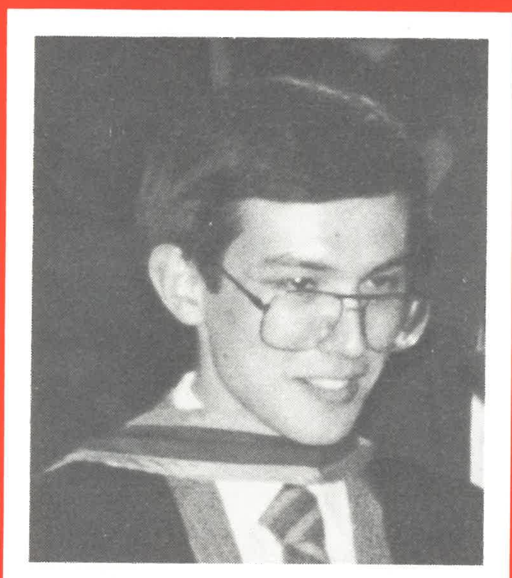
QUEEN'S MATHEMATICAL COMMUNICATOR



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REFLECTIONS ON A DOUBLE LIFE

Sel Caradus arrived at Queen's in 1964 from Auckland, New Zealand after doctoral studies at U.C.L.A. In 1979, he created a mild sensation (or at least a new topic of conversation in the Jeffery Hall lounge) when it became known that he was to be ordained as an Anglican priest. Since then he has lived a "double life", continuing to teach mathematics on a reduced time basis while also Anglican chaplain. From that experience, he writes as follows:

The study of mathematics provides intellectual stimulation and aesthetic satisfaction; to understand (or better yet, to discover) an elegant piece of mathematics is rewarding in ways which are difficult to equal. Religious commitment, on the other hand, involves an openness to wider possibilities and a struggle towards a world view which will take into account all the complexities of the universe and of human experience. The world of mathematics and the world of religious commitment seem far apart and from time to time I meet people who are surprised that I can be a part of both. While not seeking to deny the profound differences (the fundamental questions, the histories, the methodologies) which exist between the two, I find it also useful to reflect on certain similarities.

When, at a social gathering, I am asked the prototypical North American conversation opener, "What do you do?", I have the choice of two statements: "I teach mathematics" or "I am a university chaplain". It is interesting that, regardless of my choice, the responses are similar: nervous recollections of childhood experiences (distaste for the boredom of compulsory arithmetic or compulsory church-going), or guilt ("I never was any good at math" or "I'm afraid I'm not much of a church-goer"), or anger (about a ninth grade math teacher or about some alienating memory of church or synagogue). Such encounters remind me that both the mathematical and the religious enterprise are profoundly misunderstood by those who do not share in them. Mathematicians are imagined to be doing some form of high school math made more incomprehensible by additional complexity while religious people are thought to be proponents of a sterile dogmatism. That both fields might call for profound personal commitment to ongoing search is a reality which is not well understood.

In both areas, the present builds on the past. While religious traditions are often guilty of taking refuge in fixed formulations, at their best both recognize the need for continual review and clarification. Without rejecting the valid findings of the past, we see them in new ways. Both fields have histories in which exceptional individuals gifted with special insights play a primary role. And the word "gifted" is appropriate. For mathematical ability at the highest level is evidently not a result of diligence or of "works" (lest anyone should boast!) but is a mysterious phenomenon. Great mathematicians appear on history's stage without warning, often from unlikely beginnings. Equally unlikely are the origins of those who have enriched the world with religious insights.

Much more could be written on this theme but I will be content if I provoke some thought about unfamiliar possibilities. My own thinking in this area owes much to a recent work by Martin Gardner, well known for his contributions to the mathematical community through his Scientific American columns ("Mathematical Games") and also through his many books. In The Whys of a Philosophical Scrivener (Quill, New York, 1983), he reveals himself to be a "closet theist", expressing his convictions in this way:

> "Because I believe with my heart that God upholds all things, it follows that I believe that my leap of faith, in a way beyond my comprehension, is God outside of me asking and wanting me to believe and God within me responding."

While only a minority of mathematicians might express themselves in such a fashion, Gardner gives interesting reasons for his beliefs.

Thus while conventional wisdom places the mathematical and religious realms far apart, there is the possibility that they are neighbours after all.

WHAT SHOULD WE TEACH?

University departments, particularly in fields like Mathematics and Statistics, serve two masters. On one hand, we are here "to defend and uphold the faith": to preserve and extend knowledge in our chosen fields by pursuing basic research and teaching courses designed to equip at least some of our graduates to carry on after us. On the other hand, we are also charged with equipping students whose focus is in other areas - the hard sciences, the soft sciences, engineering - with the mathematical and statistical tools they will need in their chosen fields.

This dual role is one source of the title question, and "what should we teach?" is a matter of continual concern and review. We present here two pieces bearing on the question. The first, by **Dan Norman**, was spurred by a letter of Gordon Dowsley (Arts 1966); the second is extracted from the text of a lecture of **Donal O'Shea** (M.Sc. 1977, Ph.D. 1981). (In fairness to Donal, we should note that his remarks were not originally conceived as a response to our title question.) The intention is to open an ongoing symposium: responses and reflections on these issues are hereby solicited from all interested readers. In our next issue, we will present some overview of the feedback.

