DEPARTMENT OF MATHEMATICS AND STATISTICS
Summer 2026 USRA/USSRF Research Projects

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Students selected must be available to work on campus for the duration of the award (May 1-August 31) for 35 hours a week.

Project Title: Halanay Inequalities and Pinning Control
Supervisor: KEXUE ZHANG

Project Description: This project investigates the use of Halanay-type differential inequalities to analyze stability properties of large-scale and networked dynamical systems under pinning control, a strategy in which only a small subset of nodes is directly controlled to influence the behaviour of the entire network. Halanay-type inequalities provide powerful tools for studying systems with time delays, impulses, or hybrid behaviours, allowing stability and convergence rates to be established even when classical Lyapunov methods become difficult to apply. The project will develop and explore new formulations of Halanay-type inequalities tailored to networked systems, derive conditions under which pinning control ensures synchronization or stabilization, and apply the results to examples such as consensus networks, oscillator systems, or power-grid–inspired models.

Student's Role in the Project:

- Review key literature on Halanay-type inequalities and pinning control.
- Work with the supervisor to formulate simplified models that capture key features of network dynamics.
- Apply generalized Halanay-type inequalities to derive stability or convergence conditions for pinned networks.
- Perform MATLAB/Python simulations to illustrate the theoretical results.
- Prepare a short research report or poster summarizing findings for presentation at an undergraduate research event or departmental seminar.

Prerequisites (Required Background/Level of Study):

- A solid background in ordinary differential equations and linear algebra.
- Some experience with MATLAB or Python for numerical simulations.
- Familiarity with dynamical systems, control theory, or graph/network theory is an asset, though not required.

Project Title: Bayesian Inference and Computational Methods in Biomarker Study
Supervisor: WENYU JIANG

Project Description: Develop Bayesian Markov Chain Monte Carlo (MCMC) methods for analyzing extended regression models, where covariate effects may be different among subgroups of individuals identified by different biomarker characteristics.

Student's Role in the Project: Develop new MCMC computational methods utilizing Bayesian inference, for subgroup identification, and for regression models with heterogeneous subgroup-specific covariate effects. Program the proposed computational methods; evaluate the methods in simulation; illustrate the use of the methods by application to a real data set. Possibly develop an R package. Writing a report for research development.

Prerequisites (required background/level of study): Stat 361, 362, 463 are required. Stat 486 and Stat 462 are highly recommended. Proficiency in R and statistical programming.

Project Title: Mathematics of Reinforcement Learning in Non-Markovian Environments
Supervisor: SERDAR YUKSEL

Project Description: For many stochastic decision and control problems, an exact model of a system being studied is not available; and even when an approximate or exact model is available, the information at the decision maker with regard to the state being controlled is often limited and partial. Data-driven learning is an appropriate framework for such settings. The general goal of this project is to study the optimal design of decision/control policies in such stochastic systems with incomplete models and partial information via reinforcement or empirical learning methods. Depending on student interest, we will focus on one of the following, closely related, three main themes:

- (i) Optimal stochastic control with partial information: Non-linear filtering, approximations, and relations with reinforcement learning

- (ii) Multi-agent systems, stochastic teams, and games: Arriving at optimality/equilibrium under decentralized information

- (iii) Sample complexity in such learning algorithms

Student's Role in the Project: The expectation is that the student would study a wide variety of resources that will be provided, take part in regular research discussions through frequent meetings, and write a mathematically rigorous technical document at the end of the summer.

Prerequisites (required background/level of study): A strong foundation and interest in probability and analysis.

Project Title: Generative models for statistical inference
Supervisor: YANGLEI SONG

Project Description: Deep generative models have achieved remarkable success in producing high-quality images, text, and other complex data, and in recent years they have also been applied to classical statistical tasks such as conditional independence testing, distribution regression, and sufficient dimension reduction. Empirical and theoretical studies suggest that these models can still support valid statistical inference while in some cases improving performance, for example by increasing power. This project will review the relevant literature, examine how generative models can be incorporated into statistical methodology, and identify potential opportunities for further development.

Student's Role in the Project: The student will explore the key literature and discuss findings in regular meetings with the supervisor, implement and compare several existing methods, and write a final report.

Prerequisites (required background/level of study): A strong background in probability and statistics (STAT 353 or equivalent) and familiarity with at least one programming language, preferably Python.

Project Title: Statistical and AI-Based Uncertainty Quantification of Adversary Emulation in Cyber Ranges
Supervisor: DEVON LIN

Project Description: A Cyber Range is a simulated, virtual environment used for cybersecurity training, testing, and research. They are widely used to evaluate cybersecurity defenses by simulating realistic attacks in controlled environments. However, conclusions drawn from cyber range experiments are often based on a small number of runs and deterministic summaries, with limited assessment of variability, uncertainty, or model reliability. MITRE CALDERA is an open-source adversary emulation platform that automates cyber attacks using the MITRE ATT&CK framework and provides a unique opportunity for statistically principled experimentation.

