if $\vec{\beta} = 0$, just Coulomb's Law

if $\dot{\vec{\beta}} = 0$, near-field term $1/R^2$

when radiation field exists, dominates over near field far from source

as $\beta \to 1$ (for $\dot{\vec{\beta}} = 0$) field "bunches" (as studied earlier)

this term

... this is a retardation effect

Consider accelerating particle where $\beta << 1$

Then $$E_a = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e}{c}\left[\frac{\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})}{R}\right]_{ret}$$

$\hat{n}$ determined at retarded time

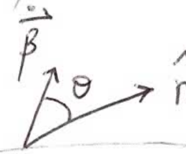
instantaneous energy flux Poynting vector

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0}\left[\vec{E}_a\times\left(\frac{\hat{n}\times\vec{E}_a}{c}\right)\right] = \frac{1}{\mu_0 c}|\vec{E}_a|^2\,\hat{n}$$

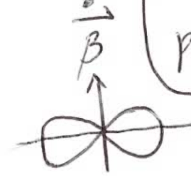
power radiated per unit solid angle

$$\frac{dP}{d\Omega} = \frac{1}{\mu_0 c}|\vec{E}_a|^2 R^2 = \frac{1}{\mu_0 c}\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{e^2}{c^2}\left|\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})\right|^2$$

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\left(\frac{1}{4\pi\epsilon_0}\right)\left|\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})\right|^2$$



$$\left|\hat{n}\times(\hat{n}\times\dot{\vec{\beta}})\right|^2 = \dot{\beta}^2\sin^2\theta$$



dipole pattern

$$\boxed{\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{|\dot{\vec{v}}|^2}{c^2}\sin^2\theta}$$

radiation polarized in plane containing $\dot{\vec{\beta}}$ and $\hat{n}$

$\int d\Omega$ total power $$\boxed{P = \frac{e^2}{2c^3}\left(\frac{1}{4\pi\epsilon_0}\right)|\dot{\vec{v}}|^2\left(\frac{4}{3}\right) = \frac{e^2|\dot{\vec{v}}|^2}{6\pi\epsilon_0 c^3}}$$

$2\pi\int$ ~~[illegible]~~ $d\cos\theta$

Larmor formula non-relativistic, accel. charge relativistic generalization of Larmor formula

P is Lorentz invariant (radiated energy per unit time zeroth component 4-vector) :

$$\Theta^{\mu\nu} = \begin{pmatrix} u & \vec{S}/c \\ \vec{S}/c & T_{ij} \end{pmatrix}$$

$$u = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} + B^2\right) = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$$

$$\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0}$$

$$\partial_\mu \Theta^{\mu\nu} = 0$$

conservation of energy and momentum

$$\partial_\mu \Theta^{\mu 0} = \frac{1}{c}\left(\frac{\partial}{\partial t}u + \vec{\nabla}\cdot\vec{S}\right) = 0$$

$$\frac{d}{dt}\int d^3x\, u = -\int d^3x\, \vec{\nabla}\cdot\vec{S}$$

$$= -\oint da\, \hat{n}\cdot\vec{S}$$

$$= -P$$

$$\therefore \int P\,dt = \int_{\text{init}} d^3x\, u - \int_{\text{final}} d^3x\, u$$

u is T^{00} time-time component, transforms like $A^0 dt$ some 4-vector, not A^μ

$$\therefore u\, d^3x = A^0\, dt\, d^3x = \underbrace{A^0\, d^4x}_{\text{transforms as time-component 4-vector}}$$

$= P\,dt$ $\therefore$ P must be Lorentz scalar

non-relativistic Larmor

$$P = \left[\frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{1}{m^2}\right)\right]\left(\frac{d\vec{p}}{dt}\cdot\frac{d\vec{p}}{dt}\right)$$

$$= \left[\qquad\right]\left(-\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right)\right)$$

$$\boxed{\text{here using } p^\mu = \left(\frac{E}{c}, \vec{p}\right)}$$

energy not $\vec{E}$ field

$$E^2 = p^2c^2 + m^2c^4$$

$$E\,dE = c^2 p\,dp$$

$$dE = c^2 \frac{p}{E} dp$$

$$dE = c\beta\, dp$$

note: $$-\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau} = \left(\frac{d\vec{p}}{d\tau}\right)^2 - \frac{1}{c^2}\left(\frac{dE}{d\tau}\right)^2 =$$

$$= \left(\frac{d\vec{p}}{d\tau}\right)^2 - \beta^2\left(\frac{dp}{d\tau}\right)^2 = (1-\beta^2)\left(\frac{d\vec{p}}{d\tau}\right)^2$$

$$P = \left[\frac{e^2}{6\pi\epsilon_0 c^3 m^2}\right]\left(\frac{d\vec{p}}{d\tau}\cdot\frac{d\vec{p}}{d\tau} - \beta^2\left(\frac{dp}{d\tau}\right)^2\right)$$

using $E = \gamma mc^2$, $\vec{p} = \gamma m\vec{v}$

$$\left[\frac{e^2}{6\pi\epsilon_0 c}\gamma^6\right]\left[\left(\dot{\vec{\beta}}\right)^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right]$$

Liénard result