PART 1: BY DAN NORMAN

Last year our Department conducted a survey, described in the previous issue of the Communicator. It revealed that probably only a minority of our graduates ever use much of the university-level mathematics and statistics they are taught. If that is so, are we teaching the right things? A letter from Gordon Dowsley (Arts 1966, now Assistant Vice-President at Crown Life) helped focus such questions for me, so with his permission I quote from his letter:

"I was at Queen's some 20 years ago and I realize that the University I attended is much different from the University today. It is also true that the skills which a student should learn at University now are more extensive and in some cases different from what would have been appropriate at that time.

"The Department of Mathematics at Queen's in those days was no different from any department in any University in that it was faced with harsh economic realities. Queen's did not have a large student body (3,200 when I started, 4,800 when I graduated) and, hence, there was a problem in filling the classes. The Mathematics Department had a very basic choice. Should it produce a high quality graduate in theoretical mathematics who would be able to proceed to graduate studies and eventually earn his Ph.D. in mathematics? The perceived need for such a course was to maintain the reputation of the Department and the University, to attract staff, etc. In many ways it was an ego need for the staff...On the other hand, most students would not be interested in proceeding to graduate school in the study of mathematics, and so the subjects they would seek would be more applied...

"The average student entering University is really not aware of [this] dichotomy which a department faces. Queen's chose a heavy theoretical approach, and to fill the classes required all of us to take far more of those courses than most of us would ever need, at the expense of courses we could have used. We, for instance, could not take a statistics course until our third year. Only one insurance course was offered. No computer courses of any sort were available. No courses in operations research were available. No courses were available in mathematical techniques for City and Urban Planning. No courses were available on such techniques in economics as input/output analysis, econometric models, etc.

"I have often regretted that the fun in mathematics disappeared for me during my years at Queen's, and although parts of each course were enjoyable, I wish that my training had been less in the "theoretical" mathematics and more in the "applied" mathematics...

"I suspect that you now have many more courses in statistics and you probably even let the students close to a computer at some time. It would be nice if you could have courses in applied mathematics relevant to the social sciences, covering such topics as gravity models in urban design, input/output tables and various optimization models. I cannot overemphasize the importance of operations research mathematics, but I would stress that the mathematics in these courses should be stressed as mathematical tools used for operations research and that mathematics is only part of operations research...

"We were not given any concept of mathematics as a human endeavour whose expansion was due to people and that had a story of its own. What was more dangerous was that we learned rules of mathematics, but were not given the scepticism that these could and should be challenged as part of the process of advancing man's knowledge."

(I should add that Mr. Dowsley said some supportive and encouraging things too, and we are glad to hear them. However, they don't cause us to think afresh so I have omitted them.)

Things have changed in many ways, some of them anticipated by Mr. Dowsley. We are now a Department of Mathematics and Statistics, with eight statisticians, and our course offerings in statistics ~~are~~ much wider than they were and include many practical courses. There is now a Department of Computing and Information Science, and our own majors are required to take at least a half-course in computing and many take more. We offer courses in mathematical optimization, control theory and linear programming with applications, and we have had a flourishing program in Mathematics and Engineering for 20 years (mentioned in Mr. Dowsley's letter, though not in the paragraphs quoted above). Our students sometimes take a half-course in operations research from the School of Business. Our introductory calculus courses for non-majors include a far wider variety of examples, including some from the social sciences. For some years, **Peter Taylor** and **Joan Geramita** have been involved in sharing the teaching of a Biology course, and both have recently been cross-appointed to that department. However, we are far from offering courses which might be described as "mathematics applied to social sciences".

I think that to a large degree we are correct to refrain from offering such courses. A university, like any other large organization, requires some division of labour. We can and do teach the elementary techniques of Lagrange multipliers, for example, and even give some very simple economic applications. But the economists are much better equipped to take up the task of using these techniques for intensive study of economic questions. (Modern university economists do use very sophisticated mathematics - and are now widely suspected of engaging in impractical theoretical pursuits themselves!)

I do, however, feel a pang of sadness, perhaps even guilt, because I imagine that Gordon Dowsley is not alone in regretting that "the fun in mathematics disappeared for me during my years at Queen's". Perhaps the fun disappears because we do not do enough to help our students discover the satisfactions of using mathematical models to help understand the world around us, but there are other points of view. In our own department, **Peter Taylor** (among others) suggests that some of the joy disappears because we are too utilitarian: we require linear algebra in first year because it is needed in other disciplines and in our own upper year courses, even though it is not intrinsically very exciting. He would like to de-emphasize it, making room for more interesting topics from elementary number theory, groups, rings and fields. (Peter is also keen on modelling and problem-solving, so his views don't necessarily conflict with Gordon Dowsley's.)

It is not in fact so easy to decide what is a good curriculum. It is, of course, reassuring to professional mathematicians to teach a curriculum which turns out professional mathematicians, but I am not sure that Mr. Dowsley is entirely correct in suggesting that that is the reason we do so. Most university mathematicians have little expertise of the sort required to teach the kind of courses Mr. Dowsley suggests. Given the constrained resources and the increased pressure to do publishable research, devoting time to acquiring such expertise is low on our list of priorities. There is also considerable inertia and we are naturally reluctant to abandon a curriculum which does have some successes for another which we can hardly imagine. It is encouraging that some experimentation does go on: **Morris Orzech** has been offering an experimental course on finite fields in our department for two years, and **Peter Taylor** is trying his hand at teaching high school this fall. In addition, within our existing curriculum, we all move from course to course over the years, so things also change in this way.

