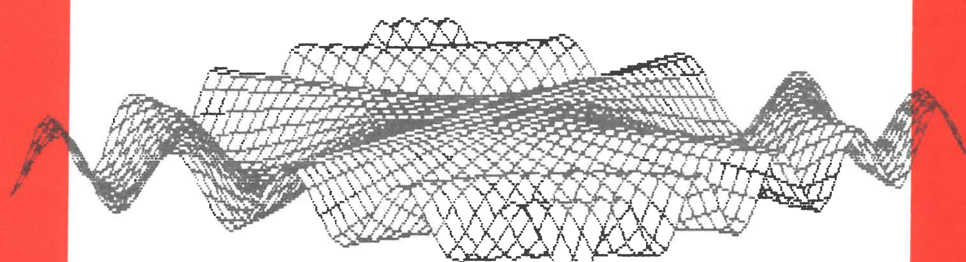
QUEEN'S MATHEMATICAL COMMUNICATOR

JUNE 1986



$z = \cos xy$

(See Page 16)

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Strange Attractors

by Leo Jonker

The following article is adapted from a talk given by Professor Jonker last November in the Undergraduate Colloquium series of Coleman-Ellis lectures.

Many natural processes are described mathematically with the help of differential equations. In each case the differential equation embodies an observed natural law, and the scientist is interested in inferring from initial conditions the future development of the process. The simplest example, perhaps, is the differential equation

$$\ddot{x} = -kx \quad , \tag{1}$$

which is used to describe both the swinging of a pendulum and the vibration of a spring. Here $\dot{x}$ and $\ddot{x}$ are used to denote the first and second derivatives of x with respect to time t. In one case the differential equation gives expression to (an approximation of) Newton's second law while in the other case it is Hooke's law. It is often convenient to rewrite (1) by letting $y = \dot{x}$. This leads to the following equivalent system of differential equations on R^2 :

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -kx \end{aligned} \tag{2}$$

In this way, many processes may be modelled by a system of ordinary first order differential equations in several variables. The general form of such a system is:

$$\dot{x}_i = f_i(x_1,\ldots,x_n), \quad i = 1,2,\ldots,n \quad . \tag{3}$$

A solution $X(t) = (x_1(t),\ldots,x_n(t))$ is a curve, or point moving with time, satisfying (3) in the sense that $\dot{x}_i(t) = f_i(x_1(t),\ldots,x_n(t))$ for each i. If we let $X = (x_1,\ldots,x_n)$ and $F(X) = (f_1(x_1,\ldots,x_n),\ldots,f_n(x_1,\ldots,x_n))$ we can write (3) more concisely as follows:

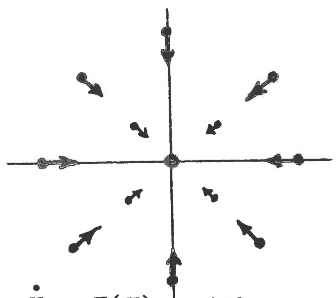
$$\dot{X} = F(X) \quad . \tag{4}$$

$F(X)$ should be thought of as a <u>vector field</u>, attaching to each point X the velocity vector $\dot{X} = F(X)$ of the solution curve through X.

In many applications the variables $(x_1,\ldots,x_n)$ live in some domain other than R^n. Such domains, called <u>manifolds</u>, can be described locally by coordinates $(x_1,\ldots,x_n)$ but <u>not globally</u> because they do not have the shape of a Euclidean space. A torus is a good example of a two-dimensional manifold, as is the sphere. In this article, then, we will be interested in ordinary first order differential equations on n-dimensional manifolds. These differential equations can be expressed in the form $\dot{X} = F(X)$ on any part of the manifold M described by coordinates $X = (x_1,\ldots,x_n)$. The function $F(X)$ should be thought of as a tangent vector field to the manifold.

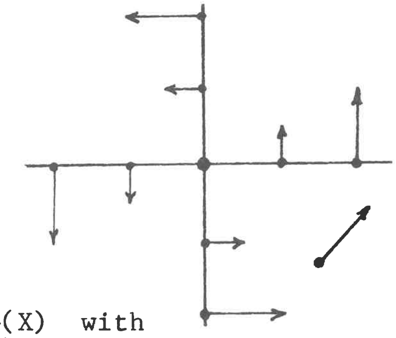
Here are some examples:

Figure 1



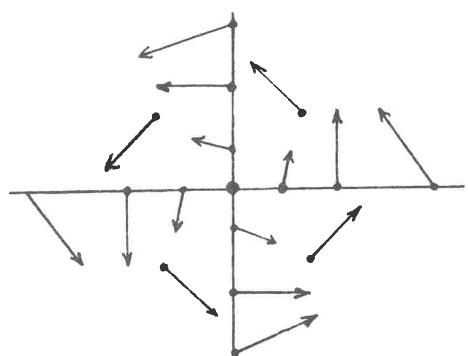
$\dot{X} = F(X)$ with
$F(x,y) = (-x,-y)$
on R^2

Figure 2



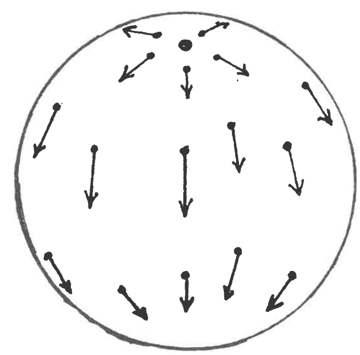
$\dot{X} = G(X)$ with
$G(x,y) = (-y,x)$
on R^2

Figure 3



$\dot{H} = H(X)$ where
$H(x,y) = F(x,y) + (r^2-r^6)G(x,y)$
where $r^2 = x^2 + y^2$

Figure 4



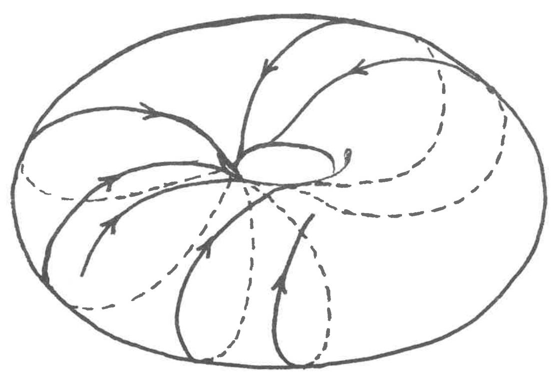
A vector field on the sphere.