This NSERC USRA project adopts a statistics and AI perspective to study CALDERA-generated attacks in a cyber range. The project integrates experimental design, uncertainty quantification (UQ), and statistical learning. The student will design reproducible CALDERA experiments, quantify variability across repeated attack executions, and develop AI-based models to detect or characterize adversary behavior. A central focus is understanding how uncertainty affects both experimental conclusions and AI model performance.

Student's Role in the Project:

- Design and conduct reproducible cyber range experiments using MITRE CALDERA

- Execute repeated adversary emulation runs under controlled and systematically varied conditions
- Collect, clean, and organize CALDERA outputs into analyzable datasets
- Apply statistical methods to quantify variability and uncertainty across runs
- Fit and evaluate predictive statistical learning models to characterize attack behavior
- Interpret results and communicate findings through written reports and presentations

Prerequisites (required background/level of study): A strong background in probability and statistics and familiarity with Python.

Project Title: Mean Field Pontryagin Maximum Principle for a mesoscopic traffic model
Supervisor: MARIA TERESA CHIRI

Project Description: The Bando Follow-the-Leader model is a microscopic traffic model in which each vehicle adapts its acceleration based on the distance to the vehicle ahead, a desired speed determined by this distance, and the relative velocity with respect to the leader. Vehicles are assumed to move along a single lane in the same direction, with each driver reacting to the behavior of the immediately preceding vehicle. The model incorporates an optimal velocity rule, prescribing how drivers adjust their speed depending on the available space ahead. This mechanism captures realistic traffic phenomena such as congestion and stop-and-go waves. The evolution of each vehicle is described in terms of its position and velocity, with acceleration driven by a relaxation toward the optimal velocity and by interaction with the leading vehicle. The project considers a mixed traffic scenario in which vehicles are divided into human-driven vehicles and autonomous vehicles.

While human-driven vehicles follow the standard interaction rules of the model, autonomous vehicles are assumed to have an additional controlled component in their acceleration, which can be designed to improve overall traffic performance. The main objective of the project is to study a limiting regime in which the number of human-driven vehicles becomes very large, while only finitely many autonomous vehicles are present. In this limit, the microscopic traffic description converges to a coupled system in which a continuum model for human-driven traffic is interacting with a finite-dimensional dynamics describing the autonomous vehicles. Within this framework, the project aims to formulate and analyze an optimal control problem for the autonomous vehicles. Given a suitable cost functional measuring the global traffic behavior, the goal is to derive necessary conditions for optimality of the control strategies, in the form of a Pontryagin-type maximum principle for the coupled continuum–discrete system.

Student's Role in the Project: The student will first become familiar with the Bando Follow-the Leader model and with basic concepts of car-following dynamics and optimal velocity models. They will then study the limiting procedure leading from the microscopic FtL system to the coupled PDE–ODE description in the regime of infinitely many human-driven vehicles and finitely many autonomous vehicles. A central part of the project will consist in analyzing the optimal control problem associated with the limiting model. Under appropriate assumptions, the student will derive first-order necessary conditions for optimality, formulating a Pontryagin maximum principle for the controlled autonomous vehicles. Depending on the student's background and interests, the project may also include analytical investigations of the resulting optimality system or numerical experiments illustrating the effect of the control on traffic flow.

Prerequisites (required background/level of study): A solid background in undergraduate calculus and ordinary differential equations is required. Basic familiarity with partial differential equations, functional analysis, or optimal control is helpful but not mandatory. The project is suitable for advanced undergraduate in mathematics or applied mathematics with an interest in PDEs, dynamical systems, and control theory.

Project Title: Boundary stabilization of non-local reaction diffusion equations
Supervisor: MARIA TERESA CHIRI

Project Description: Reaction–diffusion equations are a fundamental modeling tool for spatially distributed processes in areas such as physics, biology, and ecology. They describe the combined effect of spatial movement and local interactions, capturing how a quantity evolves over time and space. From the perspective of control theory, a key challenge is boundary stabilization, namely the design of controls acting at the boundaries of the spatial domain in order to drive the system toward a desired equilibrium and ensure its stability. While boundary stabilization is well understood for reaction–diffusion equations with local nonlinear interactions, much less is known when interactions are non-local and depend on the global spatial distribution of the system. This project focuses on boundary stabilization problems for reaction–diffusion equations of Fisher–KPP type with non-local reaction terms. The objective is to understand how non-local effects influence the qualitative behavior of solutions and the controllability properties of the system. In the non-local setting, interactions are described through spatial kernels that couple the state of the system at different locations. This leads to stationary problems that depend explicitly on space, making classical analytical techniques ineffective and leaving many fundamental properties of the solutions still poorly understood.

