Cherenkov Radiation

Gaussian units

starting with (as for density effect)

$$\Phi(\vec{k},\omega) = \frac{2ze}{\epsilon(\omega)} \frac{\delta(\omega - \vec{k}\cdot\vec{v})}{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}$$

$$\vec{A}(\vec{k},\omega) = \epsilon(\omega)\frac{\vec{v}}{c}\Phi(\vec{k},\omega)$$

$$\vec{E}(\vec{k},\omega) = i\left[\frac{\omega\,\epsilon(\omega)}{c}\frac{\vec{v}}{c} - \vec{k}\right]\Phi(\vec{k},\omega)$$

$$\vec{B}(\vec{k},\omega) = i\,\epsilon(\omega)\,\vec{k}\times\frac{\vec{v}}{c}\Phi(\vec{k},\omega)$$

$$\vec{E}(\omega) = \frac{1}{(2\pi)^{3/2}}\int d^3k\, \vec{E}(\vec{k},\omega)\, e^{i\vec{k}\cdot\vec{x}} \qquad \vec{x} = (0, b, 0)$$

$\int dk_1$ done by $\delta(\omega - k_1 v)$

$$E_1(\omega) = \frac{ize\omega}{v^2}\left(\frac{2}{\pi}\right)^{1/2}\left[\frac{1}{\epsilon(\omega)} - \beta^2\right]K_0(\lambda b)$$

$$E_2(\omega) = \frac{ze}{v}\left(\frac{2}{\pi}\right)^{1/2}\frac{\lambda}{\epsilon(\omega)}K_1(\lambda b)$$

$$B_3(\omega) = \epsilon(\omega)\,\beta\, E_2(\omega)$$

modified Bessel fns

for density effect, evaluated $|\lambda a| << 1$ with a atomic dimensions smaller than b impact parameter

~~but opposite limit~~: $\cancel{|\lambda a| >> 1}$

related to above, with $|\lambda b| >> 1$, then modified Bessel fns asymptotic form

$$E_1(\omega, b) = \frac{ize\omega}{c^2}\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]\frac{e^{-\lambda b}}{\sqrt{\lambda b}}$$

$$E_2(\omega, b) = \frac{ze}{v\,\epsilon(\omega)}\sqrt{\frac{\lambda}{b}}\,e^{-\lambda b}$$

$$B_3(\omega, b) = \beta\,\epsilon(\omega)\,E_2(\omega, b)$$

recall $\lambda^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\epsilon(\omega) = \frac{\omega^2}{v^2}\left[1 - \beta^2\epsilon(\omega)\right]$

note: λ is not wavelength

dimensions $[m^{-1}]$

$e^{-\lambda b}$ ← impact parameter in $[m]$

λ^2 is actually like k^2

$E_1(\omega, b)$ and $E_2(\omega, b)$ goes $e^{-\lambda b}$ exponentially decaying unless λ is pure imaginary

then $e^{-i\_ b}$ is a wave that propagates as far as you want to consider b

λ is pure imaginary if $1 - \beta^2\epsilon(\omega) < 0$

and if $\epsilon(\omega)$ is pure real (or mostly real, small imaginary component)

So what is $\epsilon(\omega)$? ← relative permittivity (Gaussian units) or dielectric constant

note: charged particle through medium not accelerating

$\beta^2\epsilon(\omega) > 1$

$v > \frac{c}{\sqrt{\epsilon(\omega)}}$

$n = \sqrt{\epsilon\mu}$ ← μ usually 1

$v > \frac{c}{n}$ ← phase velocity of EM fields in the medium

$\epsilon(\omega)$ is ~~real~~ complex in general; imaginary part is absorption "complex index of refraction"

← function of ω

Cherenkov radiation generated @ ω where medium is transparent

Equation from 13.37

$$\left(\frac{dE}{dx}\right)_{b>a} = -ca\,Re\int_0^\infty B_3^*(\omega)\,E_1(\omega)\,d\omega$$

$$\alpha = \frac{e^2}{\hbar c}, \text{ for } z = 1$$

$$\frac{dN}{dx\,d\omega} = \frac{\alpha}{c}\left[1 - \frac{1}{\beta^2 n^2(\omega)}\right]$$

m s^{-1}

becomes

$$\left(\frac{dE}{dx}\right)_{rad} = \frac{(ze)^2}{c^2}\int_{\epsilon(\omega)>(1/\beta^2)} \omega\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]d\omega$$

gives the frequency spectrum of Cherenkov radiation (Frank-Tamm result, 1937)

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for range ~~frequencies~~ (wavelengths) where $\epsilon(\omega) \simeq$ const.