We shall be interested in the long term qualitative behaviour of solutions of differential equations. In Figure 1 all solutions converge towards the origin. If a natural process described by this differential equation were set in motion, it would eventually settle down to rest at the origin, no matter what initial motion was given to the system. The solutions of the system indicated in Figure 2 are all circles centered at the origin. A process described by this system would exhibit periodic behaviour. The amplitude of the behaviour is determined by the initial conditions. In Figure 3 we also get periodic behaviour in the long run. However, in this case the amplitude is independent of initial conditions, as all solutions converge to the same circle $x^2 + y^2 = 1$. Figure 4 illustrates a vector field on a sphere. A solution starting at the North Pole will stay fixed; however, all other solutions will converge to the South Pole, a point of rest.

In Figures 1, 3, 4 all (or nearly all) solutions converge to either a fixed point or an invariant closed curve which then dominates the behaviour of the solutions. These sets are called attractors. An attractor is an invariant set A to which some neighbourhood of A converges as t goes to infinity. If A is a single point, then A is called a point attractor (Figures 1 and 4). If A is a closed curve, then A is called a periodic attractor (Figure 3). One might say that the attractors determine the long term behaviour of the solutions of the ordinary differential equation. The question we want to raise is this: Are point attractors and periodic attractors the only attractors possible?

The answer to this question is NO. It is possible to dream up vector fields with attractors not at all like the ones described so far. The simplest example is the irrational flow on the torus. The torus may be constructed by identifying opposite sides of a square in R^2. Putting this a little more formally, we get the torus T^2 by identifying points in R^2 whose coordinates differ by integers: $T^2 = R^2/Z^2$. If α is an irrational number, the vector field $F(x,y) = (\alpha,1)$ on R^2 gives a vector field on T^2. Moreover, a solution curve such as $X(t) = (\alpha t,t)$ on R^2 produces a solution curve (also denoted $X(t)$) on the torus when R^2 is "wrapped around" to produce T^2. Because α is irrational, no two pairs of coordinates $(\alpha t_1,t_1)$ and $(\alpha t_2,t_2)$ differ by integers. Thus the solution curve $X(t)$ is not a periodic solution. In fact, it can be shown that $X(t)$ covers T^2 densely.

Figure 5



In some sense, then, all of T^2 is the attractor for this differential equation. What is unsatisfactory about this example is that the picture changes completely if this vector field is changed ever so slightly. A small change from the irrational value of α to a nearby rational value will cause all solution curves on T^2 to become periodic. The same is true of the example described in Figure 2. A small inward component added to the vector field G will cause all solutions to converge to the origin.

This motivates the following definition:

Definition. A vector field F is structurally stable if a small perturbation of F will not change the qualitative appearance of the set of solution curves.

In many applications, not only the initial conditions, but also the parameters specifying the vector field, are determined by measurements and are therefore not completely accurate. In this situation, if the resulting vector field is not structurally stable, its predictive value is greatly reduced, and it is therefore of little interest.

Peixoto's Theorem. On a two dimensional compact orientable manifold, a structurally stable vector field can have only two kinds of attractors: stationary points and periodic solutions.

Definition. A strange attractor is an attractor which is not a fixed point or a periodic solution.

Not everyone agrees how a strange attractor should be defined, but this will do for our purposes.

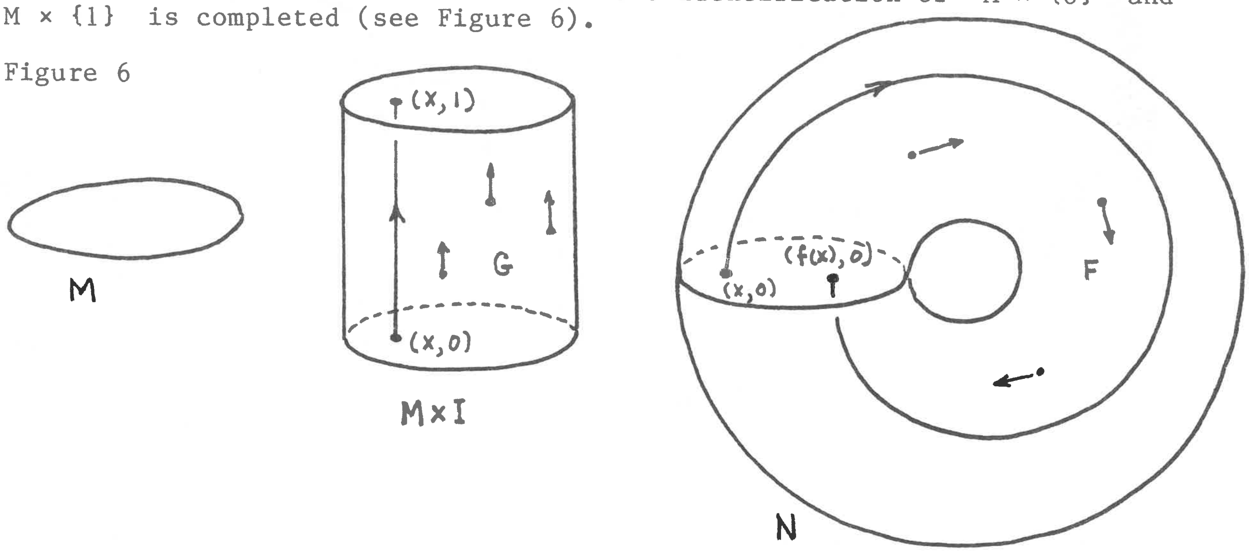
Peixoto's Theorem seems to put a damper on speculation about the existence of strange attractors, at least for structurally stable vector fields. However, for dimensions greater than 2 interesting things do happen:

Fact. If $\dim M > 2$ there are strange attractors that persist even when the vector field is perturbed.

