

QUEEN'S MATHEMATICAL COMMUNICATOR



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George L. Edgett,
about 1960

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MUSINGS OF A CALCULUS TEACHER

BY PETER TAYLOR

For many years I have been dissatisfied with my teaching, without knowing exactly what the problem was. And in my frustration, I was, at one time or another, putting the blame on everyone and everything that crossed my path: my colleagues, my students, the curriculum, the textbooks, the exams, the university, the high schools, and quite generally, "the system", whatever that is. Recently I have had a few experiences and insights which have shown me that the problem lay more with me than with anything else, that it was quite deeply rooted, that the solution might be within my grasp, but it would take a lot of imagination, possibly more than I possessed.

One of the more important experiences has been working side by side for the past three years with a very talented teacher of English, Bill Barnes. He and I have been teaching the "math and poetry" course together, and so for one and a half hours each week I would watch him closely, study his effect on our students, and try to figure out what it was that made his teaching so fine, and made his students care so genuinely about what he was talking about. For a while I got viciously sidetracked by the idea that it was mostly because his stuff was humanities and my stuff was science, and the humanities are warm and human whereas the sciences are cool and impersonal. But deep down I knew that wasn't true, and had to be a red herring. And anyway, it got so that what I was doing in the math and poetry course was just as fine as his stuff, and seemed to have the same sort of effect on the students. But I couldn't make it happen in my other courses; I didn't even know how to begin. Especially my calculus course.

Why could I not do in my calculus course what I was able to do in the math and poetry course? Presumably the reason was that I was a free agent in the latter course, and could do anything I wanted, provided it was good stuff, whereas the calculus course was an important part of a curriculum hierarchy and there were severe constraints on material, pace, testing, certification, etc. And yet, Bill Barnes teaches basic courses in the Department of English, which have to fit into the Honours English stream, and from what I've been told, those courses are every bit as fine as the math and poetry course. So what is the essence of the difference? What is it that he can do in his English 160 that I can't seem to do in my Math 121?

Well, I now think I know what it is: *he teaches what he does, and I don't!* It's that simple. Well, it may seem simple, but I guess it has a lot of difficult and perplexing ramifications.

He teaches what he does. When he brings to class a poem or a story or even an entire novel, it is something that affects him deeply and always has, something which challenged him when he first laid eyes on it and challenges him still today, something which he is happy to confront his students with again and again because it has new life each time it is read. It is, in a word, a work of art, a whole work in itself: it has integrity. He would not presume to bring anything less than that into his classroom.

But I don't. With very few exceptions, the problems in the calculus textbook I am using hold no real interest for me. They are not problems I would choose to spend time on if I had a choice, not now, nor even when I was a student. They don't appear to me to be real problems at all — they are more like technical fragments. They are certainly not works of art: they have no integrity. To put it bluntly, when I am teaching calculus, I

have a small but steady feeling that I should be apologizing to my students for the poverty-stricken nature of my material.

On the other hand, the problems I now bring to the math and poetry course are all problems that, when I first encountered them, I could not wait to find a moment to think about. And they have an enduring quality; I am constantly surprised at their capacity to yield new insights. They are indeed works of art. And they are quintessentially mathematical: not only is my activity in solving them mathematical, but the techniques I am required to develop or recall belong to the standard body of mathematical content (though they rarely fall nicely into any of our standard courses). They are, in a word, the poems of mathematics, and I always bring them to class with a feeling of great pride, not for myself (except maybe just a tad), but for my subject.

Now why don't I bring poems into my calculus course? That's a good question. Certainly the calculus contains some rich and wonderful problems, but I'm not sure I know enough of the right kind to build a course from, at least what the curriculum designers would think of as a course. And I'm not sure that I have the imagination or the time or the will to try to build a course that the curriculum designers wouldn't recognize as a course. In fact, I think the real problem has to do with the very model of science education that we have come to adopt, and now adhere to in a way that has made us lose sight of the fact that there might even be another, totally different, way to organize the teaching of science. [I see I'm getting around to blaming something other than myself after all.]

Indeed, I now believe that for better or for worse, and mainly for worse, we have decided to adopt a model of science education which can best be described as narrow, fragmentary, uninspiring, and ineffective. It's a model which is quite different from the model on which the curricula of the humanities are based, which is, at its best, holistic, artistic and quite effective. It is conceivable that the humanities might also have adopted the model we use in science, but they did not. Incredibly enough, the reason the sciences went for the ineffective model was that at some point somebody decided that science was just too important to be taught in any other way. The humanities escaped this disaster because nobody who counted in the educational world thought they were important enough to worry about.

