PHYS 832 Problem Set #3 Due: March 17, 2015

1. Jackson 12.2 b) Show explicitly that gauge transformation $A^\alpha \to A^\alpha + \partial^\alpha \Lambda$ of the potentials in the charged-particle Lagrangian (for a charged particle moving in an external EM field) generates another equivalent Lagrangian.

2. Jackson 12.18 Prove, by means of the divergence theorem in four dimensions or otherwise, that for source-free electromagnetic fields confined to a finite region of space, the 3-space integrals of Θ^{00} and Θ^{0i} transform as the components of a constant 4-vector, as implied by (12.106).

$$\int T^{00} d^3x = \frac{1}{2\mu_0}\int \left(E^2 + B^2\right) d^3x = Energy_{field}$$

$$\int T^{0i} d^3x = \frac{1}{\mu_0}\int \left(\vec{E}\times\vec{B}\right)_i d^3x = cP^i_{field}$$

3. Show that the 4-vector form for the Liénard-Wiechert potential discussed in class can be written in more familiar form for a source point charge q at $\mathbf{r}_s$ with velocity $\mathbf{v}_s$:

$$\phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(1-\vec{n}\cdot\vec{\beta}_s)\left|\vec{r}-\vec{r}_s\right|}\right)_{t_r}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0 c}{4\pi}\left(\frac{q\vec{\beta}_s}{(1-\vec{n}\cdot\vec{\beta}_s)\left|\vec{r}-\vec{r}_s\right|}\right)_{t_r}$$

where $\vec{\beta}_s = \frac{v_s(t)}{c}$, and $\vec{n} = \frac{\vec{r}-\vec{r}_s}{\left|\vec{r}-\vec{r}_s\right|}$