It is our purpose to give some examples.

To construct these examples we will first show how to construct a vector field from a mapping. Suppose M is a manifold - think of a two-dimensional disk as an example - and suppose f is a differentiable mapping from M to M with a differentiable inverse (a "diffeomorphism"). The vector field we want to construct will exist not on M but on a new manifold N which we must construct first. We take the Cartesian product $M \times I$, where I is the closed interval $[0,1]$, and we let N be the manifold resulting when the ends $M \times \{0\}$ and $M \times \{1\}$ are joined by glueing the point $(x,1)$ to the point $(f(x),0)$. If M is a disk, $M \times I$ will be a solid cylinder, and N a solid torus. If M is more complicated, N will also be more difficult to describe. However, for every manifold M of dimension n and every diffeomorphism f on M, N is a manifold of dimension $n + 1$. To construct the vector field F on N we first take G to be the vector field on $M \times I$ consisting of unit length vectors parallel to the second factor I. For example, if M is a disk, G is parallel to the axis of the solid cylinder $M \times I$. We then let F be what G becomes when the identification of $M \times \{0\}$ and $M \times \{1\}$ is completed (see Figure 6).

Figure 6



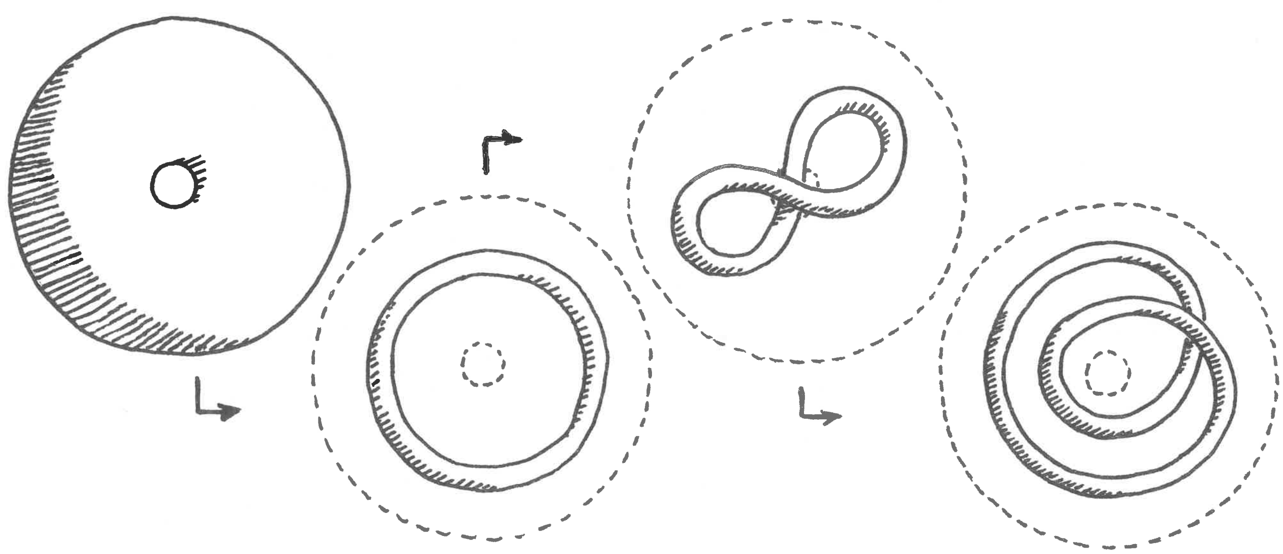
A solution curve of G is a line from $(x,0)$ to $(x,1)$. A solution curve of F is a curve in N obtained by glueing end to end indefinitely the products of such lines under the identification that produces N. It is important to note that if x_0 is a fixed point of f (that is, $f(x_0) = x_0$), then the solution curve of F starting at $(x_0,0)$ will return to $(x_0,0)$ in one unit of time, and so produce a periodic solution. If x_0 is periodic (meaning $f^n(x_0) = x_0$ for some integer n), then that solution curve will return to $(x_0,0)$ at time $t = n$, after winding around N n times. The similarities between f and F continue: If the orbit $x, f(x), f(f(x)), f(f(f(x))), \ldots$ converges to a point x_0, then the solution curve starting at $(x,0)$ will gradually converge to the periodic solution starting at $(x_0,0)$.

We see in this way that the periodic behaviour and long term behaviour of F reflect the periodic and long term behaviours of f. In particular we could look for examples of strange attractors for diffeomorphisms f. An attractor for f is quite simply a set A such that $f(A) = A$, with a neighbourhood U such that every orbit starting in U converges to A. The attractor is strange if it is not a periodic orbit (i.e. $f^n(x) = x$ for all x in A). A strange attractor for f will immediately correspond to a strange attractor of F via the above construction.

To construct our first example of a strange attractor we let M be the solid torus. This may seem a little perverse, for N will then be a rather complicated four dimensional manifold. However, given the above discussion, we will be content to look for a mapping f on M with a strange attractor, without giving thought to the precise appearance of the induced manifold N, the vector field F, and its strange attractor.