Let me illustrate with a story. The year is 2100, we have somehow overcome the nuclear threat, new and virtually unlimited sources of energy have been found, everybody has enough to eat and a stable of personal gadgets that would blow your mind, and a new technological revolution is brewing, whose nature I will not make precise except to say that it is based not on mathematics, as was the Sputnik revolution, but on poetry. It is now deemed important, if our society is to have its fair share of the fruits of the new technology, for every school child to be able to read, understand, appreciate, and even write poetry. The President of the United States promises to dedicate a sonnet to every star in the Milky Way before the end of the decade. All over the continent, educators are rising to the challenge. Poetry is being studied with a new intensity, with the objective of understanding just what are the technical skills necessary to read and write poetry. Curriculum designers are deciding how to organize these skills in a coherent, hierarchical fashion, so that what is taught in grade six can be built upon in grade seven. Textbook authors are recruited; new curricula are written; it's called the New Poetry.

One slight problem is that the right examples just aren't around. The poems in the literature are much more than is wanted or needed to illustrate a particular technique; after all, too much of a good thing can get in the way. Worse, a poem which provides an ideal illustration of some rhythmical point for the grade 7 course in metre turns out to be a dramatic monologue, and they aren't covered till grade nine. The response, of course, is to use isolated lines or stanzas to illustrate the techniques: a line of Milton here, a quatrain of T.S. Eliot there. That's all very well in principle, and textbooks of poetry begin to appear without a single entire poem in all their 400 pages. This strikes some of the more reactionary educators as somewhat unreasonable, and so poems are specially commissioned to illustrate one aspect of the craft but no others, at least none that are the province of more advanced grades. These made-to-order poems become quite popular, and with minor variations tend to appear in all the leading textbooks.

I think I've said enough; I'm sure you get the point. But I can't resist one more flourish. Educators are worried by the fact that although everybody studies poetry, they don't really seem to enjoy it; it doesn't really seem to grab the students in the centre of their being. Worse still, while boys seem not to mind too much playing mechanistic poetic games, girls are disenchanted and tend to drop out of poetic studies at the earliest opportunity, preferring instead courses such as mathematics which belong to the old technology but have now only an aesthetic significance and are taught mainly by women. A campaign is started to get more women teaching poetry...

* * *

[For another perspective on what we teach and why we teach it, the reader might enjoy the inflammatory review by Underwood Dudley in the American Mathematical Monthly, May 1987, pp.479-483. - ed.]

GEORGE LEWIS EDGETT

1900 - 1986

Professor Emeritus George L. Edgett of Queen's University died November 19, 1986, at Kingston, after a brief illness. He was predeceased by his wife Muriel, and is survived by sons Gordon and Alan, five grandchildren and a great-granddaughter.

A native of Sackville, New Brunswick, Edgett taught school for four years after graduating from Mount Allison University with an M.A. He then went on to the University of Illinois, where he received a Ph.D. in 1936; meanwhile, he had joined the Department of Mathematics at Queen's University in 1930. Edgett remained at Queen's until his retirement in 1969, and after retirement he continued to teach here part-time for several years.

Soon after his arrival in 1930 he persuaded the department to let him offer a course in statistics, and this was apparently the beginning of formal instruction in statistics in Canada. The statistics program here gradually expanded under Edgett's guidance. (In recognition of that growth, and reflecting the special nature of statistics as a discipline related to mathematics but not really part of it, we became the Department of Mathematics and Statistics in April, 1978.)

During his long career at Queen's, George Edgett introduced a great many students to statistics, and many of today's prominent statisticians and teachers of statistics were his students, among them Colin Blyth and Harold Still of this department, and Ralph Bradley, Roger Davidson, Agnes Herzberg, Ian MacNeill and John Wilkinson. Bradley and Blyth graduated from Queen's in the 1940s, and both are now Fellows of the American Statistical Association. John Wilkinson (M.A., 1952) is also a Fellow of the A.S.A., and all three went on to complete Ph.D.s in statistics. Harold Still, also an Edgett student and colleague, is completing a five year stint as Chairman for Statistics in our department; his decision to go on to a Ph.D. in statistics (1961, Virginia State University) was inspired by his experience in assisting Edgett and Daniel B. DeLury in a summer course in statistics for people from industry.