Well, all of this rambling does not move us much forward, so let me conclude with a request and a last quote from Gordon Dowsley. The request: we would very much like to hear from graduates who have ideas for topics that could be used in presenting mathematics to undergraduates in ways that have potential for getting them involved and excited about mathematics, particularly good applications.

And the quote: "I would point out the danger to the department in thinking that education in mathematics teaches students how to think. This is the last bastion of people unable to relate to the real world."

PART 2: BY DONAL O'SHEA

Donal O'Shea began school in Cornwall, Ontario, where his father managed a store. After a Jesuit secondary-school education in Buffalo, New York, and undergraduate work at Harvard, he came to Queen's where he completed a Ph.D. under John Coleman in 1981. Since leaving Queen's he has taught at Mount Holyoke College (Massachusetts), with a year at the Institut des Hautes Etudes Scientifiques (France) on an N.S.E.R.C. post-doctoral fellowship.

In 1900, Hilbert, one of the greatest mathematicians of his [or any] era, posed a list of 23 problems which covered the entire range of mathematics at the time and which, he felt, would keep mathematicians busy through the next century or two. By 1930 or so, most of the problems were solved. There followed a period of increased abstraction and fragmentation. By the early 1960's, the abstraction had become so impenetrable and the subfields of mathematics so many, that mathematicians lamented that there could never be another Hilbert, another Poincaré, or another Riemann. Mathematicians had difficulty talking to one another, much less to other scientists.

Just when the whole subject seemed sure to fly apart into specialized, disjoint disciplines, results began to appear which pointed to deep connections among disparate fields. Arcane ideas proved unexpectedly illuminating in other, seemingly unrelated contexts. Exotic number systems turned out to be related to the structure of topological manifolds. It was discovered that there were spaces, among them the four dimensional space-time in which we live, in which it was possible to do calculus in different ways. The cohomological methods developed to study topology became essential in number theory and algebra. The solutions and solvability of nonlinear differential equations, which we use to model processes in the world around us, turned out to depend subtly on the geometry and topology of finite and infinite dimensional spaces. The study of singularities produced connections with bifurcation theory, number theory, and different geometries and topologies.

By the 1970's, the interpenetration of the mathematical subdisciplines had advanced sufficiently to produce both a feeling of wholeness and a host of powerful new methods to investigate problems that had formerly been completely out of reach. The period since 1980 has seen the solution of three of the most important, most beautiful, and most intractable mathematical problems in our culture's history: the Mordell conjecture, the Bieberbach conjecture and the four-dimensional Poincaré conjecture. [More on some of this elsewhere in this issue.] At the same time, the connections between mathematics and physics have been renewed. Differential geometric constructions, which were just beginning to be investigated around 1930, now lie at the heart of the gauge theories upon which our current hopes for a unified field theory rest; homotopy theory is used in analyzing crystal defects; nonlinear methods are beginning to shed light on the structure of turbulence.

These developments have profound consequences for the way we understand our world. Recent discoveries in differentiable dynamics have taught us how the behaviour of a system can be completely determined from one point of view, but utterly random from any practical point of view. [Analogy(?): Almost all real numbers are transcendental; but we know explicitly only a handful of transcendental numbers.] The concepts and theorems of differential topology have allowed us to start to speculate about morphogenesis and pattern recognition, processes important for biology and psychology. The study of singularities and extremal structures has revealed astonishing connections between local and global properties of mathematical objects. Someday, perhaps, these methods will allow us to understand the genesis of different levels of structure in complex systems and, perhaps, even speak to problems in psychology and sociology involving the behaviour of groups of people. Rich connections among statistics, probability, and pure mathematics have begun to yield new methods for formalizing inference and finding patterns in data which had previously defied analysis. Access to the awesome computational potential of modern computers promises to bring further insights.

These extraordinary developments pose an acute challenge to college and university teachers. Failure to bring this material into the classroom robs our students of any glimpse of the most exciting intellectual achievements of our times and deprives them of the chance to use these results to further their own understanding of the world. These considerations apart, there is no more solid testimony to the durability and ingenuity of the human spirit than the mathematical achievements of our species. Mathematics is our most highly developed art, and our deepest science. As such, our students deserve our best efforts in ensuring that they have every opportunity to become as mathematically adept as they desire. Anything less betrays the ideals of a liberal arts education.

Our department [at Mount Holyoke College] has tried to meet this challenge in various ways. For instance, we established a series of elementary seminars, which are quite nontraditional in both form and content. Our goal is to enable the thoughtful student to grapple with serious mathematical ideas and see how the precision of mathematics can be a liberating speculative medium. We have tried to refashion our upper-level courses to bring out the themes and introduce the ideas which have proved so important in recent years. All this, of course, demands a great deal of effort on the part of both students and faculty. The faculty must stay abreast of these developments and must recast them in a form suitable for undergraduates. This requires a genuine creative effort. Learning a new piece of mathematics is difficult at the best of times; making it accessible to an undergraduate requires a nontrivial reworking of the main ideas and a painstaking concern with exposition. The students, for their part, must struggle with exposition which is of necessity experimental.