The mapping f is best described with the help of pictures. See Figure 7.

Figure 7



The solid torus is first shrunk into its own interior. It is then twisted to form a figure eight. Next, one of the loops of the figure eight is folded over, so that the image of the original torus is located completely in the interior of M, winding twice around the hole. We let f be a mapping on M that has the above effect. That is, f sends a point x in the original torus M to its image in the twisted figure eight torus inside M .

Now consider what happens when we iterate f . Since f(M) maps twice around the hole in M, f(f(M)) must be contained in f(M) and must wind twice around the hole in f(M) . In other words, it must map four times around the hole in M . In general, if we let $f^n(M)$ denote the image of the n-fold iterate of f , then M, f(M), $f^2(M)$, ... is a nested sequence of smaller and smaller sets, each containing the next, such that $f^n(M)$ wraps around the hole of M 2^n times. Now let $A = \bigcap_{n \geq 0} f^n(M)$.

Then for every $x \in M$ the sequence of points of the orbit x, f(x), f(f(x)), ... clearly converges to a point of A . In other words, A is an attractor.

It can be shown quite rigorously and without much difficulty that A is a <u>strange</u> attractor. This can also be illustrated (but not proved!) with the help of computer graphics. To do this we use complex numbers to obtain a possible formula for the mapping f . We will just outline the procedure here, leaving implementation to the interested reader. Let M be the subset

$$M = \{(z,w);\ |z| = 1,\ |w| \leq 1\}$$

of $C \times C$; thus M is a solid torus. Choose a constant p, $0 < p < 1/2$, and let

$$f(z,w) = (z^2, z(p+p^2w)) \quad .$$

(A little reflection shows that this formula gives a mapping f with the desired effect.) Now take an initial point (z_0,w_0) at random and have the computer draw several thousand points of the orbit of f starting at (z_0,w_0) . Since A is an attractor, most of these points will lie very close to A , so the result can be regarded as a picture of the strange attractor. The intersection of A with a radial cross-section of the torus M is a type of Cantor set, reflecting the very complicated structure of the set A .

While the above mapping is probably the simplest example <u>known</u> to give a strange attractor, there is an even simpler mapping f <u>which appears</u> to give rise to one. In this case, however, no one has been able to prove that what one sees in a computer simulation is indeed a strange attractor. In this second example the manifold M is quite simply the Euclidean space R^2 and the map is given by the formula

$$f(x,y) = (1+y-ax^2, bx) \quad .$$

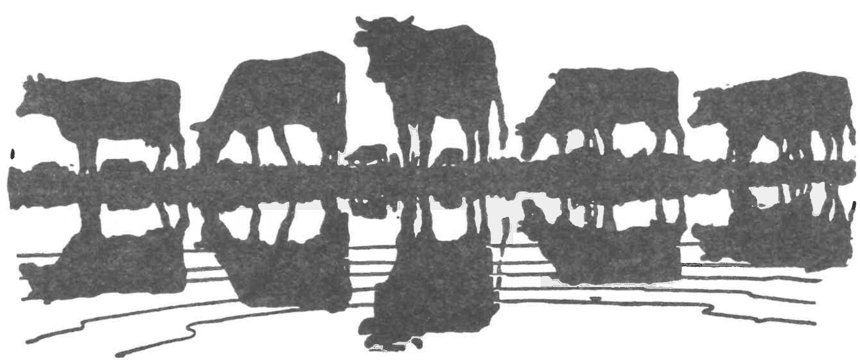
Here a and b are positive numbers whose values must lie in a certain range; $a = 1.5$, $b = 0.3$ are good choices. If you take any initial point (x,y) not too far from (0,0) and plot its orbit, you will get a complicated looking set A , known as the Hénon attractor. Successive enlargements of portions of A suggest that it is very complicated indeed, resembling somewhat the Cartesian product of a line and a Cantor set. Unfortunately, no one has succeeded in proving rigorously that the Hénon attractor is strange.

In the meantime, many other apparently strange attractors have been discovered. Most of them are not yet well understood. All of them have generated much interest among both mathematicians and physicists. The former are attracted to them for their mysterious intricacy, while the latter study them because they may provide a key to unpredictable physical systems such as turbulent fluid flow.

NEWS

The **Algebra Group** has had a busy year with visitors and seminars. We were fortunate to have **Professor Edoardo Sernesi** (Università di Roma, I) here as a visitor for the year. He gave a series of talks in the ongoing **Curves Seminar** about the **Hilbert Scheme**, an object whose geometry contains important information about continuous families of curves. The Hilbert scheme has been discussed by mathematicians for almost a century, but it is only recently that a solid understanding of it has begun to emerge. Sernesi is one of the world's best explorers of this strange terrain, and we've had a lively and interesting view of the landscape.

John Coleman remains as active as ever at age 68 despite his "retirement" in 1983. The Coleman Symposium last summer on Reduced Density Matrices (see October 1985 issue) has resulted in an invitation for John to visit China to lecture in April 1987. This Spring, John has been the driving force behind a seminar on **Groups Generated by Reflections** and such related topics as Lie algebras and Coxeter graphs (a.k.a. Dynkin diagrams). Drawing on a lifelong familiarity with these subjects, John's arguments often appealed to geometric intuition, glossing over the grisly algebraic details, a style which often called forth impassioned protests from his younger colleagues, making for an extraordinarily lively seminar. Coleman claims, "I have never before had such stimulating mathematical interaction with colleagues in all my years at Queen's".



