QUEEN'S MATHEMATICAL COMMUNICATOR

Scheduling a Golf Tournament

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Dept. of Mathematics & Statistics
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"Scheduling A Golf Tournament: An Application Of Finite Geometry"

by Norman J. Pullman

In September 1983 a local printer telephoned the Department for help with a problem.

He and 15 friends were about to spend a week at a resort. They planned to play golf in 4 foursomes (sets of 4 players) per day for at most six days. Was it possible to arrange a schedule so that each golfer played in the same foursome at least once with each other person?

He tried scheduling "by hand" but wasn't able to manage it. He thought we could use our computers to help. Actually it's rather a large problem for a direct attack by computer. Fortunately, it has an immediate solution when translated into a problem in finite geometry. In fact, his tournament required only 5 days, not the 6 days he was willing to settle for. Moreover each golfer played in the same foursome exactly once with each other golfer.

In this paper I will try to give some idea of what finite geometry is about, how it's connected with finite algebra and how these came together to solve the golf problem (and similar scheduling problems). Some unsolved problems will also be mentioned.

Let's look at the printer's problem in more detail. Each foursome brings together $\binom{4}{2} = 6$ pairs of golfers. Ultimately we want each of the $\binom{16}{2} = 120$ pairs to have played in some foursome. But four foursomes play each day, so 24 pairs play daily. Therefore we require at least $(\frac{120}{24}) = 5$ days to satisfy the condition that each pair of golfers plays together in the same foursome at least once. Can we do it in five days? Then each pair will have played in some foursome exactly once.

Here is a related problem. Trivial Pursuit is a board game for up to six players. Suppose there are v people who want us to schedule a Trivial Pursuit tournament consisting of x games per day for y days. Each day's schedule is called a round. Let's assume that all v people want to play in each round and that in each round we allow exactly 6 people per board. The games are to be scheduled so that eventually every contestant has played against all of the others exactly once. For which values of v is this possible?

Another problem of the same kind: can a wine-testing be scheduled involving 35 bottles of wine and 7 wine tasters so that in each round of judging each taster is given 5 wines to compare; enough rounds to be scheduled so that each wine is compared to each other wine exactly once?

Norman Pullman gave the final talk of the 1984-85 Coleman-Ellis lectureships on March 12, 1985. This is a shortened version of his talk.

(b) All v players compete in each round; each player in precisely one game each round.
(c) Enough rounds are scheduled so that each contestant competes against every other contestant exactly once during the tournament.

Suppose a round-robin tournament can be scheduled. Let the number of rounds be y and the number of games per round be x. Conditions (a), (b) and (c) imply that $v = kx$ and $\binom{v}{2} = \binom{k}{2}xy$. Therefore $kx - (k-1)y = 1$.

The solutions to that equation consist of all pairs of integers (x,y) such that

$$(*) \quad x = k + z(k-1) \quad \text{and} \quad y = (k+1) + zk$$

where z is any non-negative integer.

In the case $k = 2$ (which covers a lot of contests: chess, boxing, singles tennis, duelling, etc.) the values of (x,y) given in (*) are also sufficient for scheduling a round-robin tournament. I'll leave it to you to see how to schedule a round-robin tournament on $2z + 4$ contestants playing $3 + 2z$ rounds, $2 + z$ games per round, 2 players per game - for each integer $z \geq 0$.

If we could schedule a round-robin tournament for Trivial Pursuit with 6 players per game, then the number of contestants $v = 6x$ while $x = 6 + 5z$ (for some integer $z \geq 0$). Therefore v would have to be $36 + 30z$. We'll come back to this later.

If we could schedule a wine-tasting using 5 tasters, then $k = 5$ so $v = 5x = 5(5+4z) = 25 + 20z$ for some integer $z \geq 0$. So it is not possible to schedule such wine-tasting for 35 wines under the required conditions.

Rather than pursue each case: $k = 3$, $k = 4$, etc. we will see what happens if $z = 0$ (equivalently, $v = k^2$) instead. This will provide a good idea of the difficulty of the problem and the simplicity of the solution to some of the problems such as the golfers'. We want to see if there is a way to construct a round-robin tournament schedule for k^2 contestants, $k \geq 2$ players per game. Let's introduce some geometry here.

An affine plane consists of a set of "points" P and a family $\mathbf{L}$ of subsets of P called "lines" that satisfy three axioms:

A1. There is a set of four points, no three of which are on the same line.
A2. Each pair of distinct points lies in exactly one line.
A3. For each line ℓ and point p, if p is not in ℓ then there is exactly one line ℓ' containing p that has no points in common with ℓ.

