Maxwell's Equations

Lorentz force

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$ Gauss's Law

$$\vec{\nabla}\cdot\vec{B} = 0$$ no magnetic monopoles

$\boxed{\text{Gaussian units}}$

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Faraday's induction

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

Ampère's Law + Maxwell displacement current

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$\boxed{\text{SI units}}$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

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in media $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$c^2 = \frac{1}{\mu_0\epsilon_0}$$

$\vec{D}$ "electric displacement field"

$\vec{H}$ "magnetic field"

$\vec{B}$ "magnetic flux density"

↳ magnetic field

B-field, H-field

on units:

(Jackson Appendix)

$$F_1 = k_1\frac{qq'}{r^2}$$ → defines dimension and unit for $\vec{E}$

$$\frac{dF_2}{dl} = 2k_2\frac{II'}{d}$$ $\frac{\mu_0 II'}{2\pi d}$

↑ convenience

$\boxed{\frac{k_1}{k_2} \text{ units } \frac{l^2}{t^2}}$

can show $\frac{k_1}{k_2} = c^2$ ← electromagnetism & relativity

above is empirical

$$B = 2k_2\alpha\frac{I}{d}$$ → defines dimension and unit for $\vec{B}$

$\boxed{\frac{E}{B} \text{ units } \frac{l}{t\alpha}}$

$$\vec{\nabla}\times\vec{E} + k_3\frac{\partial\vec{B}}{\partial t} = 0$$

after defining $\vec{E}$ and $\vec{B}$ units, how do they relate to each other (Faraday's induction)

can show $k_3 = 1/\alpha$

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{x}}{|\vec{x}|^3}$$

Ampère's Law

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I$$

Gaussian units

$k_1 = 1$; $k_2 = 1/c^2$; $\alpha = c$; $k_3 = \frac{1}{c}$ $\rightarrow$ E/B ~~has~~ is dimensionless ("same units")

SI units

$k_1 = \frac{1}{4\pi\epsilon_0}$; $k_2 = \frac{\mu_0}{4\pi}$; $\alpha = 1$; $k_3 = 1$ $\rightarrow$ E/B has units $\frac{m}{s}$

$$[\text{statvolt}/\text{cm}] \equiv [\text{gauss}]$$

$$[10^{-4}\ \text{T}]$$

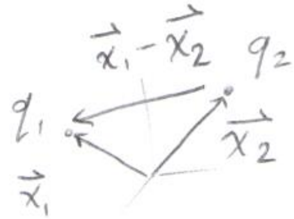
$$[\text{statvolt}] = [299.792\ \text{V}] = \frac{c}{10^6}\ \text{V}$$

in m/s

Electrostatics $\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \vec{\nabla}\times\vec{E} = 0$

$\vec{E}, \rho$ time independent

$\vec{J}, \vec{B}$ everywhere 0



$$\underset{\text{due to 2}}{\vec{F}_{\text{on } 1}} = k\, q_1 q_2 \frac{\vec{x}_1 - \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|^3} \qquad k = \frac{1}{4\pi\epsilon_0} \quad \boxed{\text{SI}}$$

electric field $\quad \underset{\text{due to 2}}{\vec{E}(\vec{x})} = k\, q_2 \frac{\vec{x} - \vec{x}_2}{|\vec{x} - \vec{x}_2|^3}$

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} q_i \frac{\vec{x} - \vec{x}_i}{|\vec{x} - \vec{x}_i|^3}$$

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(x') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3}\, d^3x'$$

$\rho(\vec{x})$ charge density

for point charge at $\vec{x} = \vec{x}_1 \quad \rho(\vec{x}) = q\,\delta(\vec{x} - \vec{x}_1)$

Dirac delta function

$$\frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} = -\vec{\nabla} \frac{1}{|\vec{x} - \vec{x}'|}$$

$|\vec{x} - \vec{x}'| = (x^2 + x'^2 - 2\,\vec{x}\cdot\vec{x}')^{1/2}$

$\vec{\nabla}|\vec{x} - \vec{x}'|$ acts on $\vec{x}$ $= \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$

see Larry's notes or $\quad$ or $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\hat{r}$

$\vec{\nabla}\times\vec{E} = 0 \quad$ curl free

conservative vector field

Scalar potential

$\vec{E} = -\vec{\nabla}\Phi$, then $\vec{\nabla}\cdot\vec{\nabla}\Phi = -\frac{\rho}{\epsilon_0}$

$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}$ Poisson equation

charge-free region $\quad \nabla^2\Phi = 0$ Laplace equation

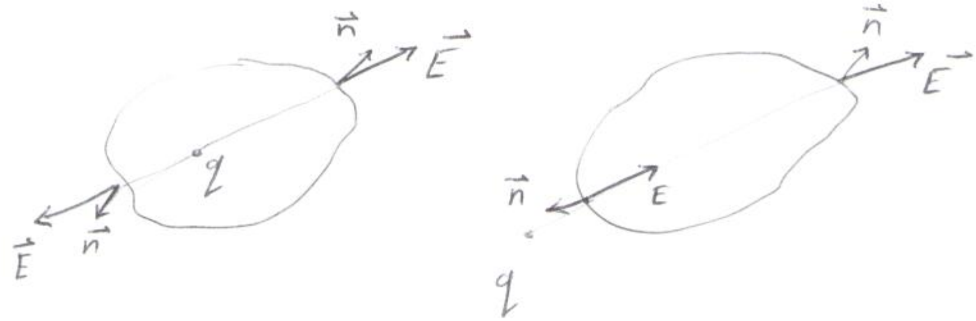
Aside

* divergence theorem

$$\oint_S \vec{E} \cdot \vec{n}\, da = \int_V \vec{\nabla} \cdot \vec{E}\, d^3x \qquad \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

Gauss's Law $\quad \oint_S \vec{E} \cdot \vec{n}\, da = \frac{1}{\epsilon_0} \int_V \rho(\vec{x})\, d^3x$

$$d\Omega = \frac{da \cos\theta}{r^2}$$



$$\vec{E} \cdot \vec{n}\, da = \frac{q}{4\pi\epsilon_0} \frac{\cos\theta\, da}{r^2} = \frac{q}{4\pi\epsilon_0} d\Omega$$

$\int d\Omega = 4\pi$ if q is inside

$\int d\Omega = 0$ if q is outside

from linearity it follows

continuing... $\quad \vec{E}(\vec{x}) = \frac{-1}{4\pi\epsilon_0} \vec{\nabla} \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$

so $\quad \Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$

work done moving charge from A to B

$$W = -\int_A^B \vec{F} \cdot d\vec{l} = -q \int_A^B \vec{E} \cdot d\vec{l} = q \int_A^B \vec{\nabla}\Phi \cdot d\vec{l} = q \int_A^B d\Phi$$

$$= q(\Phi_B - \Phi_A)$$

$$\int_A^B \vec{E} \cdot d\vec{l} = -(\Phi_B - \Phi_A)$$

$$\therefore \oint \vec{E} \cdot d\vec{l} = 0$$

* Stokes's Theorem : $\quad \oint_C \vec{E} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{E}) \cdot \hat{n}\, da$