George Edgett was named a Fellow of the American Statistical Association in 1963, and in 1980 the Statistical Society of Canada honoured his contribution "to the emergence of statistics as both a science and an applied discipline" by electing him (with four others) as its first honorary member. During the 1930s and 40s Edgett developed Bachelors and Masters programs in Statistics at Queen's, and he directed many Masters students who went on to complete doctoral studies. In the late 1940s and 50s he developed statistics courses for engineering students, medical students, and graduate students in other fields.

During his Mount Allison days, Edgett made a name for himself as a hockey player, and after coming to Queen's he coached junior hockey as well as the Queen's hockey team; he also served for many years on Queen's Athletic Board of Control. During the Second World War he served as an officer in the Royal Canadian Air Force where (among other things) he taught navigation. He was a church elder (Chalmers United, Kingston) and an active mason, a member and former grand master of Queen's Lodge No. 578.

Edgett's former colleagues remember him with warmth; he is missed. His legacy, including a generation of statisticians who were his students, lives on as an enduring monument to his pioneering work for statistics in Canada.

STATLAB

Until the mid 1950s, George Edgett was the only statistician in the Department of Mathematics at Queen's, and he was responsible for virtually all the statistical consulting done here. Among the first faculty members to share the teaching and consulting of statistics with him were M. T. Wasan (from 1959), John W. Wilkinson (1957-59) and Harold A. Still (1953-55 and 1965 to the present). Toward the end of the 1960s the number of faculty members in statistics began to increase rapidly and at present there are nine faculty members in the department whose primary responsibility is statistics.

Indeed in the late 1960s and early 70s many departments at Queen's went through a period of rapid growth, and as programs in all disciplines expanded, so also did the demand for statistics teaching and consulting, the latter in connection with both student projects and faculty research in other fields. There were still no formal arrangements for statistical consulting; anyone looking for help had to find a willing statistician and interest him in his problem.

George Edgett had pressed continually for a Statistical Laboratory at Queen's over the years. In 1968 the Computing Centre had hired Louis H. Broekhoven to help researchers with statistical computing, and when Donald G. Watts joined the Department of Mathematics in 1970 he drew on his experience in establishing a statistical consulting service at the University of Wisconsin to urge the implementation of a similar centralized service at Queen's. These efforts culminated in the establishment, in 1973, of the Queen's STATLAB, with Don Watts as its first Director.

Initially, STATLAB was located in Rideau Hall, to emphasize its independence of the Department of Mathematics and its availability to all members of the university community. It was relocated in Jeffery Hall in January 1977, whence it continues to serve researchers in a wide range of disciplines. In 1979, STATLAB was renamed the George L. Edgett Statistical Laboratory, in honour of Edgett's profound and pioneering contributions to statistics in Canada.

The following statement of purpose, taken from a recent STATLAB report, describes STATLAB's role:

The main purpose of STATLAB is to provide a statistical consulting service to the Queen's community, serving graduate students, researchers, undergraduates, and administrative personnel.

STATLAB is motivated by a desire to increase the quality and utility of information derived from statistical data generated by the Queen's community, and to help users of STATLAB to become less dependent on its services.

GEORGE L. EDGETT SCHOLARSHIP FUND

Former students, colleagues and other interested parties are invited to contribute to a fund, now being set up, named in honour of George Edgett and intended to provide scholarships for outstanding undergraduate students in statistics.

At present, Queen's has no scholarships designated specifically for statistics students. Since Edgett's courses at Queen's were the beginning of formal instruction in statistics in Canada, it is fitting that his contribution be honoured in this way.

Cheques destined for this scholarship fund should be made payable to Queen's University and sent to:

G. L. Edgett Scholarship Fund
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario K7L 3N6

Receipts for income tax purposes will be issued.

CMS MEETINGS

The Canadian Mathematical Society held its annual Summer Meeting at Queen's, May 28-30. More than two hundred mathematicians converged on Kingston for the gathering, which featured invited talks by eminent mathematicians, special sessions devoted to papers in particular areas of mathematics, events with mathematical education as their focus, a business meeting, and various social events. Several publishers had booths in the registration area to display their latest wares, at the textbook and research monograph level. As always, the meetings provided ample opportunities for the participants to get together informally to re-establish working connections and exchange ideas and professional gossip. For many (if not all) practising research mathematicians, this kind of cross-fertilization is a major stimulus in their work.