NEWS AND LETTERS

Rick Mollin (K.C.V.I. 1967, University of Western Ontario B.A. 1971 and M.A. 1972, and Queen's Ph.D. 1975) is an Associate Professor at the University of Calgary, where he has been since 1982. He has also won a Killam Award for 1986 for a research project in Algebra. These prestigious awards are given to support "scholars of exceptional ability engaged in research projects of outstanding merit". Three Queen's professors - Fred Cooke and Jerry Wyatt in Biology and M.C. Urquhart in Economics - currently hold Killams.

After leaving Queen's in 1975 Rick was a "gypsy scholar", holding positions at six Canadian universities in seven years, before settling at Calgary. His letter expresses appreciation both for his varied academic experience ("staying in one place for all of one's professional career breeds complacency and regionalism") and for the stability of his present position ("coming to Calgary has allowed me time to work without concern about next year's contract"). Part of Rick's "gypsy" period was spent at Queen's on an NSERC U.R.F. - more information about these fellowships appears elsewhere in this issue ("New Appointments").

Rick organized a Number Theory seminar as part of the Canadian Mathematical Society's annual meeting in December 1985. He is now planning an ambitious international Number Theory conference to be held in June 1988 in Banff, and invites inquiries from interested Communicator readers.

Our High School Math Seminar series got off to a fine start on September 30 with some 50 senior high school students from Belleville, Napanee, Kingston, Gananoque, Brockville, and Prescott. At the opening session the students played some 400 games of snap (played with two decks of cards) of which 258 terminated in a "snap", giving a snap-probability of 0.645. The rest of the two hour session was spent in calculating the theoretical snap-probability, which turns out to be $1 - 1/e \cong 0.632$. The sessions are led by **Peter Taylor** and continued on a biweekly basis until November 25.

Peter Taylor has been teaching a section of Grade XIII Calculus at LCVI this fall. He claims to have successfully adopted all the hallmarks of a high school teacher including the box of kleenex on the right-hand corner of his desk. Seven weeks into the curriculum he reported that "my ideals are intact 'but they sure have changed'".

Ira Demsey (M.A. 1950) writes as follows:

"At long last I have retired. Received an edition of the Communicator on my very last day in the teaching profession. Am writing this note of thanks, and suggest you may discontinue sending them to me now. My intention over the next few years is to visit the world's great opera houses and concert halls."

W. Bart Lewalski (Mathematics and Engineering 1985) writes as follows:

"In February, I started working for Lynch Communication Systems in Reno, Nevada. So far, my job has turned out really well. I was first placed with a senior engineer to help him test a certain digital circuit. Then (to my surprise) I was put on a project for a pay phone application for the System 300 for Taiwan. I managed to have a prototype built and tested hours before one of our managers departed for Taiwan for testing. So far, the circuit works in all but one situation. We're now working to remedy that...The more I get into working and finding out about the education of my colleagues (especially the recent grads) the more I become convinced that Applied Math [at Queen's] was the only way to go."

Sally Cockburn (B.Sc. 1982, M.Sc. 1984) wrote last summer, from Botswana, as follows:

"My last few months at Queen's in the summer of 1984 were just about the most hectic of my life. I got married, defended my Master's thesis and left for Botswana in the space of five weeks. My husband, **Onno Derlemans** (B.A. 1983, M.A. 1984), defended his thesis a mere five days before we left the country. We began teaching at a private secondary school, Maru a Pula, less than a week after we arrived in Gaborone. Our first few terms were far from easy. We have not had to cope with the problems usually associated with teaching in Africa - lack of facilities, tropical diseases, military coups, corrupt administrations, - but we've had to battle with less exotic challenges like classroom discipline and adapting to a British exam-oriented school system. These problems have been minor, however, compared with the rewards we have reaped during our two-year stay. Aside from the numerous holidays we've been able to take - to Malawi, Zambia, Zimbabwe, Kenya and within Botswana itself - we have appreciated our geographical position for the insight it has given us into the southern African political situation. Gaborone has been raided at least three times since we arrived, making the regional conflict very real for us."

Sally and Onno have subsequently left Botswana to begin doctoral studies at Yale University.

DAVID KENNETH PICKARD (1945-1986)

The Department was saddened and shocked by the early and unexpected death, on July 28, 1986, of David Pickard, who joined us in July 1985.

David held B.Sc. and M.Sc. degrees from Mount Allison (1967) and Stanford (1971) respectively, and a Ph.D. in Statistics from the Australian National University (1977). Before coming to Queen's he spent eight years as a member of the Statistics Department at Harvard, where he won three university-wide prizes for excellence in teaching. His research was in the areas of stochastic processes and applied probability, particularly Ising models and sedimentation.

We have lost a valued colleague and a friend.

THE IRENE MACRAE PRIZE

Alan An Yuan Thompson of Halifax, Nova Scotia, became the first recipient of the recently established Irene MacRae Prize in Mathematics and Statistics at the May 31, 1986 Queen's convocation.

The prize, establish by **Margaret Crain** (B.A. 1927) in memory of **Irene MacAllister MacRae**, is awarded at graduation to the departmental medalist. Since graduation, Alan has gone to M.I.T. for graduate studies where he was promptly given an advanced calculus course to teach. At graduation he was also the winner of the Prince of Wales prize, given annually to the student graduating with a B.A. Honours degree with "the best academic records at Queen's".