When a pair of lines have no points in common they are said to be parallel. As you can see, the Euclidean plane is an affine plane. You can verify that a round-robin tournament schedule for $v = k^2$ contestants ("points"), k players per game ("lines") is an affine plane. A "round" corresponds to a parallel pencil of lines. That is, a family of mutually parallel lines that contains all the points of an affine plane. It turns out (see e.g. [5], p. 371) that in every finite affine plane, every line has the same number of points. That number is called the order of the

Here is a general framework for the preceding examples. Suppose a certain sport requires k players per game and v people want to participate in a tournament for that sport. A round of a tournament consists of several games, no two of which involve the same players. If a round consists of x games, then kx people compete in that round because each game involves k players. For a tournament to be called a round-robin, three conditions must be met.

(a) Each game involves exactly k players $(v>k>1)$.
(b) All v players compete in each round; each player in precisely one game each round.
(c) Enough rounds are scheduled so that each contestant competes against every other contestant exactly once during the tournament.

Suppose a round-robin tournament can be scheduled. Let the number of rounds be y and the number of games per round be x . Conditions (a), (b) and (c) imply that $v = kx$ and $\binom{v}{2} = \binom{k}{2}xy$. Therefore $kx - (k-1)y = 1$.

The solutions to that equation consist of all pairs of integers (x,y) such that

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THEOREM 1. A round-robin tournament schedule for k^2 players, k to a game, exists if and only if there is an affine plane of order $k \geq 2$. []

But how can we construct an affine plane? Let's take a hint from the Euclidean plane. You learned to model it by taking the ordered pairs of real numbers as "points". The "lines" are the solution sets to the equations $y = mx + b$ and to the equation $x = c$. One parallel pencil of lines consists of the vertical lines (those with equations of the form $x = c$), the other parallel pencils are the lines with common slope m . If you take the trouble to verify that this system is an affine plane (that is, check that the three axioms hold), then you will find that the only algebraic properties of the real numbers that you use are the properties of any field.

This leads to a method of contructing some affine planes. Take a field F . The set of all its ordered pairs are the "points". The "lines" are the solution sets to $x = c$ and to $y = mx + b$. If F has k elements, then the affine plane will have k^2 points and so it will have order k .

Now you can see how easy it was to solve the golfers' problem. All that was needed was a field with 4 elements. As you may know, (see e.g. [1], p. 127), a field of k elements exists if and only if k is a power of a prime, e.g. 2, 3, 4, 5, 7, 8, 9, 11, ... So in general,

THEOREM 2. An affine plane of order k exists if k is a prime power.[]

Here are the addition and multiplication tables of a 4-element field.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

We construct the required 5 parallel pencils (rounds of golf) from these tables. For example, Thursday's round corresponds to the pencil $y = 2x + b$. This consists of the lines whose equations are: $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, $y = 2x + 3$. Each line $y = 2x + b$ corresponds to one of Thurday's foursomes. For instance, the line with equation $y = 2x + 1$ is $\{(0,1),(1,3),(2,2),(3,0)\}$, according to the tables. To present the printer with a cleaner schedule, I associated golfer 1 with (0,0), 2 with (0,1), ..., 16 with (3,3). So the second foursome on Thursday consists of golfers 2, 8, 11 and 13.

PENCIL (round)	EQUATIONS OF THE LINES IN THE PENCIL	POINTS ON THE LINE (players in the foursome)	LIST OF FOURSOMES for the day			
	x = 0	[00,01,02,03]	1	2	3	4
x = c	x = 1	[10,11,12,13]	5	6	7	8
(Monday)	x = 2	[20,21,22,23]	9	10	11	12
	x = 3	[30,31,32,33]	13	14	15	16
	y = 0	[00,10,20,30]	1	5	9	13
y = b	y = 1	[01,11,21,31]	2	6	10	14
(Tuesday)	y = 2	[02,12,22,32]	3	7	11	15
	y = 3	[03,13,23,33]	4	8	12	16
	y = x	[00,11,22,33]	1	6	11	16
y = x + b	y = x + 1	[01,10,23,32]	2	5	12	15
(Wednesday)	y = x + 2	[02,13,20,31]	3	8	9	14
	y = x + 3	[03,12,21,30]	4	7	10	13
	y = 2x	[00,12,23,31]	1	7	12	14
y = 2x + b	y = 2x + 1	[01,13,22,30]	2	8	11	13
(Thursday)	y = 2x + 2	[02,10,21,33]	3	5	10	16
	y = 2x + 3	[03,11,20,32]	4	6	9	15
	y = 3x	[00,13,21,32]	1	8	10	15
y = 3x + b	y = 3x + 1	[01,12,20,33]	2	7	9	16
(Friday)	y = 3x + 2	[02,11,23,30]	3	6	12	13
	y = 3x + 3	[03,10,22,31]	4	5	11	14

According to Theorems 1 and 2 we can schedule a round-robin tournament on k^2 players, k to a game ($k \geq 2$) if there is a field with k elements. Notice that Theorem 2 is an "if -" theorem, not "if and only if ".

To this day, no one knows if the converse to Theorem 2 is true. Is there an affine plane of order k for some k other than a prime power? No such number k has been found -- yet.