Professors Eddy Campbell and Leslie Roberts of this department were members of the committee which organized the scientific program, and Grace Orzech looked after local arrangements such as housing for the participants. The invited talks were on Computational Complexity and Constructive Proofs (S. A. Cook, University of Toronto), Ramanujan's Theory of Theta Functions and Modular Equations (Bruce Berndt, University of Illinois), Homotopy Groups and Commutative Rings (S. Halperin, University of Toronto), Orthogonal Representations of Graphs (L. Lovasz, Eötvös Loránd University, Budapest), and Differential Operators over Singular Varieties (J. T. Stafford, University of Leeds). Vice-Principal Duncan Sinclair opened the meeting with a short speech welcoming the participants on behalf of the University.

A special feature of the annual summer meeting is the Jeffery-Williams Lecture; it was inaugurated in 1968 by the CMS (then the Canadian Mathematical Congress) to recognize mathematicians who have made outstanding contributions to mathematical research. This year the recipient was L. Nirenberg (Courant Institute, New York); his talk, on Nonlinear Elliptic Partial Differential Equations, described some of the recent progress in this field.

The five Special Sessions devoted to short papers in specialized areas were in Noncommutative Rings, Theoretical Computer Science, Homotopy Theory, Several Complex Variables, and Numerical Optimization.

In the direction of mathematics education, one evening was devoted to a fruitful panel discussion, organized by Peter Taylor, on the topic, "Where Are We Ten Years After the Coleman Report?" The reference is to Background Study #37, "Mathematical Sciences in Canada", a document put together in 1976 by Klaus Beltzner, John Coleman and Gordon Edwards, reporting on a study undertaken for the Science Council of Canada. The panelists were Professors Ed Barbeau (University of Toronto), Joan Geramita of this department, Bernard Hodgson (Laval), Bernard Madison (Arkansas), Tony Thompson (Dalhousie) and David Wheeler (Concordia), with Jack Fernley of Air Canada and Maria Klawe of IBM, San Jose. (Gordon Edwards, incidentally, was a graduate student here (Ph.D. 1972) whose thesis was in the area of Grothendieck-style Algebraic Geometry; he has since gone on to become a prominent and articulate spokesman for anti-nuclear organizations in Canada, and is now President of the Canadian Coalition for Nuclear Responsibility.)

A second feature in mathematics education was the address on "Calculators with a College Education?" by Herbert S. Wilf, currently Editor of the American Mathematical Monthly. His talk, delivered to a joint meeting of the CMS and the Canadian Mathematics Education Study Group, examined the implications, for how and what we teach, of some recent technological advances in computing and calculators.

For many of the mathematicians in attendance, these three days of CMS meetings also provided a chance to renew old ties with Queen's and Kingston. Queen's was the home of the first CMC Summer Research Institute in 1949, and has hosted it many times since, and over the years these summer institutes (and other meetings and conferences) have attracted a generation of Canadian mathematicians here.

NEWS

Jim Woods became a Fellow of the Royal Society of Canada at its annual meeting recently; the honour of election to Fellowship in the R.S.C. is a significant recognition of professional achievement.

Leslie Roberts spent January, February and half of March at the Tata Institute of Fundamental Research, near Bombay, working with research collaborators in algebraic geometry and K-theory.

Mark Green of Belleville, who graduated from our Mathematics and Engineering program in May with first class honours, also took several other honours. He was awarded the Governor-General's medal, given to the student with the highest standing throughout the four year program in the Faculty of Applied Science, as well as the Professional Engineers' Gold Medal (awarded by the Association of Professional Engineers of Ontario) and the University Medal in Mathematics and Engineering. Mark has won a Commonwealth Fellowship to pursue graduate studies at Cambridge.

OPERATOR SYMPOSIUM

The fifteenth annual Canadian Symposium on Operators and Operator Algebras, organized by Ole Nielsen and Jim Woods, was held in our department during the week of May 18-22. Forty specialists in the field were in attendance; most were from Canada, but eight came from the United States and two from Europe. Several of the participants had previous Queen's connections as students, post-doctoral fellows, or professors. There were twenty five lectures during the five day gathering.

Queen's has long been known as a center for work in operator algebras, and much of the important new work done in this field in the early 1970s was done here. This was due principally (but not entirely) to the presence in our department, during 1974-75, of Alain Connes, who was subsequently given an honorary LL.D. by Queen's (1978) and a Fields Medal (1982) for his pioneering work. Ironically, Alain's year here, working on operator algebras, fulfilled part of his military service obligation as a citizen of France.

