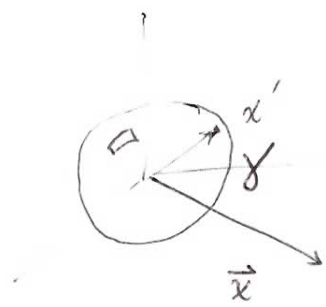
Solution of Laplace equation outside a sphere with potential $\Phi(a, \theta', \phi')$

$$\Phi(\vec{x}) = \frac{1}{4\pi} \int \Phi(a, \theta', \phi') \frac{a(x^2 - a^2)}{(x^2 + a^2 - 2ax\cos\gamma)^{3/2}} d\Omega'$$



$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta \sin\theta' \cos(\phi - \phi')$$

if $\Phi(a) = V$,

$$\frac{Va}{4\pi} \int \frac{(x^2 - a^2)\, d\cos\theta'\, d\phi'}{(x^2 + a^2 - 2ax[\quad])^{3/2}}$$

Jackson 2.4 Point charge is placed distance $d > R$ from centre of an equally charged, isolated conducting sphere of radius R.

(a) Inside what distance from the surface of the sphere is the point charge attracted rather than repelled?

(b) Limiting value of F when the point charge is located a distance $a = d - R$, if $a \ll R$?

Jackson 2.4 a) discussed in "tutorial"

conductor: $\vec{E}$ is zero inside, same Φ inside, $\vec{E}$ is normal to surface, Φ discontinuity due to surface charge density

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{|\vec{x}-\vec{y}|} - \frac{Rq}{d\,|\vec{x}-\frac{R^2}{d^2}\vec{y}|} + \frac{q\left(1+\frac{R}{d}\right)}{|\vec{x}|}\right]$$

- start with grounded, conducting sphere
- bring q near, charge flows and sphere becomes $-\frac{a q}{y}$
- remove ground wire, then charge sphere with $Q + \frac{a q}{y}$ to bring to total charge Q

(in this problem Q is q ... Then use Coulomb's force law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} q^2 \left(\underbrace{\frac{-R/d}{\left[d - \left(\frac{R}{d}\right)^2 d\right]^2}}_{\text{attractive}} + \underbrace{\frac{1+R/d}{d^2}}_{\text{repulsive}} \right)\hat{r}$$

far from sphere $R/d \to 0$: repulsive

when $\frac{R}{d} \to 1$: force becomes attractive like method of images, charge near conductor dominant

solve for R/d when above $= 0$