The following information about **Irene MacRae** was graciously supplied by her daughter, **Mrs. R.B. Tackaberry** (B.A. 1941):

"**Irene MacAllister MacRae** grew up in the picturesque village of Chute-à-Blondeau on the Ottawa River between Montreal and Ottawa, where her father owned the local grain mill and operated a general store. She attended high school in nearby Hawkesbury, graduating with honours and participating actively in student affairs as president of the literary society and founder of the glee club. The love of music played an important role throughout her life; she was an accomplished pianist and much in demand as an accompanist. She entered Queen's in October 1911 in Honours Mathematics and Physics, graduating cum laude in 1914. During her years at Queen's she was involved in a wide range of student activities, serving as treasurer of Levana, vice-president of the choral society, president of the Queen's Y.W.C.A., and vice-president of the mathematics club.

"Following graduation, **Irene MacAllister** moved to Ottawa where she took a teaching position at the Ottawa Ladies' College. A year later she married **Alexander Ernest MacRae** (B.Sc. 1914), who later served on the Queen's Board of Trustees for some forty years and was granted an honourary LL.D. in 1945. [The A.E. MacRae Award in Creative Leadership, established "to promote the practice of effectively appraising action from the point of view of others concerned", is also presented annually. Mr. MacRae served also as president of the General Alumni Association.]

"**Irene** and **Alexander MacRae's** association with Queen's has developed into a family tradition: their five children, and several of their grandchildren, are also Queen's graduates. Mrs. MacRae maintained an active personal interest in Queen's all her life, and often made her Ottawa home available for alumnae functions."

Margaret Crain (née Farnham; B.A. 1927), who established the prize in memory of Irene MacAllister MacRae, is also part of a Queen's connection which has become a family tradition: among her relatives who are Queen's graduates are a brother and two daughters.

PROBLEM CORNER

Our October 1985 issue contained several checkerboard problems proposed by **Colin Blyth**, who has written up some solutions and hints. Interested readers are invited to inquire or submit their own solutions. Good problems or queries, with or without solutions, are always welcome. Just one new one for now: let W be a 4-dimensional subspace of a 5-dimensional vector space V. What is the probability that a point of V chosen at random lies in W?

THE NEW M.SC. IN STATISTICAL CONSULTING

BY J. TERRY SMITH

This is an exciting time for statisticians. The "data avalanche" precipitated by the computer continues unabated and the advent of the microcomputer has put tools for data collection and analysis into the hands of millions of new do-it-yourself statisticians. These factors, in a climate of growing markets for statistical services, are challenging our assumptions about the training of statistical practitioners.

We have known for years that the practice of statistics involves more than the identification of the best statistical method to solve the problem. Those who use statistics to solve problems for others (as well as many applied mathematicians) know that even the most elegant solution is wasted if the client to whom it is offered lacks the understanding, the motivation, the resources, the freedom, or the ability to use it. Likewise, the solution, however elegant, is irrelevant if it answers the wrong question, and identification of the right question often requires great skill at listening and questioning. The human dimension of problem-solving pervades much of the work of practicing statisticians, and failure to take it into account frequently leads to unproductive results.

Our Department has met this problem head-on. In the fall of 1985 we offered the first course in Canada to address the learning and improvement of statistical consulting in a coherent and comprehensive way. A method for improving the quality of statistical consulting, developed at Florida State University by Douglas Zahn and his colleagues, was adapted and reshaped, and became our course STAT 869, Statistical Consulting. The course begins with examination of what constitutes effective consulting, and formulation of criteria for assessing a consulting session. The next step is to collect data on consulting practice by videotaping consulting sessions; the student consultant then analyses these data with the help of an experienced coach in order to identify and alter aspects of the consultant's performance that diminish consulting effectiveness. This method has been used with success by Zahn and others in workshops for industrial and government statisticians in the U.S.

The new M.Sc. degree in Statistical Consulting, offered for the first time this year, attempts to integrate the statistical and interpersonal aspects of statistical practice. In addition to the course described above, the program requires a course in linear and nonlinear models and a consulting project. Additional courses are chosen from computational data analysis, applied multivariate statistics, stochastic processes, categorical data analysis, time series, sampling theory, and modelling of steady-state processes. The program is usually completed in one calendar year.

Jeffery Hall, the venue for the program, houses a range of supporting facilities which include generous access to an IBM 3081 mainframe computer, microcomputers with accompanying hardware and software accessories, videotaping equipment, the George L. Edgett Statistical Laboratory providing statistical consulting for the university, and an excellent mathematics and statistics library.

If you are interested in this program, or know of others who may be, the person to contact is **Harold Still**, Chairman for Statistics, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario, K7L 3N6.

ON BABYTALK, MONEY, NUMBNESS AND TWEEDS

The words we use every day without much thought are rich storehouses of forgotten meanings and associations. Here are a few true stories about the strange relations of some mathematically-minded words.