TWO PROBLEMS AND A SOLUTION

Jim Whitley passes on the following problem, which may qualify as a calculus poem: Suppose a piece of wire of length L is cut into n pieces; the i^{th} piece has length x_i and is used to make a regular polygon with n_i sides and having area A_i . Imagine this done in various ways, fixing n and the n_i but letting the x_i vary; let A_{\min} be the minimum total area $\sum_{i=1}^n A_i$ attainable. Show that $A_i = \frac{x_i}{L} A_{\min}$. (Hint: show first that a regular polygon with n sides, whose perimeter is P , has area $\frac{P^2 \cot(\pi/n)}{4n}$.)

Amending the Constitution: When a constitutional amendment is proposed, each of the 10 provinces can vote Yes, No, or Abstain. The amendment is adopted provided there are at least 7 Yes votes, and each block of provinces (the 4 Western provinces, the 4 Atlantic provinces, the 2 middle provinces) contributes at least one Yes. Of the $3^{10} = 59,049$ possible outcomes of a vote, how many result in an adopted amendment? (The answer is a number which is famous in another context!) The law also stipulates that once an amendment is adopted it is binding on provinces which voted Yes or Abstain, but not on those voting No. Hence it is of interest to know also: of the votes resulting in an adopted amendment, how many include 3 No's? 2? 1? None?

A Solution: Our last issue posed the problem of determining the probability that a point chosen at random in a 5-dimensional vector space lives in a given 4-dimensional subspace. The obvious answer $4/5$ is WRONG. Think of replacing 4, 5 by some other m, n with $m < n$. (Examples with $0 \leq m \leq n \leq 3$ are probably the easiest to visualize, although physicists will naturally prefer $m, n = 3, 4$.) So the answer, for any m, n with $m < n$, is 0, --if the ground field is infinite. (Once one sees the result, it's easy enough to write down a real proof.) If the field is finite, say with q elements, the probability is clearly q^{m-n} .

THE MATHEMATICS AND ENGINEERING PROGRAM

BY BILL WOODSIDE

Twenty years ago the first class of mathematical engineers graduated from Queen's. Since then over three hundred students have completed the program. The class which graduated this Spring is the 21st; the program has come of age. It is still unique in Canada, and as far as we know, in North America. This article traces its history, describes it as it is now, and makes a few guesses about its future.

Early Days

Jacke Hogarth, who played an important role in getting the program started, has just retired. Recently he shared with me some of his reminiscences. He joined the department in 1959, coming from the Defence Research Board in Ottawa, with the grand title of Abitibi Professor of Engineering Mathematics. The president of Abitibi Pulp and Paper Company had provided \$100,000 to each of the Big Four universities (Toronto, Queen's, McGill and Western) to establish professorial chairs. This sum was

intended to pay the bulk of the Abitibi Professor's salary for a period of ten years! The four chairs were all in different fields, including forestry at Toronto, engineering mathematics at Queen's. It was Ralph Jeffery (for whom our building is named) who lured Hogarth to Queen's.

In 1962, influenced by his own undergraduate training in engineering physics at Toronto, Hogarth proposed to the then newly appointed Head of Mathematics, John Coleman, the establishment of a joint program in mathematics and engineering. A few weeks later Coleman attended a meeting of the Committee of Heads of Applied Science departments, chaired by Hugh Conn, Dean of Applied Science and Head of Mechanical Engineering. At this meeting Dean Conn asked the question "Why doesn't Queen's have a program in mathematics and engineering?" He had recently heard of such a program at a university in Belgium. Coleman, ever the astute politician, returned to Hogarth saying "better that the idea originate with the engineers" and destroyed all evidence of Hogarth's proposal. (Now it can be told!) And so, with the blessing of the engineering faculty the program began accepting its first students in the fall of 1964. There were two options then: the forerunner of the present Control and Communications option with electrical engineering, designed by Hogarth, and the Thermosciences option with mechanical engineering, planned by Conn. Accreditation as a full-fledged engineering program by the provincial body of professional engineers faced some opposition. Hogarth pays tribute to Jim Ham (then a professor of electrical engineering at Toronto, later Head, then Dean of Graduate Studies and finally President of the University of Toronto) who went to bat for the program and provided much encouragement and support. He also recalls some initial hesitation from some engineering professors at Queen's; for example the Electrical Engineering department was reluctant to allow Maths and Engineering students to take a 4th year electrical course without all the usual prerequisite courses. The M & E students ended up with an average on the course 21% higher than the regular electrical students. The program attracted very good students right from the beginning. In those days, with provincial examinations at the end of Grade 13, it was much more difficult than it is now to win an Ontario Scholarship; nonetheless, four of the seven students in the second M & E class were Ontario Scholars.

