} J_\alpha A^\alpha$$

$$= -\frac{1}{8\pi} g_{\alpha\mu} g_{\beta\eta} \partial^\mu A^\eta \partial^\alpha A^\beta - \frac{1}{c} J_\alpha A^\alpha$$

$$\frac{\partial \mathcal{L}}{\partial \partial^\kappa A^\lambda} = -\frac{1}{8\pi} \left(\delta^\mu_\kappa \delta^\eta_\lambda \partial^\alpha A^\beta + \delta^\alpha_\kappa \delta^\beta_\lambda \partial^\mu A^\eta \right) g_{\alpha\mu} g_{\beta\eta}$$

$$= -\frac{1}{8\pi} \left(g_{\kappa\alpha} g_{\lambda\beta} \partial^\alpha A^\beta + g_{\mu\kappa} g_{\eta\lambda} \partial^\mu A^\eta \right)$$

$$= -\frac{1}{4\pi} \partial_\kappa A_\lambda$$

$$\frac{\partial \mathcal{L}}{\partial A^\lambda} = -\frac{1}{c} J_\lambda$$

so we have $\partial^\kappa \partial_\kappa A_\lambda = -\frac{4\pi}{c} J_\lambda$

valid in Lorentz gauge

$$P = \frac{2}{3}\frac{z^2e^2}{m^2c^3}\left(\left(\frac{d\vec{p}}{d\tau}\right)^2 - \left(\frac{dE}{d\tau}\right)^2\right)$$

$$\frac{dE}{d\tau} = \gamma z e \vec{E}\cdot\vec{v} \qquad \vec{E} = \frac{Ze}{((vt)^2+b^2)^{3/2}}(vt, b, 0)$$

$$= \gamma Z z e^2 v^2 \frac{t}{((vt)^2+b^2)^{3/2}}$$

$$\frac{d\vec{p}}{d\tau} = \gamma z e\vec{E} = \gamma z Z e^2 \frac{(vt, b, 0)}{((vt)^2+b^2)^{3/2}}$$

$$P = \frac{2}{3}\frac{z^4Z^2e^6}{m^2c^3}\gamma^2\left(\frac{1}{((vt)^2+b^2)^2} - \beta^2\frac{v^2t^2}{((vt)^2+b^2)^3}\right)$$

$$W = \frac{4}{3}\frac{z^4Z^2e^6}{m^2c^3}\frac{\gamma^2}{v}\left(\int\frac{du}{(u^2+b^2)^2} - \beta^2\int\frac{u^2du}{(u+b^2)^3}\right)$$

$$= \frac{\pi}{12}\frac{z^4Z^2e^6}{m^2c^4}\frac{1}{\beta}\gamma^2\underbrace{\left(4-\beta^2\right)}_{=3\gamma^2+1}$$

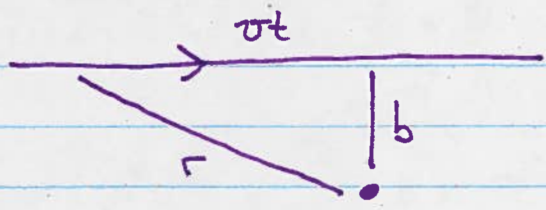
b) set $b = \frac{2zZe^2}{mv_0^2}$

$$\Delta W = \frac{\pi}{3} \frac{z^{\cancel{4}\,2}\cancel{Z^2}\cancel{e^6}}{\cancel{m^2}c^3\cancel{v_0}} \frac{m^{\cancel{3}}v_0^{\cancel{6}\,5}}{8\cancel{z^3}Z^{\cancel{3}}\cancel{e^6}}$$

$$= \frac{\pi}{8\times 3} \frac{z}{Z} \frac{mv_0^5}{c^3}$$

dimensionally the same off by numerical factors which is perhaps not surprising.

14.7



$$r = \left((vt)^2 + b^2\right)^{1/2}$$

$$a = \left(\frac{zZe^2}{(vt)^2 + b^2}\right)\frac{1}{m}$$

$$P = \frac{2}{3}\frac{z^2e^2}{m^2c^3}\frac{z^2Z^2e^4}{[(v_0t)^2+b^2]^2} = \frac{2}{3}\frac{z^4e^6Z^2}{m^2c^3}\frac{1}{v_0^2t^2+b^2}$$

$$\Delta W = 2\int_0^\infty P\,dt = \frac{4}{3}\frac{z^4Z^2e^6}{m^2c^3}\int_0^\infty \frac{dt}{[v_0^2t^2+b^2]^2}$$

integral is $\displaystyle \frac{1}{v_0}\int_0^\infty \frac{du}{[u^2+b^2]^2}$ $\qquad u = v_0 t$

$$= \frac{\pi}{4v_0b^3}$$

$$\Delta W = \frac{\pi}{3}\frac{z^4Z^2e^6}{m^2c^3v_0}\frac{1}{b^3}$$