PHYS 832 Final Exam Thursday, April 23, 2015

Take-home exam. Duration: 24 hours. You may use Jackson and one other EM textbook. You may consult your written course notes and the course webpage. You may not use the Internet for anything else except looking up "ordinary" math such as integrals or trig identities (that's OK), or looking up the value of a fundamental constant (that's OK too).

Honour Code: by writing your name on your answers that you hand in, you are declaring that you have abided by the instructions specified above. Good luck!

1. (7 points) Because this is an open-book, take-home exam, you get an "essay" question. Explain to me (showing that you understand fully, by writing down words and a few key equations and formulae) the Lagrangian for the electromagnetic field. Start by writing down the Lagrangian density (you don't have to derive it or copy the derivation in Jackson). Explain every term. Then, describe what you do with it to get "equations of motion". What are the equations of motion? How do you use the equations of motion (i.e. describe an approach you can use to solve them) when you have the general case of time-varying potentials and you want to determine the behavior of the EM field (i.e. how the field evolves with time, at all points in space, hence its "motion").

2. (5 points) a) Cherenkov radiation is generated by a 7 GeV (total energy) proton travelling through water (n = 1.33, over the relevant wavelengths in this problem). How many Cherenkov photons are emitted per centimeter of path, between 400 nm and 600 nm? How much energy does this correspond to? Use $m_p = 1\ \mathrm{GeV}/c^2$ for the proton rest mass.

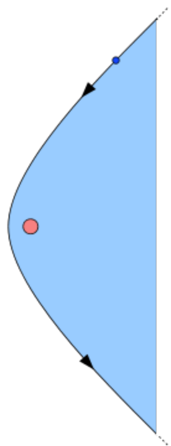
   (5 points) b) Calculate the collisional energy loss per centimeter of path (meaning the instantaneous *dE/dx*) for the same 7 GeV proton in water. A useful quantity is the mean excitation potential of H_2O: $\hbar\langle\omega\rangle = 75\,\mathrm{eV}$.
   Be sure to state what approximations you used and/or what effects you could ignore. Also note: the *m* in the energy loss formula (from my notes) is the electron rest mass, and not the heavy charged particle's rest mass. Also, feel free to look up the classical electron radius on the Internet to get its value (in m, or cm) and use that to help your calculation.

   (1 point) How does the energy loss from Cherenkov radiation compare to the *dE/dx* energy loss (collisional)?

3. (5 points) A point charge q crosses the origin at t = 0, moving with: $\boldsymbol{v}(t) = \beta c\,\hat{\boldsymbol{x}}$. At spatial coordinate (3,4,0) [in meters] and at time ct = 5 m, what is the scalar and vector potential Φ and $\mathbf{A}$ for the EM field caused by the moving point charge?

(1 point) For the problem above, at the specified coordinate and time, does the "radiation field" dominate or does the "Lorentz-transformed Coulomb field" dominate, if you wanted to determine **E** and **B** fields? Note: don't spend time evaluating **E** and **B**!

4. (5 points) A relativistic electron (charge –e) incident on an atomic nucleus (charge +Z) is accelerating towards the nucleus (being attracted to it), makes its closest approach (at a distance b), and then flies off, moving on a hyperbolic path. Don't worry too much about the details of the hyperbolic path.



At the point of closest approach, the electron has total energy E. What is the total instantaneous power – write the formula using known constants and terms that can be derived from variables that have been given – radiated by the electron at this point of closest approach?

5. (3 points) Starting with the EM stress-energy tensor (symmetric), explain how the energy density of EM field (at x^α) can change with time (i.e. increase or decrease) if there is a source current density coupling to the field.

6. (5 points) a) Consider a charge e moving along the z-axis: $z(t) = a\cos(\omega_0 t)$, just like in Problem Set #4, only this time it is relativistic motion. Derive the expression for the angular distribution of the instantaneous power radiated per unit solid angle: $dP(t)/d\Omega$.

   (3 points) b) For $\omega_0 t = \pi/4$, sketch the angular distribution of the radiation for large γ, and find the angle at which the radiated power is maximum, θ_{max}.

7. (3 points) Cosmic-ray electrons in the galaxy, with energy 5 GeV, can produce synchrotron radiation as they spiral around the galactic magnetic field. There is a distribution of electron energies, of course, and a full radio spectrum of synchrotron emission that has been measured; but, let's simplify this problem and consider just 5 GeV electrons producing a frequency spectrum of synchrotron radiation that has an upper cutoff at 4.2 GHz. Use this information to estimate the strength of the galactic magnetic field (in tesla, or in gauss).