- A Group Generating A Reflection -

Coxeter Visit

Professor H.S.M. Coxeter (University of Toronto) is undoubtedly the world's foremost geometer, still going strong in his late seventies, and we were privileged to have him visit on May 5-6. An inkling of Coxeter's zest and scope can be gleaned from the way he introduces his book Regular Complex Polytopes (Cambridge University Press, 1974): "This book has occupied much of my time and attention for nearly twenty years ... I have made an attempt to construct it like a Bruckner symphony, with crescendos and climaxes, little foretastes of pleasures to come, and abundant cross-references. The geometric, algebraic and group-theoretic aspects of the subject are interwoven like different sections of the orchestra." Coxeter gave a lecture on Quaternions, sprinkled with anecdotes and historical highlights, for a general audience, and a long and masterful talk to the Reflection Group seminar. We hope to have him back for a longer visit soon.

Colin Blyth reports on recent work: two papers with a collaborator in New Mexico (look for a slick derivation of Stirling's formula in the May 1986 American Mathematical Monthly), one on Fiducial Probability with a collaborator at Carleton University, and others on Cauchy Convolutions, Approximate Binomial Confidence Limits, and Binomial and Poisson Confidence Intervals, the latter with **Harold Still** of this Department.

Donald Watts was one of the keynote speakers at the Conference on Influential Data Analysis held in Sheffield, England, April 8-11. He also gave an invited address at the Mathematics and Statistics Workshop with Government, Science and Industry at Dalhousie University, April 25-26. In addition, he has been invited to speak at the Tenth International CODATA Conference to be held in Ottawa in July. CODATA is sponsored by the International Council of Scientific Unions Committee on Data for Science and Technology, which deals with data of importance to science and technology, quantitative data on properties and behaviour of matter, quantitative data and characteristic values of biological, geological, and astronomical systems and other experimental and observational values in all areas of science.

David K. Pickard, whom you met in our previous issue (October 1985), is completing his first full year with our Department. During the past year he has given Colloquium talks in Michigan and at Cornell and York Universities as well as here at Queen's. He contributed a paper about a three-parameter Markov model for sedimentation to the First International Congress of PARTEC (Particle Technology) in April, and another entitled "Do Particles Interact? How Can You Tell?" to the Statistical Society of Canada in May.

Putnam Competition

The William Lowell Putnam Mathematical Competition has been written every year (except for a 3 year break during WWII) since 1938. It is the premier university-level mathematics contest in North America, and each university puts forward its best undergraduates.

The 46th annual exam was written last December, and 264 universities contributed teams, each team consisting of three preselected individuals. The Queen's team - Neale Ginsburg, Ian Carmichael and Jim Beatty - placed 23rd. Neale and Jim are in their third year and Ian is in his second. These three, and another second-year student, Krishna Ragogopal, all stood in the top sixth of all contestants. We congratulate them on their performance; the competition is stiff indeed.

The Queen's competitors were coached, in a series of problem-solving sessions, by Professors Peter Taylor, Leo Jonker and David Gregory.

The highest ranking Queen's has achieved in the Putnam in recent years was an 8th place finish in the 1983 exam. In that year, four Canadian universities placed among the top ten - quite unusual. Only three Canadian unviersities have ever placed first: Toronto won the first exam in 1938 (John Coleman was a member of the team) and won again in '40, '42 and '46. Queen's won in 1952 and Waterloo won in 1974.

WHAT DOES ONE DO WITH A DEGREE IN MATHEMATICS OR STATISTICS?

This article describes the history, and summarizes the results, of a questionnaire sent recently to about 700 of our graduates, in hope of eliciting some answers to this question.

Some time ago, when our undergraduates asked us to put on an information evening on careers in mathematics and statistics, we found we had little information on our graduates' careers. So last fall we sent a questionnaire to about 700 bachelor's graduates going back to 1960. We used our Communicator mailing list; it includes mostly graduates from Arts and Science with majors in mathematics or statistics, or graduates from our Applied Science program in Mathematics and Engineering, but there are also some general B.A.'s with a concentration in mathematics. There are many graduates for whom we do not have an address.

We received 216 replies, and 100 envelopes were returned because the addressee had moved. To report the results, occupations have been grouped; in some cases, where a person has done various things, he or she has (somewhat arbitrarily) been assigned to one group which describes either the longest-running or the most recent occupation. (In particular, many women had spent significant periods as mothers and homemakers, but where they have had some extensive paid employment, it seemed most useful for this survey to record their paid employment.)

There are additional reasons why readers should be cautious in extrapolating from the results reported here. There are few responses from Arts and Science graduates after 1977 (except for '81), partly because we have few addresses for them. There are also factors which might make some people more likely to respond than others, for example professional involvement in education, or particularly happy memories of Queen's. The high percentage of respondents with higher degrees is perhaps not truly representative.

The results were as follows:

14 in Actuarial Work:

Included were three insurance company vice-presidents and one federal deputy minister.

46 in Computers and Systems Analysis:

There is great variety in this group: junior programmers, data-base managers in large firms and government, managers of software development in large and small firms, consultants, a creator of entertainment software.

25 in Engineering (other than computing/systems):

Included is one Arts and Science graduate with a Ph.D. Again, lots of variety.

21 in Other Business Areas:

Retail sales, accountants, senior managers, Vice-President of a large New York financial institution.

41 School Teachers:

Mostly in high school mathematics (some now promoted to administration), 4 in public school, 1 in nursery school.

22 University teachers:

6 in mathematics, 4 in statistics, 3 in economics, 2 in operations research, 2 in computing, 2 in electrical engineering, 1 in chemistry. (Not included are Professors Gregory, Rice and Taylor of our own Department, or Professor Higginson of our Faculty of Education, whom we neglected to survey.)