Matrix is Latin for womb, the essential thing about a matrix being its capacity to contain things (its entries). In late Latin, matrix comes to mean also a public register (again the idea of a form to contain things), hence matriculation (registration) of students. The root from which matrix comes is the Indo-European ma-, surely as old as babytalk and therefore older than language. (Thus the origins of linear algebra are not only prehistoric, but prelinguistic!) The same root ma- gives rise to mammal and mother (via Latin mater) as well as matter. The latter word may belong more to physics than mathematics, but the story is too good not to tell. Mater was used for the trunk of a tree (the "mother" of the branches) and came to mean first timber, and then any hard substance, or matter. It probably matters to no one that it is from the same association with timber that the (well-wooded) island of Madeira takes its name, hence also the wine made there. (Next time you find yourself quoting the famous line of Flanders and Swann, "Have some Madeira, m'dear!", remember its connection with linear algebra!)

Mathematics comes not from the same maternal root ma-, but from the Greek μαθαινω (manthanien), to learn, with derivative mathema (that which is learnt) and its plural, mathemata. (Compare "pathe mathos" - man learns through suffering - from Aeschuylus' Agamemnon.) Digging back before the Greek verb one finds that μαθαινω comes from the same Indo-European root men- (to think) from which we have such related words as mind, mental, mention, reminisce, and maniac. Automaton (that which thinks, hence (?) acts, for itself) is from the same root, as is money! The derivation of this latter word is truly worth mentioning: men- (to think) gives rise to mon- (to cause to think, hence to warn, as in admonish). Moneta thus becomes an epithet of Juno, reflecting her warning role in Roman mythology. Hence the coinage struck at the Roman temple of Juno becomes known as money, and places where coins are made are mints.

Number has all sorts of interesting relations. It turns out, amazingly enough, to be no accident that it's spelled the same as number (more numb)! The Greek verb (δια)νεμω (nemein), to distribute, gives nomos, anything distributed, and is related to Latin numerus, number, for counting out that which is distributed. Meanwhile the same Greek nemein becomes the German verb nehmen and the Middle English nimen, both meaning to take. The past participle of nimen was nume, taken or seized, whence our word numb (feeling taken away). The Middle English verb nimen, to take or steal, was later shortened to the verb to nim, which you might recognize as the name of a mathematical game involving taking. Nimmer, a word which used to mean thief, has gone obsolete, but its derivative nimble remains. The Greek root nemein, to distribute, is also behind Nemesis, goddess distributor of retribution, represented for many of our students by the dreaded duo, ϵ and δ. Nemein also gives us nomad, via the association of distributing with roaming. Roaming still further, nemein gives nomos, first anything distributed, later anything sanctioned by custom, for example local coinage, hence numismatic!

The transformation of Greek nemein (to distribute) to Middle English nimen (to take) is a lovely instance of a phenomenon which happens often in the evolution of language: words often give birth to descendants with the opposite meaning. (Sometimes it even happens simultaneously: a high mountain and a deep ocean are both, in Latin, described by the adjective altus.) In language formation, as in thought generally and mathematical thought in particular, we seem to rely equally on recognition of similarity (the making of metaphors and analogies) and the association or pairing of opposites (polarity, orientation, duality).

Speaking of ϵ and δ, duo is, of course, etymologically linked with two. Some surprising relatives of two are twill, a "two-thread" fabric (compare drill, something our students sometimes complain of too much of, but also a "three-thread" fabric) and tweed. The latter is a confusion of tweel, the Scottish form of twill, with the Tweed River, which winds through a manufacturing region of Scotland where it (tweed cloth) is made. Twin (as in twin prime) is related to two, as are twine (a double or twisted thread) and twisted (made of two strands twined together) and twig (originally, anything forked or divided in two) and between. Dual and duality are obviously linked with two, as is duel, a fight between two persons.

This brings us to one and units and ... but those are other tales for other days.

NEW APPOINTMENTS

Ernst Kani, Assistant Professor and NSERC University Research Fellow, joined our Department in July 1986, with B.Sc. and M.Sc. degrees from the University of Toronto, and a doctorate from the University of Heidelberg, Germany, 1978. He has been the equivalent of an Assistant Professor at Heidelberg since 1980. His main research is in number theory and algebraic geometry.

(University Research Fellowships ("URF's")are awarded by NSERC to outstanding Canadian researchers in various fields, usually for five years, with the goal of enabling them to establish a research program while getting some teaching experience. A further objective of the program is to keep Canadian researchers in Canada until larger numbers of regular positions become available through retirements at Canadian universities.)

Bill Ross, Assistant Professor, joined us at the same time with B.Sc. and M.Sc. degrees from the University of Saskatchewan and a Ph.D. from Queen's, 1984. He has extensive experience at the Saskatchewan Research Council in data analysis and statistical consulting. His Ph.D. thesis won the Pierre Robillard award for the best Canadian thesis on a statistical topic.

ALGORITHMS -- PART 2

MULTIPLICATION IS NO HARDER THAN SQUARING

The first part of this series (October 1985 issue) dealt with what everybody knows computer scientists studying algorithms do: come up with new algorithms. In this part we discuss a simple result in a different vein. Consider the problems of squaring an n-bit binary integer and multiplying two n-bit binary integers. Is it possible that one of these may be done asymptotically more quickly than the other? Let

$S(n)$ = worst-case time to square an n-digit number
$M(n)$ = worst-case time to multiply two n-digit numbers .