Returning to the Trivial Pursuit round-robin tournament on 36 players, 6 to a game. Can that be scheduled? If so then (according to Theorem 1) there would be an affine plane of order 6. Six is not a prime power so Theorem 2 doesn't help. Here is a theorem that will enable us to decide. It was discovered thirty-six years ago, see e.g. [5], p. 402, note Theorem 3 on p. 373.

THEOREM 3. (Bruck & Ryser)

If an affine plane of order k exists and $k \equiv 1$ or $2 \pmod 4$, then k is the sum of two squares. []

Since $6 \equiv 2 \pmod 4$ but $6 \neq a^2 + b^2$ for any integers a, b, it follows that there is no affine plane of order 6. So there is no way to schedule the 36-player Trivial Pursuit round-robin tournament.

Applying Theorems 2 and 3 we see that the first few integers k for which it is known that an affine plane of order k exists are 2, 3, 4, 5, 7, 8 and 9. These are all prime powers. Six was disqualified by Theorem 3. Ten isn't a prime power. But $10 \equiv 2 \pmod 4$ and $10 = 1^2 + 3^2$. So Theorem 3 doesn't exclude 10 as a possible order of an affine plane.

To this day, no one has constructed an affine plane of order 10 or shown that it cannot exist, although many people have worked on the problem.

We saw that there is no round-robin tournament on 36 players, 6 to a game. According to (*), the next possible value of v for a 6 person game is 66.

PROBLEM. Can a round-robin tournament involving 66 contestants 6 to a game, be scheduled (so that there are 11 games per round and 13 rounds) in which each contestant meets every other contestant exactly once?

REFERENCES

[1] Combinatorial Theory, by Marshall Hall, Jr., Blaisdell Publ. Co., New York, 1967.

[2] Topics on Tournaments by John W. Moon, Holt, Rinehart and Wilson, New York 1968.

[3] Combinatorial Mathematics, by Herbert John Ryser, Jr., The Carus Math. Monographs, No. 14, Math. Assoc. of America, publ. by John Wiley & Sons, Inc., New Jersey 1963.

[4] Projective Planes, by Daniel R. Hughes & Fred C. Piper, Springer-Verlag, Berlin 1972.

[5] Combinatorics: A First Course by Anne Penfold Street and Walter D. Wallis, Charles Babbage Research Centre, Winnipeg 1982.

Queen's University High School Mathematics Seminar Series 1985-86

This is a series of biweekly problem solving sessions for senior high school students which runs at Queen's University in the Fall and Spring. In part, the series is designed to provide enrichment material for motivated students, but only in part. A large part of its original conception was to effect a change in the student's conception of how mathematics is learned. The sessions are dedicated to the principle that to do mathematics you don't need to be given "the word", that you can find your own words, and in fact, that that's really the only way to understand what you're doing. Thus mathematics is viewed, not as content but as process, and the teacher is viewed not as authority but as resource.

Emphasis is on exploration of problems (one problem per session) by the participants in small groups with periodic reports of progress at the blackboard. The problems are carefully chosen to provide enjoyable and challenging activity for the students, and to expose them to valuable mathematical techniques and/or interesting mathematics.

The seminar meets Tuesday afternoons at 2-week intervals. In 1985/86 there will be 6 meetings in the Fall and 4 in the Spring. **The Fall dates are September 24, October 8, 22, November 5, 19, and December 3. The Spring dates are April 8, 22 and May 6, 20.**

The sessions are organized and run by **Peter Taylor.** The first session this year was attended by 40 students and teachers from Prescott, Brockville, Smith's Falls, Almonte, and Ernestown. The students were given the first four perfect numbers: 6, 28, 496, and 8128, and asked to find another. These four were almost certainly known by Euclid, but a fifth was not convincingly found for 1500 years after his time. [Regius in 1536 produced the fifth perfect number]. After a number of imaginative guesses they found it. It is 33, 550, 336. **Note:** A number is **perfect** if it is the sum of its proper divisors, e.g. $28 = 1 + 2 + 4 + 7 + 14$.

The Coleman Symposium by Bob Erdahl

In late August Queen's Mathematics and Chemistry Departments hosted an international conference on reduced density matrix and density functional theory. Seventy participants from over 15 different countries came to honour **John Coleman** for his contributions to reduced density matrix theory, one of the two inter-related topics of the Symposium. He formulated and studied some of the difficult mathematical problems associated with this theory.

The Conference was of interest to theoretical chemists, physicists and mathematicians who specialize in the theory of interacting quantum particles such as the electrons in an atom or in a solid, the protons and neutrons in the nucleus of an atom, the pions surrounding a pair of nucleons or even quarks. Thus the Conference was interdisciplinary. We had three theoretical chemists from mainland China and many solid state and nuclear physicists from Europe.

