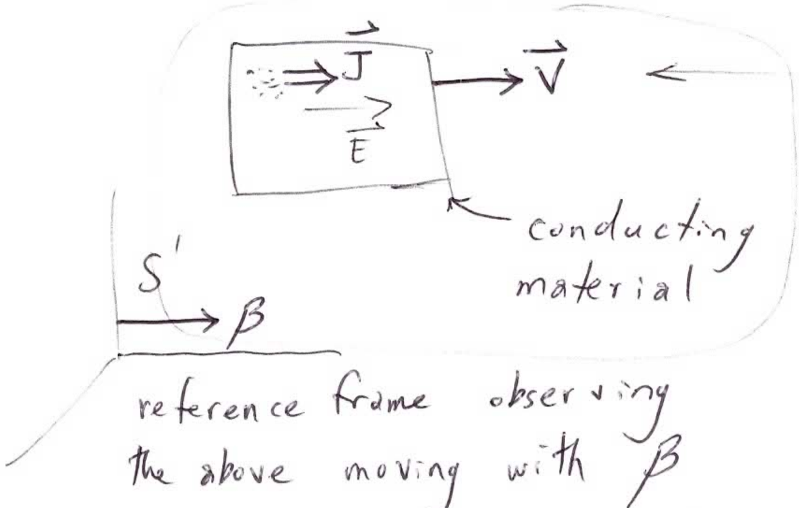
Jackson 11.14 b) possible to have pure $\vec{E}$ in one frame be seen as pure $\vec{B}$ field in another frame?

$$F_{\mu\nu} F^{\mu\nu} = 2(B^2 - E^2/c^2) \quad \text{Lorentz invariant scalar!}$$

in pure $\vec{E}$ field frame < 0

" " $\vec{B}$ " " > 0

Jackson 11.16 covariant Ohm's Law $\vec{J} = \sigma \vec{E}$



The material could be moving

4-velocity of the conductor

$$U^\mu = (\gamma c, \gamma \vec{v})$$

sol$^{\underline{n}}$ go to reference frame where conductor is at rest, $\gamma = 1$

$$U'^\mu = (c, 0)$$

we note: $F'^{\mu\nu} U'_\nu = (0, \vec{E}')$

right-hand side of Ohm's Law could be $\sigma F'^{\mu\nu} U'_\nu$

however J'^μ has charge density zero component

$J'^\mu = (c\rho', \vec{J}') \Rightarrow$ want to relate this to $\sigma F'^{\mu\nu} U'_\nu = (0, \vec{E}')$

note: $J'^\nu U'_\nu = c^2 \rho'$ (Lorentz scalar) subtract this from J'^μ

$$J'^\mu - \frac{J'^\nu U'_\nu \, U'^\mu}{c^2} = \sigma F'^{\mu\nu} U'_\nu$$

$$(c\rho', \vec{J}') - \left[\frac{c^2\rho'}{c^2}\right](c, 0) = (0, \vec{J}')$$

!! is Ohm's Law

and that's in covariant form, valid in all frames (so don't need to specify being in the S' rest frame of the conducting material)

$$\boxed{J^\mu - \frac{1}{c^2}(J^\nu U_\nu)U^\mu = \sigma F^{\mu\nu} U_\nu}$$

$$J^\mu = \underbrace{\sigma F^{\mu\nu} U_\nu}_{\text{conduction}} + \underbrace{\frac{1}{c^2}(J^\nu U_\nu)U^\mu}_{\text{convection}}$$

$c^2 \rho'$ it's charge density

velocity times velocity i.e. convection

recall Lorentz force law

$$\frac{dp^\mu}{c\, d\tau} = q F^{\mu\nu} U_\nu$$

covariant Ohm's Law is like Lorentz force

$F^{\mu\nu} U_\nu$ includes current induced by moving conducting material in $\vec{B}$ field (Hall effect)

$F^{\mu\nu} U_\nu$

$q(\vec{E} + \vec{v} \times \vec{B})$