Since the first of these problems is a special case of the second, it is clear that $S(n) \le M(n)$. However, we can derive a converse. The essential ingredient is the identity

$$ab = \frac{((a+b)^2 - a^2 - b^2)}{2} \quad .$$

Subtraction takes linear time in the number of bits; squaring must take at least linear time. Therefore there is some constant k_1 such that

$$Sub(2n+2) \le k_1 S(n)$$

where $Sub(n)$ is the (worst case) time to subtract two n-bit binary numbers. Now squaring an $n + 1$ bit number takes at most linear more time than squaring an n bit number. This may be shown by breaking up the multiplication process as follows. Let i be a one-digit number, and x an n-digit number. Then

$$(i2^{n+1}+x)^2 = i^2 2^{2n+2} + 2ix2^{n+1} + x^2 \quad .$$

The extra multiplications i^2 and ix may be done in linear time showing that the $n + 1$ bit squaring may be performed using only linear more time than the n bit squaring. Therefore, there exists some constant k_2 such that

$$S(n+1) \le k_2 S(n) \quad .$$

Division by two may be performed on binary numbers in constant time by deleting the right-most digit, hence for large n it can be ignored. Putting the pieces together we may state:

$$M(n) \le (k_2 + 2k_1 + 2) S(n) \quad .$$

Therefore, if anybody comes up with a better algorithm for squaring numbers, this algorithm can also be used to multiply two numbers in the same amount of time, up to a constant factor. Since there is no proof that the current best algorithm for multiplication is optimal, this result could be of practical as well as theoretical value: if a fast algorithm for squaring could be found, it would yield an equally fast algorithm for multiplication.

Mike Swain
B.Sc. 1984

THE MATRIXPAD™ MATRIX CALCULATOR

In our previous issue you read about the Applied Science Microcomputer Project, and the CALCULUS-PAD™ program provided to Applied Science students for use in Calculus. Another program distributed on the same basis is MATRIXPAD, a matrix calculator designed specifically for use by students and instructors in introductory Linear Algebra courses.

In developing MATRIXPAD I had in mind the model of a calculator, particularly one with a stack for storing information. The MATRIXPAD default screen shows a four-level stack of matrices -- two of these are totally visible; the other two are revealed by information about their shape, but to display their entries a special key must be depressed.

The bottom matrix in the stack can easily be edited, both as to shape and content, using the keyboard. Because MATRIXPAD is intended as a classroom and textbook adjunct it accepts input such as 2/3, -1/2, etc. and can provide corresponding rational answers (or decimals if the user prefers). MATRIXPAD handles matrices of size up to 6 by 6.

Once a matrix is entered it can be moved on the stack, or arithmetic operations can be done on it, by hitting a single key. For example, the matrix can be inverted, transposed, or reduced to its row-echelon form; its determinant can be found, a Gram-Schmidt orthogonalization can be done on its rows, and it can be saved on disk and recalled. Binary operations such as addition, multiplication or projection of the rows of one matrix onto the subspace spanned by the rows of another can also be carried out on the matrices in the stack.

Because of the basic importance of elementary row operations I decided to include a mode for doing row operations one at a time. Immediate feedback is provided on attempts to do illegal operations, and on successful completion of row-reduction. Coupled with a printing feature, this stepwise procedure facilitates the generation of classroom examples, and of student work which is easy to check. Feedback on attempts to do illegal operations is not limited to this setting -- it is provided in general, for instance if the user tries to multiply incompatible matrices.

Response from students and instructors to MATRIXPAD and CALCULUS-PAD has been quite favourable, reflecting an acceptance of, and perhaps a need for, tools which can keep the tedious aspects of certain calculations from interfering with their practice and illustration in the classroom.

- **Morris Orzech**

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 & 0 \\ 3 & 0 & 6 & -1 & -3 \\ 1 & 1 & 1 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

BOOK REVIEW - BY COLIN BLYTH

A HANDBOOK FOR SCHOLARS

M.-C. VAN LEUNEN

PAPERBACK $15.95, KNOPF 1985

From the introduction: "The built-in limitations of scholarly prose are no excuse for bad writing. Bad scholarly prose results, as all bad prose does, from laziness and hurry and muddle...The scholar should aim at clarity and force and grace, but so should every writer...For help with punctuation, spelling, grammar, diction, style, organization, logic, and rhetoric, look elsewhere. For help with citations, quotations, footnotes, references, and reference lists, you have come to the right place."

In spite of the disclaimer, this book has a lot of excellent advice about style, and especially about mathematical style because of the many mathematical examples. If you are seriously interested in making your mathematical writing more readable and more publishable, this book is a MUST. The following are some quotes that made me realize "There, lacking the grace of God, went I":

Scholarly writing, like all writing, improves as it becomes more concrete and less abstract.

"Reference" is not a verb. At the least, you should correct it to "refer to".

"The lemma above", not "the above lemma". I never heard anyone try to say "the below lemma", but that's wrong too. "Above-mentioned" is a vile locution.

The "we" that means "you and I, reader" is a tricky one. "We see", "we observe", and "we notice" are dangerous. The reader who doesn't see, doesn't observe, doesn't notice resents the pretense that he does. This "we" is just another version of the rhetorical error often found in "obviously" and "of course". Anything you label obvious had better be very obvious indeed, or your reader may think you're high-hatting him. (It's obvious to ME, but you peasants probably need to be told.)

"And so, then, these results are therefore ...". If you have fallen into this habit, try excising all connectives and then restore only those you need for emphasis.

Begin at the beginning and go on till you come to the end; then stop. That's excellent advice, and every scholar should heed it. There is no rule of composition that says you must end with a summary of everything that's come before - honestly there isn't.