One of the important mathematics courses taken in 2nd year by all students in the program involved a heavy dose of linear algebra, which was then considered rather specialized; it was taken, if at all, at the 4th year level by pure mathematics students. A common complaint was "Why do we have to take all this linear algebra?" Later, on a field trip to M.I.T., the students emerged from a lecture in the Lincoln Lab, broad smiles on their faces, saying "Now we know". I remember that course well. I arrived at Queen's in 1966 and one of my first teaching assignments was Math 290, the M & E tutorial for all the 2nd year mathematics courses; the content was linear algebra and abstract structures, advanced calculus, and differential equations. I learned a lot that year. Nowadays, of course, we expect all engineering students to master the basic concepts of linear algebra in first year. Motivation is still a problem. I wonder what it was that finally convinced those students that day at M.I.T.; probably something to do with the Kalman filter.

The Present Program

Bruce Kirby became Chairman for Engineering Mathematics in 1970 and during the subsequent ten years, under his expert guidance, the program developed into roughly its present form.

Students in all ten engineering programs at Queen's study the same fairly standard collection of mathematics and science in their first year: chemistry, computing science, engineering graphics, geology, linear algebra, calculus, mechanics, electricity and magnetism. Towards the end of their first year they choose an engineering program (chemical, civil, electrical, mechanical, etc.) which they follow for the next three years. One of these programs is Mathematics and Engineering (ME). Within each program there may be several options. The original two in ME have grown to six: control and communications, computer science, applied mechanics, thermosciences, process control, and structural engineering. One of these is selected by the student entering ME and represents the engineering component of his/her program. The mathematics component, for all options, consists of courses in pure and applied mathematics up to the level of an honours degree: algebraic structures, advanced linear algebra, advanced calculus and real analysis, differential equations, applications of numerical methods, probability, statistics, complex analysis, methods of applied mathematics, an engineering mathematics project, and elective courses chosen from a variety of fourth year and graduate level courses in optimization, combinatorics, modern analysis, statistics and numerical analysis.

The engineering courses are taken in the appropriate engineering department. For example, the courses in the control and communications option include electric circuits, digital systems, engineering electricity and magnetism, introduction to systems and control, communications systems, electronics, microprocessor system principles, control systems engineering, digital communications and computer networks, linear systems, filtering and estimation, and further engineering electives. Most of these courses are taken in the electrical engineering department. Details of the curriculum can be found in the Calendar of the Faculty of Applied Science.

In addition to the courses in mathematics and engineering, students are required to take complementary studies courses in the humanities, social sciences and administrative studies (for example, all students take a course in engineering management and economics.) The total program consists of about 500 units, of which about 120 are taken in first year, 160 are mathematics courses beyond the first year level, 160 are engineering courses and 60 are complementary studies.

Evolution of the Program

The program is administered by the Chairman for Engineering Mathematics in the Department of Mathematics and Statistics. The department curriculum committee consists of five professors and six students (two each from the 2nd, 3rd and 4th years). Student input to the committee is vital; several of the more recently introduced options in the program were created in response to student interest and initiative. Proposed changes are passed along to the Faculty of Applied Science Curriculum Committee for approval. Besides changes generated internally, many changes come about as a result of annual meetings with the Advisory Council on Engineering, a body whose aims are to link the engineering departments more effectively with the engineering profession and with industry, and to provide criticism, advice and assistance to the departments from sources outside the university.

Graduates of the program receive a degree in engineering. The program is fully accredited by the Canadian Council of Professional Engineers. To maintain that accreditation it operates under curriculum guidelines laid

down by the Canadian Engineering Accreditation Board of the CCPE. The guidelines impose constraints on the mix of the mathematics, science, engineering, design and synthesis and complementary studies components. Curriculum development is thus a complex process.

Problems

Being a relatively small and non-traditional engineering program produces several recurring difficulties. Courses cannot be designed exclusively for the program. Most of the mathematics courses are also taken by students specializing in mathematics in the Faculty of Arts and Science. More importantly, however, the engineering courses are designed for the "regular" engineering students and so cannot take advantage of the stronger mathematical background of the ME student. Thus the fit between the mathematics and engineering courses is seldom optimal; there is not as much mutual reinforcement between the two components as we would like. Difficulties with laboratory work sometimes arise; our students, being interested in the more theoretical aspects of their engineering discipline, sometimes fail to appreciate the importance of the practical side and occasionally feel less competent in this area.