$\frac{\text{energy loss in Cherenkov}}{d\omega}$ goes as ω

per unit frequency

$\frac{\#\text{ of photons}}{d\omega}$ goes as const.

energy of photon is $\hbar\omega$

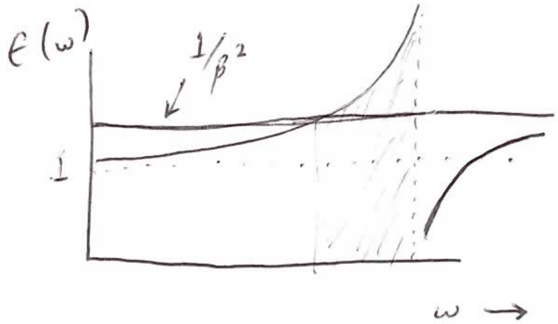
* Cherenkov spectrum (# of photons) is flat in ω in regions where $\epsilon(\omega) \simeq$ const.

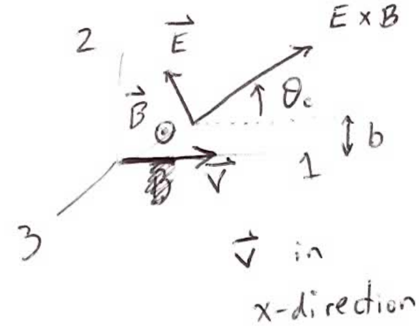
$$\frac{\omega\lambda}{2\pi} = c \;;\quad \omega = \frac{c}{2\pi}\frac{1}{\lambda} \;;\quad \frac{d\omega}{d\lambda} = -\frac{c}{2\pi}\frac{1}{\lambda^2}$$

$$\frac{\#\text{ of photons}}{d\lambda} = \frac{\#\text{ of photons}}{d\omega}\frac{d\omega}{d\lambda} \text{ goes as } \boxed{\frac{1}{\lambda^2}}$$

Cherenkov light is blue or UV (i.e. more emitted at short λ)

[also because many media $\epsilon(\omega)\uparrow$ as $\lambda\downarrow$ gives more Cherenkov photons at low λ]





direction of propagation is $\vec{E} \times \vec{B}$

E_1
E_2
B_3

$$\tan \theta_c = -\frac{E_1}{E_2}$$

then using far fields from above can show

$$\cos \theta_c = \frac{1}{\beta \sqrt{\epsilon(\omega)}}$$

$\beta^2 \epsilon(\omega) > 1$ also gives physical angle for $\cos \theta_c < 1$

Cherenkov radiation is 100% linearly polarized in plane $\vec{v}$ and direction of propagation of radiation

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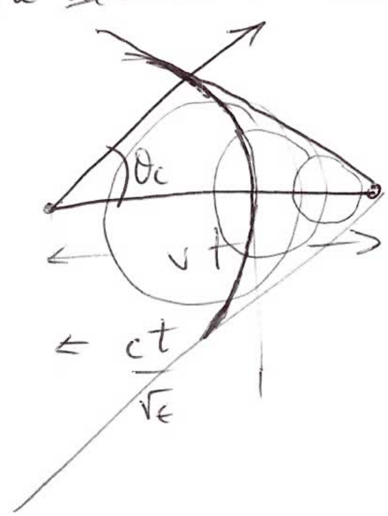
same angle ~~derived~~ illustrated using standard Huygens construction



"shock" wave front moving in direction given by Cherenkov angle θ_c