47 Other:

This group includes 2 farmers, 2 M.D.'s and 2 medical students, 4 lawyers and 1 law student, 2 statisticians, 2 people in the armed forces, 8 working for various levels of government, and a large group of recent graduates still in school (library science, priesthood, physics, chemical or aerospace or electrical engineering, computing, statistics, mathematics, medieval English [thesis topic: "Legendary Metalsmiths"]).

The questionnaire did not ask explicitly about some of the following matters, so some graduates may not have given relevant information. In some cases, the information was implied rather than stated, so some of the numbers below are not very reliable.

- 66 responses were from women; 25 indicated varying periods primarily as mothers or homemakers.

- it appears that 27 out of the 216 have Ph.D.'s, another 38 have master's degrees in mathematical science, physics or engineering, 11 have M.B.A.'s, 7 have other master's degrees, 2 have M.D.'s, 4 have Ll.B.'s, and of course, many of the teachers have B.Ed.'s; about 20 of the respondents were still in graduate school (some part time).

- 56 graduates of the Mathematics and Engineering program replied; 12 are in computing/systems, 24 are in other engineering roles (some now quite senior in engineering management), 5 are in business (marketing, management consultant), 2 are professors, 9 recent graduates are still studying.

The questionnaire asked about the use made of mathematics and statistics in careers. Except for the university teachers, actuaries, and some engineers, few of the graduates have made much use of specialized mathematical knowledge, except that many (in various occupations) have found some knowledge of statistics important. Some also commented that the style of thinking nurtured by mathematical training was useful.

Additional comments from graduates varied according to experience and, probably, temperament. Many pointed out that some computing skills had been essential in their careers. Others felt undergraduates should be advised to develop communication and writing skills. A number felt that some exposure to business and/or economics courses was very useful. A large group urged that undergraduates not become too specialized, remaining open to a wide range of opportunities and pursuing their real interests.

Finally, let me thank all those who helped by replying to our questionnaire.

- Dan Norman

Microcomputers for Queen's Engineering

In April 1984, the Faculty of Applied Science agreed to introduce microcomputing as a fundamental component of its undergraduate program. A review of the events leading to this decision appeared in the June 1984 issue of this aperiodical. This note summarizes progress since that decision.

The microcomputer project was to be implemented by "strongly encouraging each entering engineering student to acquire a microcomputer recommended by the Faculty". The Faculty formed a Microcomputer Committee to decide on an appropriate microcomputer, and to coordinate its purchase by first year students and its introduction to the academic program. During the 1984-85 academic year this group solicited and reviewed tenders from a variety of vendors of IBM compatible PCs. ZENITH Data Systems was selected as the recommended vendor, and their entry level model 148 was offered to students at a total cost of $1800. At registration in September 1986, 373 of our entering engineering students took delivery of a ZENITH model 148 or 151. This represented about 90% of Science '89. Of those remaining, some brought their own computer to Queen's and the others elected to use microcomputers available on campus. (A cluster of 24 had been placed in Jackson Hall primarily to meet this need.)

Another aspect of the introduction of microcomputing is the provision of software - reliable, easy-to-use computer programs to perform specific computing tasks - complementary to the academic program. Because the development of educational software is a relatively new activity, little information and technology is available. While some software is available, good software tends to be very expensive; to provide a copy of such software to each student could be exorbitant. For the IBM compatible PCs, there is little available. One way to provide good academic software to complement our courses at a reasonable cost to the student would be to develop software at Queen's.

To promote the development of software, the Faculty distributed 36 microcomputers to academic departments in the summer of 1984, and provided ten student assistants during the summer of 1985. The idea was that faculty would design software appropriate for their courses and supervise students in implementing these designs using programming tools available with the systems. With these resources, a number of faculty became involved in software development aimed at our incoming class during the year prior to their registration.

Most of the development was based on prior perceptions of what might work, rather than well-documented experience. Even so, some of the software developed was quite useful to the students. More information on some of this software -- Calculus Pad (tm) and Matrixpad (tm) -- can be found elsewhere in this issue. With a continuing effort by both faculty and students, it seems likely that additional useful software for our first year program will be developed.

Software for courses in second and subsequent years seems to be easier to develop. For such courses, the problems can be more substantial, and as a result require considerable computation. By relegating such computation to a computer using convenient software, the student can spends a greater fraction of his time thinking about the design and interpretation of problem solutions. This, in turn, is conducive to the development of better engineering skills.

During the past year, our first year students have used their microcomputers primarily in connection with their computing courses. This has been due partly to the limited variety of software available, and partly to the scarcity of time available, given their other academic requirements. It is expected that with accumulating experience in utilizing software in connection with courses, and with the development of new software, our first year students will make more extensive use of their microcomputers in the years ahead. It now seems that the major use of the microcomputers will occur in upper year programs. Even those students who made little use of their microcomputers in the first year expect that with further work on their micro this summer they will become more fluent in fast access to computing tools, and will be more inclined to use microcomputing in their courses next year and thereafter.

First year engineers not the only people involved. A total of 1400 ZENITH PCs were sold at Queen's in the past twelve months. The price has been reduced substantially for the next bulk purchase, and so more are likely to be sold in the future. With widespread access to IBM compatible PCs, the volume of software available and the expertise will both grow. As a result, microcomputing in engineering will become both easier and more attractive.

- Jim Verner

photo: Lisa Lowry

Teaching Excellence Award

Peter Taylor of this Department and Bill Barnes of English are co-winners of this year's Teaching Excellence Award, given by ASUS (Arts and Science Undergraduate Society) in recognition of "outstanding achievements in promoting innovative approaches to learning". This award is based in part on a unique interdisciplinary course, Mathematics and Poetry, which they offer together. For more information about their course (which is scheduled to take flight again in January 1987) and some of Peter's reflections about it, see the following page.

"We must rely on our scientists to help us find the way through the near distance, but for the longer stretch of the future we are dependent on the poets."