[Footnote: The above-mentioned advice to "begin at the beginning ..." is, of course, the instruction the King of Hearts gives the White Rabbit in the trial scene near the end of Alice in Wonderland. Van Leunen uses it without identifying the source. This seems odd in a book which promises "help with citations, quotations, footnotes, references, ..." Perhaps she thought the source so well known that no reference was needed. -Ed.]

THE FIELDS MEDALS AND ICM 86

The most prestigious international awards in mathematics, the Fields Medals, are named after a Canadian, Professor John C. Fields of the University of Toronto. He became interested in establishing such awards at the time of the International Congress of Mathematicians (ICM) in Toronto in 1924, and left money in his estate to endow them. The first Fields medals were awarded in 1936 at the ICM at Oslo, and 2 to 4 awards have been given at each ICM since then. The prizes are intended to recognize promise as well as achievement, so the selection committees have always chosen mathematicians under the age of 40.

ICMs normally occur every four years, and the 1986 ICM took place at Berkeley, California in August. Nearly 4,000 mathematicians from around the world attended, including at least 8 from this department: **Leo Jonker**, **Paulo Ribenboim**, **Leslie Roberts**, **Jim Woods**, **Eddy Campbell**, **Dom de Caen**, **Dan Norman**, and **Jan Minac**, a graduate student who recently completed a Ph.D. under Ribenboim. Three Fields Medals were awarded: to Simon Donaldson of Oxford University; to Gerd Faltings, originally from West Germany, now at Princeton; and to Michael Freedman of the U.S.A., now at the University of California at San Diego. There was also a new prize, the Nevanlinna Prize, awarded to a young mathematician for work in the information sciences. The first recipient was Leslie Valiant of the U.K., now a professor at Harvard.

Gerd Faltings was awarded his medal for work with vast implications in algebraic geometry and number theory, including proofs of several important conjectures (e.g., of Tate and Shafarevich) about abelian varieties. One consequence of this work is a proof of the famous <u>Mordell conjecture</u>: any curve of genus greater than one, over a number field K, has only finitely many K-rational points. In particular this implies that lots of interesting Diophantine equations have at most finitely many solutions. For example, it follows from Faltings' affirmative resolution of the Mordell conjecture that the Fermat equation $X^n + Y^n = Z^n$, $n > 3$, has <u>at most finitely many</u> integer solutions. (Of course, the question whether there are <u>any</u> solutions remains open!) The history of the Mordell conjecture is essentially the history of number theory and algebraic geometry since about 1920; the interested reader should consult the lead article in the June 1986 issue of Notices of the American Mathematical Society.

Freedman and Donaldson won their awards for work on four-dimensional manifolds, bearing on (among other things) the <u>Poincaré conjecture</u>, according to which a compact n-dimensional manifold with the same homology groups as the n-sphere "is" the n-sphere. Here "is" may mean "is homeomorphic to" ("continuous Poincaré conjecture") or "is diffeomorphic to" ("differentiable Poincaré conjecture"). Two-dimensional manifolds were classified long ago, and, curiously, the next case to be settled satisfactorily was the case of dimension greater than four, around 1960. In the 1970's, Thurston developed the tools for classifying 3-manifolds, winning a Fields medal for his work. (The Poincaré problem remains open in dimension 3, however.) Freedman's work provides a classification of 4-manifolds under homeomorphism, essentially by establishing a correspondence between classes of manifolds and certain quadratic forms which exist only in dimension 4. A result of this classification is the proof of the continuous Poincaré conjecture in dimension 4.

Donaldson built on Freedman's work, adding ideas from modern physics such as gauge theories and their relations to connections, instantons, and Yang-Mills fields, to obtain a <u>differentiable</u> classification of compact 4-manifolds. The result is surprisingly different from the results in other dimensions, and the differentiable classification also turns out to be quite different from the continuous one. The work of Freedman and Donaldson also has consequences for non-compact manifolds; one of the more surprising of these is the existence of a 4-manifold which is homeomorphic to 4-dimensional Euclidean space but not diffeomorphic to it.

Valiant's work is in complexity theory, which studies such questions as "how long" a given algorithm (or class of algorithms) will take to produce a result: does the time increase linearly with the number of variables? in polynomial fashion? exponentially? The work which won him the Nevanlinna Prize involves Chomsky's context-free grammars, n-superconcentrators, Turing machines, search problems, etc. and is an intricate blend of ideas from mathematical logic, graph theory, and algebra. The interweaving of diverse fields within mathematics, and the interplay among mathematics, computer science, and physics, much in evidence in the work of all four prize winners discussed here, were in fact repeated themes throughout ICM 86, and are the hallmark of much of the best work in mathematics today.

An interesting article in The New York Times, August 12, 1986, reports on a media interview with the four medalists. In response to the question whether they used computers in their work, Freedman and Donaldson gave the same one-word answer: "No". Faltings' response: "Perhaps it could reduce some sorts of tedious work for us, but it doesn't do the thinking." Personally, he says, he doesn't use one. And then, according to the N.Y. Times account, "eyes turned to Leslie G. Valiant of Harvard University, winner of the recently established Nevanlinna Prize for Information Science, whose work centered on computer algorithms. 'Maybe I should clarify my own position', Dr. Valiant said. 'I don't use computers either.'"

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