As a result of the relative youth and innovative design of the program, not enough industrial recruiting people are fully aware of what it has to offer. Most have a clear understanding of programs such as engineering physics and the traditional programs in civil, mechanical, electrical and chemical engineering. But Mathematics and Engineering is something new. The name of the program is not perfect; however, such alternatives as mathematical engineering or engineering mathematics are not quite appropriate either. Brochures describing the program have been sent to industrial and government personnel managers from time to time, and some of these problems may be less severe than before.

The Students

The program is clearly an ambitious one; the load is heavy, the material is not easy and the attrition rate is relatively high. Success requires genuine ability, hard work and perseverance. By and large, the program attracts good students. Our traditional rivals in attracting the best students from first year are engineering physics and electrical engineering. The number entering ME in recent years has fluctuated between 20 and 35; engineering physics enrolments average about 40 with electrical engineering ranging as high as 110. These are drawn from a first year class which numbers between 380 and 400 students. Interestingly, although only 10% of all graduating engineers at Queen's graduate with first class honours (cumulative weighted average of 80% or higher in all courses in the final two years of the program), the figure is 20% for Mathematics and Engineering, 17% for Engineering Physics and 14% for Electrical Engineering. (These figures are for the graduating classes from 1979 to 84 inclusive.) This year the top three graduating students in the entire Faculty of Applied Science at Queen's were all Mathematics and Engineering students. They are Mark Green, Jim Beattie and Richard Pawlowicz.

In recent years about 30% of graduating ME students have gone on to graduate work or further education, with the remainder joining the work force. The field of specialization of those going to graduate studies varies widely. Some examples: - This year Mark Green will be going to Cambridge University to continue his studies in Structural Engineering and

Jim Beattie will be staying at Queen's for graduate work in physics. Last year Peter Oskarsson went to mechanical engineering at McGill, Bill Beatty to Aerospace Studies, Duncan de Chastelain to Mathematics and May Chan to Electrical Engineering, all at University of Toronto. (There are in fact three others currently working with the Systems Control Group in Electrical Engineering at Toronto: Karen Rudie, Charles Arnoldi and Glenn Reed.) Other top students in recent years include Steve Norman (doing electrical engineering at Stanford), Greg Wilson (computer science at Edinburgh), Peter Giardetti (chemical engineering at McGill), Marilyn Lightstone (mechanical engineering at Waterloo), Sarah Frisken (biomedical engineering at Carnegie-Mellon), Amanda Hubbard (plasma physics at Imperial College, London), Paul Tseng (operations research at MIT) and Shawn Monaghan (electrical engineering at Queen's).

The Faculty

The success of the program obviously depends very heavily on the faculty members associated with it. Outside the department, we rely on the many engineering professors who teach courses, run laboratories and provide supervision for fourth year engineering mathematics projects. The cooperation of the engineering departments has been first-rate right from the start. In addition, encouragement from the Office of the Dean of Applied Science is important. Dean Conn in the beginning and more recently Deans Uffen and Bacon have been strong supporters. Within the department, Lorne Campbell, Jon Davis, Bob Erdahl, Ron Hirschorn, Bruce Kirby, Dan Norman, Cedric Schubert and Jim Verner have all made (and continue to make) major contributions. [also Bill Woodside! - ed.]

The Future

The immediate future brings an accreditation visit this fall. Preparations are being managed by Dan Norman, the new Chairman for Engineering Mathematics. In the longer term, given a continuing supply of good students, the main challenge appears to be that of fitting into the curriculum all the courses needed by a mathematical engineer in the year 2000, within the framework of a four-year program, subject to accreditation guidelines. Every so often the thought of a five-year program surfaces. More than twenty years ago the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America undertook a study of the need in the U.S. space program for "technical manpower specifically prepared for a strong combination of mathematics and engineering - in addition to the more customary converts from a great variety of backgrounds". Meetings with NASA and space industry representatives "confirmed the eminent need for the projected product. At the same time it became clear that many other industries, such as electronics and communications, would have an equal interest in a mathematical engineer with the same basic background but possibly somewhat different specialization. All these industries face innumerable engineering problems with a common need for extensive and sophisticated mathematical analysis. For the solution of such problems it is no longer true that the prime requirement is a good physical intuition - rather, one also needs a well developed mathematical intuition. Thus the mathematical engineering program must involve a heavy concentration in mathematics, but with a choice of topics that will give a useful basis for applications as well as solid grounding in theory."