- Lewis Thomas,
The Medusa and The Snail

LOOKING FOR A WINTER HALF COURSE? TRY

Mathematics and Poetry

MATH 391*
Winter 1984-85

Monday 7:00-10:00 pm
Jeffery Hall Room 101

A poet who wished to entice his listeners towards the rewards of poetry would give them a short, tempting poem and have them read it, listen to it, play with it, study it, talk about it, and seek to understand how it is that the poet has achieved his remarkable effect.

And this is also the way a mathematician should entice his listeners. The "poems" he should choose are elementary problems or intriguing patterns which cry out to be played with and understood.

But this is rarely done in mathematics courses. Too often the student is presented with a comprehensive set of lectures about a fixed topic which exposes the heart of the matter whether he is ready for it or not - rather like a systematic review of all Milton's poems. That's fine for the committed scholar, but probably not for anyone else.

In this course, each three hour class will consist of two equal parts. In one, a poem will be presented, savoured, and discussed, and in the other the process will be repeated with a pattern of numbers or shapes. The emphasis will be on student exploration and discussion. The objective is to give the student greater understanding and appreciation, and increased technical proficiency, both as poet and as mathematician. Each student will be required, for each of the two subjects, to submit 3 or 4 papers which present a discussion or analysis similar to that done in class.

The course is normally open to students in the third or fourth year of study of either the sciences or the humanities. It will be taught by Peter Taylor of the Department of Mathematics and Statistics and Bill Barnes of the Department of English. Interested students must preregister with one of these instructors before December 7, 1984.

The above "course description", taken from our advertising pamphlet last year, will serve to introduce this unusual course. It has been offered three times now, and is now institutionalized to the extent that it appears in the Calendar and is offered, on demand, every second year. It's not MATH 391* anymore, but IDIS 298* [Interdisciplinary Studies: there are three courses at Queen's now, the others being IDIS 200 (Women's Studies) and IDIS 299 (Modern European Theatre)].

The course was conceived about four years ago, out of the idea that the activity of a poet wasn't fundamentally different from that of a mathematician, at least on the level of things like language, pattern, technique and beauty. I thought it might be interesting to explore that idea with a group of adventurous students. Not explicitly, of course; I never had any desire to talk about similarities and differences. I wanted simply to juxtapose the two activities in the same classroom and see what happened. I also had the feeling that while the mathematics would mostly be directed by me, the poetry could be something that came from the class; thus we'd each have something to offer and thereby the glaring asymmetry between student and teacher (more glaring, I think, in the natural sciences than in the social sciences or humanities) might be softened. So in that first year, we did mathematics for some 2 hours each week, and the students took the 30 minutes at the end to take turns reading selected poems, occasionally their own. And it worked pretty well, I guess. Well, the students thought so, and that counts for something. Certainly it was interesting to watch the science and humanities students interact (there were about half of each).

But I was dissatisfied. I felt more leadership was needed with the poetry. I wanted to make the same kind of "progress" in coming to grips with the poetry, as I was aiming to make with the math. And for that I felt we needed more authority. [And here I have always felt the classroom to be too authoritative - alas!] I wanted the math and poetry to be completely parallel. I was aware that it would mean abandoning the idea that the poetry could belong to the students, but I thought it would be worth it.

So I looked for a colleague, and with a real stroke of luck, I met Bill Barnes of the Department of English. I was not five minutes into my description of what I was up to, and why and how, before he signed on. We have now taught the course together twice, and each time I have learned a huge amount.

In one sense we are kindred spirits. We approach the class in the same manner: direct and unsophisticated, seeking to free rather than burden them, seeking to be one of them (as indeed we are for one another), or making them one of us (which is not quite the same). Yet we both retain a keen sense of the dramatic: we are not above putting on a show when the moment is right as if to emphasize that we are the ringmaster and they the awful (full of awe) audience.

In another sense, a sense I find it hard to get hold of, we are quite antipodal. But only a poet would dare put it in words (let me try): he is earth and I am stone; he is river and I am lake; he is rain and I am snow (or is it the other way around? I keep changing my mind); he is crab and I am scallop (sorry, I couldn't resist that one); he is owl and I am squirrel (flying squirrel: let me not be too modest).

One after another, each Monday evening, we do our own thing. I have learned an enormous amount about poetry from him, but more than that, from being a student of his, a student who is also a fellow professor, I have learned a lot about teaching, mainly about the importance of involving students as intimately as possible in what's going on.

But this has been a source of great frustration for me. Consider the picture: my fellow teacher has produced a moving poem about love and loneliness, and has spent an hour wrestling with it, and getting the students to read parts of it in their own way, to relate it to their own feelings, to try to say it themselves, to suddenly see how it is that the poet has created a sharp impression with just the right placing of rhythm or word. And when the hour is up, we have all shared a love and a loneliness that has belonged to each one of us but is somehow magnificently bigger than us all. And then it's my turn.

How can I follow an act like that? Even the most highly polished geometrical gem, even the most perfect of perfect numbers, must surely pale beside that shared moment of love and loneliness. I stand presumptuously at the front, watching them return with their cardboard cups of vendamatic coffee. They sit and stare quizzically at me, wondering if I, or perhaps my stuff, will be up to scratch. And I wonder too.

And so I try to match my colleague. To see if my stuff can have the same impact on them as his does. So I choose my problems carefully, and I resort to tricks. I entice them into thinking in a simple way about my simple problem, and then pull a long-eared counterexample out of my six-cornered hat to impress them that the world of numbers has its subtleties too. And they smile and shake their heads in puzzlement, and I know I have them, and the hour will go alright. But...but when I ask them questions, they do not say as much to me as they say to him.

But I'm working on that.

