PHYS 832 Problem Set #4 Due: April 9, 2015

1. Jackson 14.4 Using the Liénard-Wiechert fields, discuss the time-averaged power radiated per unit solid angle in non-relativistic motion of a particle with charge *e*, moving
   a) along the z-axis with instantaneous position $z(t) = a \cos \omega_0 t$,
   b) in a circle of radius R in the x-y plane with constant angular frequency ω_0.
   Sketch the angular distribution of the radiation and determine the total power radiated in each case.

2. Jackson 14.26 Consider the synchrotron radiation from the Crab nebula. Electrons with energies up to 10^{13} eV move in a magnetic field of the order of 10^{-4} gauss.
   a) For $E = 10^{13}$ eV, B = 3 x 10−4 gauss, calculate the orbit radius ρ, the fundamental frequency $\omega_0 = c/\rho$, and the critical frequency ω_c. What is the energy hbar ω_c in keV?
   b) Show that for a relativistic electron of energy E in a constant magnetic field the power spectrum of synchrotron radiation can be written
   $$P(E,\omega) = \text{const}\left(\frac{\omega}{E^2}\right)^{1/3} f\left(\frac{\omega}{\omega_c}\right)$$
   where f(x) is a cutoff function having the value unity at x=0 and vanishing rapidly for x >> 1 [e.g., $f \cong \exp(-\omega/\omega_c)$], and $\omega_c = (3/2)(eB/m)(E/mc^2) \cos\theta$, where θ is the pitch angle of the helical path.
   c) If electrons are distributed in energy according to the spectrum
   $$N(E)dE \propto E^{-n}\, dE$$
   show that the synchrotron radiation has the power spectrum
   $$\langle P(\omega)\rangle\, d\omega \propto \omega^{-\alpha}\, d\omega$$
   where $\alpha = (n - 1)/2$.
3. We won't have time to cover Thomson scattering in the lectures, so I want you to study it from Dr. Widrow's notes and from Jackson. Then, consider Thomson scattering of EM waves by free electrons. Unpolarized EM radiation incident from the –x axis direction on a free electron and unpolarized EM radiation incident from the +y axis direction on the same free electron gets scattered to an observer in the +z direction (i.e. 90 degree scattering). Then, show that if the intensity ratio of the two sources is, say, 3:1 (i.e. radiation from the –x direction is 3 times more intense than the radiation from the +y direction), the result is the observer in the +z direction sees *polarized* radiation. That's how Thomson scattering polarizes the CMB (cosmic microwave background), by "encoding" quadrupole anisotropy as linear polarization.