A reduced density matrix is a function of the coordinates of a pair of particles and with it a theorist can describe, for example, a representative pair of electrons which is surrounded by and interacting with a large number of other electrons. A reduced density matrix is the quantum mechanical analogue of a correlation function. Using such a function many of the physical properties of a system of many electrons can be described. But it is a difficult mathematical problem to give a useful characterization of such functions. Not any function will do. To be a reduced density matrix the function under consideration must, in the case of electrons, accurately reflect the Pauli-principle, named after the famous Swiss physicist Wolfgang Pauli. Pauli articulated the famous principle after postulating that quantum particles are indistinguishable - it makes no sense to talk about one particular quantum particle in a collection of identical particles. The electrons surrounding a nucleus are strongly attracted towards the nucleus by its large positive charge, but the Pauli-principle prevents them from condensing on the surface of the nucleus. The Pauli principle puts limitations on how close the electrons can approach one another. It is the indistinguishability of quantum particles which allows one to consider a representative pair of quantum particles and hence makes reduced density matrix theory possible. Pauli's principle is one of the cornerstones of quantum theory.

The Symposium was organized by Bob Erdahl of the Department of Mathematics & Statistics and Vedene Smith of the Chemistry Department. Close to 50 papers were delivered over a three day period. Many of these papers dealt with ideas which were pioneered by John Coleman.

Recent Appointments

David Pickard, a Harvard statistician has just been appointed to the Department of Mathematics and Statistics at Queen´s. David obtained his B.Sc. degree at Mount Allison University in 1967, followed by an M.Sc. at Stanford in ´71 and a Ph.D. at the Australian National University in ´77. For the past 8 years he has been teaching in the Statistics Department at Harvard. Accompanying him are his wife Dale, who brings along some welcome experience in mathematics education, and 2 children Damon and Darcy.

David has had a busy summer: besides buying a house and moving to Queen´s, he spoke at the Joshi Symposia at London, Ontario and gave an invited address on inference in image processing at the ASA/IMS meeting in Las Vegas.

Dan Norman has been appointed Acting Head of the Department for one year effective July 1, 1985. The present Head **Lorne Campbell** is enjoying a well-earned sabbatical year at the Statistical Laboratory in Cambridge (U.K.) until Christmas, and after that in Chapel Hill, North Carolina.

Joan Geramita is paying for her sabbatical last year in Boston by taking over as Chairwoman of Undergraduate Studies for a three year term. Joan has always been one of our most effective counsellors (measured by soundness and sound) and her skills will now be put to a definitive test.

Malcolm Griffin is beginning a one year appointment as senior statistician in **The George L. Edgett Statistical Laboratory** (Queen´s STATLAB for short). While STATLAB normally deals with statistics problems arising from on campus research, occasionally problems off campus are dealt with and he would be interested in hearing from Communicator readers about statistical problems (547-5935).

Statistics News

The non thesis Master´s degree in statistics is now in its third year. Although the course is turning out well-qualified students the statistics group is continuing to add courses which improve the offerings in applied statistics. The two most recent additions are:

STATISTICAL COMPUTING 862*: The use of the major statistical packages SAS, BMDP, GLIM and SPSS for a wide variety of statistical analyses and comparison of their areas of strength and weakness.

STATISTICAL CONSULTING 869*: This course provides students with a broad base of skills required for statistical consultants. The first third concentrates on interpersonal aspects - factors which effect the liklihood of the consultants advice being taken. The middle third deals with problem solving strategies while the last third involves videotaping and analyzing simulated consulting sessions.

For graduates with some mathematics either now involved in statistical consulting or wishing to begin, this program is an attractive one. We will be glad to hear from any prospective students.

Algorithms

by Mike Swain

The study of algorithms is part of computer science, so why should mathematicians be interested? Well, there are many problems in the subject which give interesting insights just like good math problems. A fair number of them do not require an initial preparation in computer science.

An algorithm is a method for carrying out a task. Algorithms are the problem-solving methods which result in computer programs. Algorithm design is carried out by many computer scientists, but there are a group who specialize in looking for new algorithms to pervasive problems, analysing their running time, and discovering relationships amongst them. These computer scientists are said to be in the field of algorithm design and analysis.

When analysing an algorithm for its running time it is often the case that you are interested only in its asymptotic properties, as the number describing the size of the input grows to very large values. That is the approach I will take here. This type of analysis is called ´big oh´ analysis and relies on the following definition:

Def. A function $f(n)$ is said to be $O(g(n))$ if there exists some number N such that for all $n > N$ $f(n) \leq Kg(n)$ for some constant K.

The function $f(n)$ used in this article will always be the worst case running time of the algorithm as a function of the measure n of the size of the input.

An elegant illustration of the power of algorithm design may be found in Jon Bentley´s "Programming Pearls" article from the Sept. 1984 issue of the Communications of the ACM (Association for Computing Machinery). He describes the following problem:

The Maximum Subvector Problem

You are given a vector X of N real numbers. The problem is to find an algorithm which gives as its output the maximum sum found in any contiguous subvector of the input.