Student's Role in the Project: The student will first acquire familiarity with reaction–diffusion equations and with the classical Fisher–KPP model, focusing on basic well-posedness and qualitative properties of solutions. They will then study boundary control and stabilization techniques for parabolic equations, with particular attention to boundary feedback laws and Lyapunov-based methods. A main objective of the project is to analyze how non-local reaction terms affect stabilization strategies. The student will investigate the stationary problem associated with the non-local Fisher–KPP equation and explore the difficulties arising from the explicit spatial dependence introduced by the kernel. Depending on the student's background and interests, the project may include analytical results on stability or controllability, as well as numerical experiments illustrating the effect of boundary controls in the presence of non-local interactions.

Prerequisites (required background/level of study): A solid background in undergraduate calculus and ordinary differential equations is required. Familiarity with basic concepts from analysis and linear algebra is expected. Previous exposure to partial differential equations or control theory is welcome but not required. The project is suitable for advanced undergraduate students in mathematics with an interest in dynamical systems and applied analysis.

Project Title: q-numbers
Supervisors: IVAN DIMITROV, CHARLES PAQUETTE AND DAVID WEHLAU

Project Description: The idea of defining “deformations” of integers and binomial coefficients goes back to Euler (1760’s) and Gauss (early 1800’s). These early definitions were related to applications to analysis and number theory. For example, q-binomial coefficients count the subspaces of a vector space over a finite field. Because of this long history, it is surprising that until recently there has not been a reasonable definition of q-rational and q-real numbers. In 2020 S. Morier-Genoud and V. Ovsienko proposed a definition of q-rational numbers as certain rational functions of q and in 2022 they extended the definition to q-real numbers as certain (formal) Laurent series. S. Morier-Genoud, V. Ovsienko along with other mathematicians have proved a number of properties of q-numbers that suggest deep connections with other areas of mathematics (representation theory, geometry, etc.). However, the theory of q-numbers is still in its infancy and there are more open questions than affirmative results. Here are a few examples of open questions that we want to study:

(a) The order on q-rationals inherited from the the order on the rational numbers has an independent definition in terms of q-rationals only. More precisely, it can be defined through an order on the rational functions of q which respssent the q-rationals. Does a similar statement hold for the order on q-reals inherited from the order on the real numbers? In other words, is there an order on Laurent series that restricts to the order on q-reals?

(b) So far, operations (addition and multiplication) on the sets of q-rationals and q-reals have not been defined. Any progress in this direction will be a substantial achievement.

(c) For an irrational x, the corresponding q-version is defined as the limit of the q-versions of a sequence of rational numbers that converges to x. Even though this limit is in the q-adic topology of (formal) Laurent series, it turns out that it is actually a convergent series. The question of how the radius of convergence depends on x is an open question.

(d) Even more surprisingly, if x is a rational number, there are two possible limits of q-versions of sequences of rational numbers that converges to x. If the sequence approaches x from above, then the limit is q-version of x; however, if the sequence approaches x from below, the limit is a q-number strictly less than the q-version of x. In other words, we may think of each rational having two q-versions. Here is a question: what is in the gap between these two versions?

The goals of the project is twofold. First, we want to explore different open questions related to q-numbers, to form conjectures, and prove new results. Second, we want to explore how AI can be employed in mathematical research. Problems related to q-numbers offer many opportunities for experimentation and we intend to explore how AI systems can be employed in generating examples and forming conjectures.

Student's Role in the Project: Students will be expected to generate experimental data (including using appropriate AI systems). Based on the data, the group will work on forming conjectures and proving results about q-numbers. The format of the research will normally consist of two meeting per week of the entire group and daily collaboration between the two students involved in the project.

Prerequisites (required background/level of study): Background related to MATH 110, 120, 210, 281 will be used extensively. Also experience with AI and coding will be a plus.

Project Title: Partially observed Markov processes with imperfect/incomplete covariate information
Supervisor: FELICIA MAGPANTAY

Project Description: Mathematical modeling is a powerful tool for studying complex systems. It allows us to test the consistency of our hypotheses on the mechanisms governing the system's dynamics and determine if there are informative structures in noisy observations. A common issue that arises in modeling is incomplete and imperfect covariate information. For example, when fitting disease models to epidemiological time series, we may have missing data on vaccine coverage. Since such models are usually very sensitive to the vaccine coverage input, this unfortunately means that resulting forecasts can be inaccurate and lead to suboptimal distribution of resources by public health agencies. One way to work with this is to treat vaccine coverage as another type of observation to be fitted (instead of as an exact covariate for the model). This allows us to model errors in the measurements of these covariates and also diagnose shortcomings of the model. In this project we will explore various tools that can be used for this such as B-splines and stochastic bridges.

Student's Role in the Project: Students will begin by learning the theory behind partially observed Markov processes, how to implement inference methods numerically and high-performance computing using R. We will try to develop some algorithms to tackle the problem discussed above and test them on both simulated and real data.

Prerequisites (required background/level of study): Probability (STAT 268 or equivalent, STAT 353 preferred) and familiarity with R. Some exposure to stochastic processes (MTHE/MATH/STAT 455 or equivalent) would be an asset but not required.