- Peter Taylor

Calculus Pad^{tm}

As noted in the article (page 12) on Microcomputers for Queen's Engineering, a major aspect of the microcomputer project is the development of software, both for complementing the traditional educational techniques and providing tools which may be used in professional activity.

One of the software tools developed is CALCULUS PAD(tm). This is an integrated package of programs to process functions. It is designed to assist students in their study of calculus, but is quite likely to be helpful in other areas of a student's program as well.

This program is designed to be run on an IBM compatible PC (such as the ZENITH models 148 and 158 owned by most of our first year students). After typing the initial command [CALC], the skeleton of a co-ordinate graph with scales is displayed on the screen. Of the six options available from this panel, it is usual to begin by creating a function using the Edit mode. A typical function such as

F(X) = X*SIN(X) + X^2

may then be graphed, changed, or differentiated exactly. Such a facility by itself is likely to give a great deal of confidence to a beginning Calculus student who has little facility with any elementary functions other than very simple expressions. The additional features allow considerable experimentation. For example, a zoom facility with the graphics system admits detailed study of small portions of the display window. The exact derivatives may be used to obtain and graph partial sums of a Taylor series of a function.

Another tool this package provides enables students to study functions of two variables. For any function which may be represented as an elementary function of two variables, the tool will provide an isometric graph. The axes may be rotated, tilted, compressed or expanded for views from various points of three-dimensional space. Some functions which provide interesting graphs are discussed in the March, 1986 issue of The College Mathematics Journal (Vol. 17, No. 2, pp. 172-181). On our front and back covers you will find graphs of two of the functions mentioned in that article. These graphs were printed directly from the CALCULUS PAD(tm) program using the PRINT SCREEN facility. Other functions defined mathematically in that article could be graphed with this program.

If you have access to an IBM compatible PC, you may acquire this program from the Calculus-Pad Coordinator in this department. The cost is $25 each (plus $20 for each additional copy ordered at the same time) for a 5.25" disk and documentation. Each copy is for use on only one microcomputer at any one time.

A software tool for linear algebra, MATRIXPAD(tm), also developed here, will be featured in our next issue (Fall 1986).

- Norm Rice & Jim Verner

MORE NEWS

Grace Orzech has been promoted to Associate Professor, effective July 1, 1986. She has been a full-time member of this Department since 1972. Her research interests are in algebra; she has published papers in Category Theory, Matrix Theory and Algebraic Structures. She is also co-author (with Morris Orzech) of a monograph on "Plane Algebraic Curves" (Marcel Dekker, 1981).

Dan Norman has been Acting Head of the Department during 1985-86 while **Lorne Campbell** has been on sabbatical leave. Once Lorne returns (later this month) Dan will replace **Bill Woodside** as Chairman for Engineering Mathematics for a two-year term.

Recent graduates will remember **Jennifer Read** (formerly Jennifer Affleck), a secretary in the Department since October 1980. On April 17 at 4:22 a.m. her daughter **Mikaela** arrived, weighing in at 7 pounds, 14 ounces. Jennifer reports that Mikaela routinely sleeps through the night and is a joy in general; she expects to be back at work next October.

Atiyah Visit

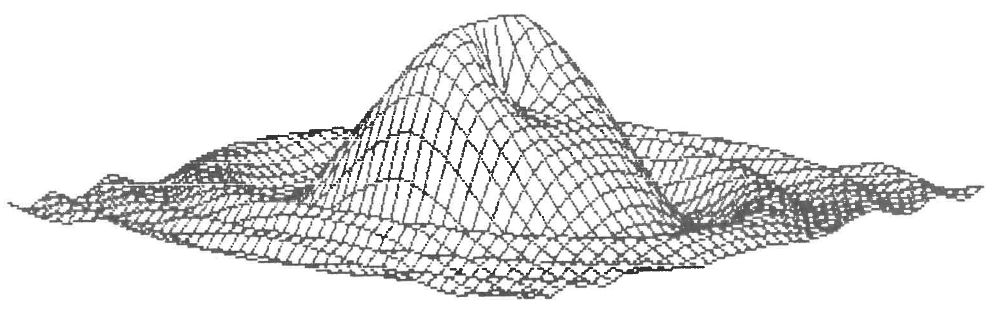
On Monday, April 7, 1986, the Department was honoured by a visit by Professor Sir Michael Atiyah of the Mathematical Institute at Oxford. Professor Atiyah is a world-renowned topologist, famous for (among many things) his discovery of topological K-theory and for his part in the Atiyah-Singer index theorem. He was awarded the Fields Medal (the mathematical equivalent of the Nobel prize) at the International Congress of Mathematicians held in Moscow in 1966. He is currently a Royal Society Research Professor, and is as well-known for the beauty and clarity of his mathematics as for its significance.

Professor Atiyah is currently on a tour of Eastern Canada which includes talks at Dalhousie, McGill, Carleton, and York Universities, as well as Queen's. His talk here, on "The topology of rational surfaces", was a tour-de-force. In it he reviewed some recent work of S.K. Donaldson, a former student of his and now a colleague at Oxford, who is famous for his discovery of exotic differentiable structures on R^4.

A DOUBLE APPEAL

Queen's Mathematical Communicator is sent free to all interested graduates. Production costs (printing and mailing, labour not included) of approximately one dollar per copy are met from the same hard-pressed Departmental budget which pays for marking, supplies, course notes, etc. If you want to help keep the Communicator coming, you are invited to send a small donation from time to time. Cheques may be made payable to: The Communicator, Queen's University. Thanks to those who have already contributed!

Our goal is summed up in our title; your letters are our lifeblood. Write us with news, views, a brief book review, and we'll put you in the next issue.



$$z = \frac{\sin(2x^2+y^2)}{x^2+y^2}$$

(See Page 16)

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