If X contains only positive numbers the solution is simple: it is the sum of all the components of the vector. However if X contains negative numbers the negative numbers may outweigh positive entries around them.

Computer scientists have a cute term for an algorithm which solves a problem in the most straight-forward but time-consuming way. They call such an algorithm ´naive´. A naive algorithm which correctly solves the maximum subector problem goes like this: For each pair of integers L and U, $1 \leq L \leq U \leq N$ compute the sum $X(L)+...+X(U)$, which I will denote $sum(L,U)$. Then simply find the maximum of all of these computations, by looking at all L between 1 and N, and for each of these at all U between L and N, and for each of these computing $sum(L,U)$.

It is easy to show that the three nested loops we get result in a running time that is N^3. That is, in the worst case the algorithm would take at most KN^3, with N being the number of elements in the vector and K some constant independent of N. There are two simple ways that this could be made $O(N^2)$ by cutting out one of the loops. They are quite different in their approach. One involves noticing that

sum(L,U) = sum(L,U-1) + X(U) . An $O(N^2)$ solution that takes advantage of this is to go through the L and U loops as above, but to carry along the current value of sum(L,U) and update it with the above formula as U increases, keeping track of the max sum obtained so far. The other approach is somewhat different and uses the identity sum(U,L) = sum(1,L) - sum(1,U-1) . I leave it to the reader.

Finally, a linear time algorithm, one that is O(N) . A mathematician would see this immediately, right? Only those slow computer scientists have to go through all the prolog I went through above!? In fact it was a statistician who came up with the algorithm, reportedly in one minute.

This algorithm scans the array from 1 to N keeping track of the largest subector seen so far. It uses the following two assertions.

If we let
max_so_far(U) = the sum of the max. subvector in the range 1,U
and max_end(U) = the sum of the max. subvector ending with element U
then:
max_so_far(U) = max(max_so_far(U-1), max_end(U)) (1)
and max_end(U) = max(max_end(U-1) + X(U), 0) (2)

Number (1) is true because the maximum subvector in the range [1,U] either lies completely within [1,U-1] , in which case it is max_so_far(U-1) , or ends with position U , in which case it is max_end(U) .

To prove number (2): if the maximum subvector ending with element U is non-trivial then it must have sum ≥ 0 . I will show by contradiction that its sum is max_end(U-1) + X(U) .

The maximum subvector without its last element is a subvector ending in position U-1 . Suppose that is had sum less than max_end(U-1) . Then the vector with first elements with sum max_end(U-1) and last element X(U) would have a greater sum, which is a contradiction.

And so the linear-time algorithm goes through values of U from 1 to N keeping track of both max_so_far(U) and max_end(U) , and finally returns max_so_far(N) .

This algorithm is asympotically optimal (i.e. in the 'big oh' sense) because every element of the input vector must be examined to be sure of finding the correct subvector. Just to examine every element takes O(N) time, putting a lower bound on the possible running time of any algorithm which solves the problem.

Mike Swain appeared on the cover of the June '84 Communicator as a member of Queen's Putnam team which place 8th that year in North America. He graduated in 1984 in Math and Physics and is now doing a Ph.D. in Computer Science at the University of Rochester. I quote from a recent letter of his.

"My favorite topic is algorithms. I can recommend a book (a short paperback) by R.E. Tarjan, "the data structures man" as my algorithm's professor calls him. It is called "Data Structures and Network Algorithms" (SIAM 1983) and explores some classic problems, mostly graph-theory problems, in a clear and elegant way.

Concerning applications of computers, one of the biggest uses of computers around the department is for text processing. With our text editing and laser printer facilities we can produce book-quality material, **good** book-quality too. I imagine it is very expensive for a normal university department to do this; we seem to get the laser printers thrown in with equipment grants from places like Xerox.

The other great application is game-playing. The current craze is something called "Hack". It is an updated and expanded version of "Rogue". They both take place in the many levels of a dungeon filled with monsters. The dungeon evolves from game to game; for instance if I had died fighting a dragon in one of the caves you might find my ghost there the next game. "I´d rather be sailing" you say. Well so would I. I am amazed that that isn´t the universal opinion.

I still have a **lot** to learn about computer science, and am still very slow at writing "real" programs. These "real" programs arise in systems programming and AI symbol-manipulation situations. They involve an order of magnitude more of complexity and thinking about data structures than scientific programming usually demands. Computer vision itself is more like what I am used to. In fact I´ve managed to co-author a conference presentation in computer vision. We find out this month if it was accepted.

The first-year class is quite a closely-knit group. In fact a number of us are in the same office. I feel that some of us suffer some cabin fever sometimes but I feel privileged to be associated with all of them. On average, my classmates are very broad culturally, that is they have a knowledge of the arts as well as computer science.

Queen´s Students Win Awards

Sylvia ´Suzie´ Monson, a Kingston homemaker and mother, graduated on June 1 with the highest average ever achieved by a part-time student. Suzie not only graduated with a 96% average in her specialty of Mathematics and Statistics, but also won the medal in that program and the Kathleen L. Healey Prize for part-time students. She is also the recipient of a National Science and Engineering Research Council scholarship for M.Sc. studies which she is pursuing this year in Mathematics at Queen´s.

She received a BA from Queen´s in 1963 as Suzie Oliver, then married and had children before returning to Queen´s in 1979 in part-time studies.

The Ph.D. thesis of **William Ross** has won the 1984 Pierre Robillard award for the best Canadian Ph.D. thesis in Statistics. The award was announced last June at the annual meeting of the Statisical Society of Canada. The thesis was entitled "Measuring influence in nonlinear regression" and was supervised by **Don Watts**. Bill has now returned to Saskatchewan to continue his work with the Saskatchewan Research Council.

News

From Don Watts

I had a most enjoyable and stimulating 8 months leave of absence at the Rohm and Haas Company Research Laboratories in Spring House, PA (near Philadelphia). Valery and I were impressed with how beautiful the State is, and how much more sunshine there is there than in Kingston. As a consequence, on the weekends we managed to put 2500 miles on our bicycles doing "breakfast runs" to nearby towns and villages.

Work at Rohm and Haas was most stimulating. I got to work with chemical engineers, polymer chemists, agricultural chemists and toxicologists on a wide variety of problems. I will maintain contact with the people there, since this year I will be on reduced responsibility, spending part of my time working on problems for Rohm and Haas. I hope to be able to involve graduate students in some of these neat industrial problems.

During the past year, **Douglas Bates** (Ph.D., Queen's, 1978) and I were accorded the honour of presenting the Technometrics Paper at The American Statistical Associate (ASA) Meetings in Las Vegas. I was also invited to present a paper at the Delaware Chapter of the ASA, at the Philadelphia Chapter of the ASA, at the 25th Anniversary of the Department of Statistics at the University of Wisconsin, at the Statistical Society of Canada meetings in Winnipeg, and was keynote speaker at the first annual Graduate Seminar in Statistics and Biostatistics held in Waterloo. In August, to top the year off, Valery and I took a 1000 mile bicycle tour from Kingston to Montreal to Quebec City to Ottawa and home.

From Tony Geramita

I spent last year on sabbatical at Brandeis University in Massachusetts. I took courses and followed seminars in Algebraic Geometry and gave several talks, both in the "Surfaces" seminar and at the "Fellowship of the Ring" seminar.

I was invited by **Donal O'Shea**, one of our former Ph.D. students, to speak at the Joint Geometry Colloquium given by U. Mass, Mt. Holyoke and Amherst College.

I also was invited to give a series of lectures at Northwestern University in Evanston in February (I arrived on the coldest day of the year in Chicago).

I went to Oberwohlfach for a week in May followed by a week in Torino where I gave 2 lectures in the Politechnico di Torino.

I spent a lot of time working with Jim Carrell (U.B.C.) and Peter Russell (McGill) editing the Proceedings of The Algebraic Geometry Conference at U.B.C. in August 1984.

This sounds awfully dull, but I learned some things this year that I've struggled with for too long and it's hard to explain the thrill that gives. Also, so much of the sabbatical was: cleaning our uncleanable house in Newton, driving the kids to their too numerous activities and recovering from too much sickness; it's hard to describe trying to do this in a city where you have few friends and no network.

From Robin Giles

I gave an invited talk on "The concept of grade of membership" and chaired a session on the same general topic at the First I.F.S.A. Congress (IFSA = International Fuzzy Sets Association) in Mallorca, Spain in early July. I also gave an uninvited talk there on "The problem of connectives in fuzzy reasoning.

Two weeks later I gave (at the Assocation of Symbolic Logic Meeting at Stanford, CA) a talk on "A non-classical logic for reasoning with beliefs".

Excerpts from a letter of **David Morel** (B.Sc. ´73, B.Ed., M.Sc. ´76) to Peter Taylor, January 1985:

I´ll first attempt to fill in the past 8-9 years. Following Teacher´s College and a one-year stint at Trinity College, Port Hope, I married and moved to Elliot Lake. My wife, Peg, already had a job teaching Phys. Ed. in Elliot Lake and I managed to convince the principal that hiring me would be best for the school and her. We have two children now, Emily (2 1/2) and Skeets (1 1/6) both very healthy and lovable.

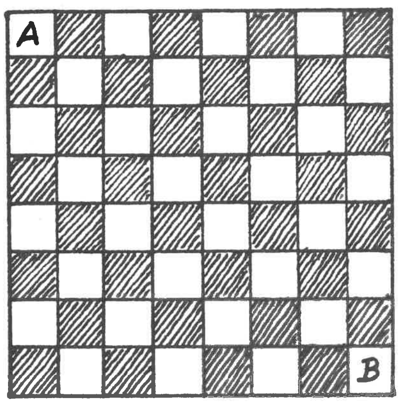
Living in a mining town was never quite the full experience of life, but by marathon running, and competitive cross-country skiing we managed to maintain a positive attitude to health, life and school. No racquet courts, save badmonton, to be found, but rumors always exist of construction. Teaching itself can be somewhat of a rollercoaster experience, but we are both maturing yearly, and have held on to an open outlook with students as individuals, curriculum changes, administration, and the system.

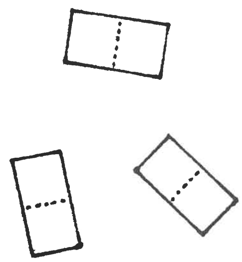
Two years ago, we made the move to the bush. We are now situated on the shores of a moderately-sized lake, as the only permanent residents. We have neighbours within a km, but no more for 5 km. There are cottages on the lake but none are visible to us. In short, it is our dream come true! We do have about 40 km to travel to work, no telephone, and about 200 m of driveway to keep open, but each night´s peace and serenity make it all worthwhile. Until this move, we were anxious to leave Elliot Lake, or at least didn´t feel settled. Many times I considered returning to school for a Ph.D. or at least the M.Sc. in Statistics that I once read about at Queen´s. But I was never, nor am I now, convinced that more education would necessarily lead me to a job with greater satisfaction. But I do still miss that feeling of a problem solved, and a new challenge to tackle. Enough said, this moment does find us in a wonderful winter setting with all healthy, and a new term about to begin.

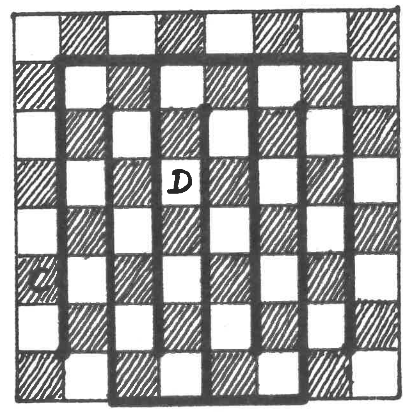
This next term, I will be allowed to teach Calculus for the first time. I hope to once again use your "Exploratory Problems in Calculus and Functions". I have been able to use a small selection of these with my Grade 12´s in the past, and have found them very worthwhile. It certainly is an effort to push them toward understanding, but a few always learn so much from these projects, that I consider the effort more than worthwhile. As noted, I have chosen to do this in a project set-up, as I always feel pushed by the curriculum. But I do, throughout all advanced classes, try to encourage the student to use his "conceptual apparatus" to solve problems.

Problems

Covering Checkerboards with Dominoes. Consider an 8×8 checkerboard and a supply of 1×2 dominoes:







If two opposite corner squares like A and B are deleted, the reduced board cannot be covered with 31 dominoes, for it contains unequal numbers of squares of the two colors, while each domino covers one square of each color.

On the other hand, if one square of each color is deleted, the reduced board can always be covered with 31 dominoes. Ross Honsberger in **Mathematical Gems** (Math. Assoc. Amer. 1973) gives Gomory's elegant proof: Divide the board into a single closed path as shown. Deleting one square of each color like C and D divides this into two paths (or leaves it as one) which can be covered with dominoes because each path contains an even number of squares and we can always get around the corners.

If 2 (or more) squares of each color are deleted, it is easy to give examples in which the reduced board cannot be covered by dominoes. Of course, none of this depends on the squares being colored – the colors can be merely something we introduce as a convenient way of keeping track of parity.

Problem 1: Settle the same questions for an $m\times n$ checkerboard.

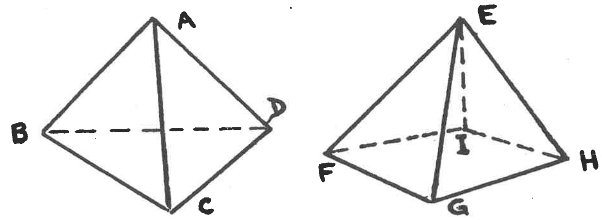
Problem 2: Take an $m\times n\times r$ rectangular solid space divided into $1\times 1\times 1$ cubes, and delete some of those cubes. Can the reduced space be packed with $1\times 1\times 2$ building blocks? By assigning two colors alternately to the cubes, the easy negative results for the plane apply without change: if unequal numbers of cubes of the two colors remain, the packing is impossible. Can the positive results for the plane also be extended to the solid?

Problem 3: Returning to the two dimensional problem, are there interesting results for deletion of more than 3 squares, with the deletions restricted to being rectangles? Think of building a wall with $1\times 1\times 2$ cement blocks, leaving openings for doors and windows. The blocks can be laid horizontally or vertically but cannot be cut.

Colin Blyth

Solutions to Problems of January 1985

Problem 1. (An Educational Testing Service "Scholastic Aptitude Test" question.)

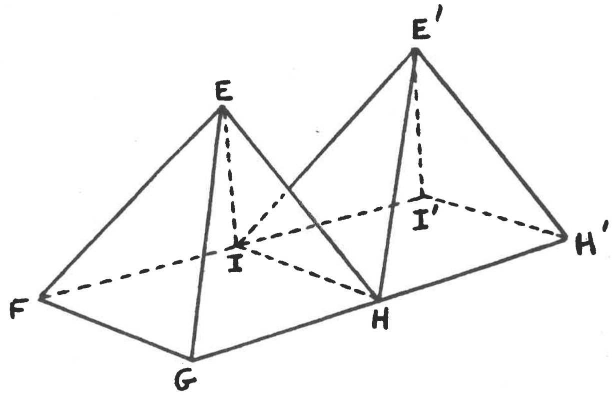


In the regular tetrahedron ABCD and pyramid EFGHI shown above, all faces except FGHI are equilateral triangles of equal size. If face ABC were glued congruently to face EHI , how many faces would the resulting solid have? (Colin Blyth)

Solution by Colin Blyth

For this question, Educational Testing Service overlooked the possibility of coplanarity, and incorrectly scored "7" as the right answer. In fact, the right answer is "5". [**Time** 30 Mar. 1981, **Newsweek** 6 April 1981, **Science** June 1981.]

This answer is instantly obvious if you think of **two copies** of the pyramid, sitting side by side on a table:



Then $EE' = GH$: the points E and E′ each lie directly above the center of a square base. Therefore EE′HI is a copy of the tetrahedron ABCD . Clearly EE′HG is a plane, so EE′HG makes a single face of the new solid, not two faces EHG and EHE′ . So the new solid has 5 faces.

Problem 2. A fisherman is 1/3 of the way across a long, narrow, high railway trestle when he hears a train coming behind him at 60 m.p.h. (constant). He starts running instantly at his top (constant) speed, and can just save his life by running to either end of the bridge. How fast can he run? (Colin Blyth)

Solution by Colin Blyth

This is a very old problem, but still a good one. If he runs back toward the train he has to run 1/3 the length of the bridge; if he runs away from it he has to run 2/3 the bridge length. Therefore he must be able to run $2/3-1/3 = 1/3$ the bridge length while the train is crossing it, so his speed must be $60/3 = 20$ m.p.h.

Problem 3. Find the next term in this sequence.

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)(x^2+xy+y^2)^0$$

$$(x+y)^5 = x^5 + y^5 + 5xy(x+y)(x^2+xy+y^2)^1$$

$$(x+y)^7 = x^7 + y^7 + 7xy(x+y)(x^2+xy+y^2)^2 \quad .$$

(Peter Taylor and Doug Dillon.)

Solution by Peter Taylor

I guess one way to state the problem is to ask for what p the equation

$$(x+y)^p = x^p + y^p + pxy(x+y)(x^2+xy+y^2)^{(p-3)/2}$$

holds for all x and y. There are probably some clever things to do with this with algebra and number theory, but nobody showed me any, so I'll solve it the way a calculus student might. Setting $x = y = 1$ we get $2^p = 2 + p3^{(p-3)/2}$ which can be rewritten as

$$\frac{p3^{(p-3)/2}}{2^{p-1}} = 1 - \frac{1}{2^{p-1}} \quad . \qquad (*)$$

I show that (*) cannot hold if $p > 7$. Regard the LHS of (*) as a continuous function of p. An examination of the derivative of its logarithm shows that it decreases in p for $p \geq (\ell n2) - (\ell n3)/2 \approx 6.95$. Clearly the RHS increases in p. So they can be equal at most once in the range $[6.95,\infty)$. In fact they are equal at $p = 7$ and therefore never again. So the only possible p are $p = 3$, 5 and 7.

A Continuing Financial Appeal

The Department budget is hard pressed. For example, this year, we have had to drastically cut back on marking services for undergraduate courses. We have come up with a number of creative ways to reduce both the negative impact on the students and the extra time that will have to be spent by the staff, but one of the negative impacts we can do nothing about is the loss of experience and income (small but certainly not negligible) to our undergraduate markers.

The costs of the Communicator are met from the same budget as marking, casual teching, course notes, supplies and a myriad of other things that keep the teaching life of the Department running. If you want to help keep the Communicator coming why not send a small donation? It costs approximately one dollar to print and mail a single copy (labour costs not included) so periodic contributions of, say, \$5 every three years or \$10 every 5-6 years would certainly pay for issues received.

Along with your contribution, send along a piece of news, an opinion or a problem, and we'll put you in the next issue! Cheques should be made payable to The Communicator, Queen's University.

AND: Our thanks to those many readers who have already contributed.