The above quotations are from "Mathematical Engineering, A Five Year Program" published by the CUPM in October 1966. They could be taken as a

prescription for our program, which had its beginnings four years earlier. With the imminent shortening of the Ontario high school program, at least for some students, the time may be ripe for a serious consideration of a five year program in mathematics and engineering, perhaps with the awarding of a bachelor's degree at the end of four years, as at present, and a master's degree at the end of the fifth.

One of the strengths of this program has been its versatility and flexibility, and there is every reason to believe that it will continue to provide an excellent education for able students choosing a career in mathematics and engineering.

ISAAC LEE G'ZINTA: A TRIBUTE

The great nineteenth century mathematician, Isaac Lee G'Zinta, was born in the southern United States, of Russian Jewish emigrants, thus explaining his odd combination of Christian names. "Zack", as he was known to his friends, was a child prodigy who showed an early and pronounced gift for number theory, especially elementary divisibility problems. His life's work, all done in complete isolation from the mathematical world of the time, has recently been recognized to show tremendous genius, even though much of it was mere duplication of known results. One renowned mathematician recently said in an interview, "He was another Ramanujan, except no Hardy ever discovered him". However belatedly, G'Zinta is now receiving the acclaim due his genius: in honour of his work, the American Mathematical Society has called on its members to change the name of a standard mathematical symbol. The symbol $|$ for "divides" has been renamed the G'Zinta symbol, so instead of saying "two divides four" for the expression $2|4$, mathematicians throughout the United States are now called upon to say "two G'Zinta four", or even "two Zack Lee G'Zinta four". Whether the new nomenclature will catch on remains to be seen....

-Lawrence Howe

(Lawrence Howe is a graduate student currently completing a masters thesis under the supervision of Leo Jonker, in which he illuminates Gauss' "General Investigations of Curved Surfaces" (1827) by translating Gauss' infinitesimal arguments into the formalism of non-standard analysis. This fall he will begin doctoral work at Brandeis University, in the field of Algebraic Geometry.)

(G'Zinta was an unusually tall man, and in addition to his work in number theory he made some pioneering contributions in the field of surveying. In the early days of that science, his name and stature were combined to form a unit of measurement; a cornfield, for example, might be described as sixty G'Zinta-heights by a hundred and twenty. Imagine a troop of surveyors, suffering from hay-fever, tromping around the fields calling out measurements to one another, and you will understand how this erstwhile unit of measurement came, in time, to be used as a response to a sneeze...)

A BINARY OPERATION ON AN UNCOUNTABLE SET WITH A UNIQUE NON-ASSOCIATIVE TRIPLE

BY W. R. LORIMER

It is well-known that, given the table for a binary operation, one can tell at a glance whether the operation is commutative by checking whether the table is symmetric about the main diagonal. However, there is no such easy test for associativity. In investigating this (as an undergraduate at Dalhousie 3 years ago), I conjectured that if a binary operation were not associative, it must have at least 2 non-associative triples. By an amazing fluke I stumbled on the following counterexample:

	0	1	2
0	0	0	0
1	0	1	2
2	0	0	2

To simplify the verification, note that any triple containing 0 will be 0 ; any triple not containing 2 will be the min function. So the only cases we need consider are:

$$\begin{aligned}
 (11)2 &= 2 = 1(12) \\
 (12)1 &= 0 = 1(21) \\
 (21)1 &= 0 = 2(11) \\
 (22)1 &= 0 = 2(21) \\
 (21)2 &= 0 \neq 2 = 2(12) \\
 (12)2 &= 2 = 1(22) \\
 (22)2 &= 2 = 2(22)
 \end{aligned}$$

More generally, define a binary operation on the set $[0,1] \cup \{2\}$ by the table

	0	x	y	z	1	2
0	0	0	0	0	0	0
x	0	x	x	x	x	0
y	0	x	y	y	y	0
z	0	x	y	z	z	0
1	0	x	y	z	1	2
2	0	0	0	0	0	2

where $0 < x < y < z < 1$. As before, one need only check non-zero triples containing at least one 2; the unique non-associative triple is $(21)2 \neq 2(12)$.

How much is known about this question? Has anyone seen this table or other tables with the same property? Is it necessary that an operation with a unique non-associative triple be non-commutative?

Remarks: (1) If a commutative operation with a unique non-associative triple exists, that triple must be of the form aba (proof: exercise).

(2) In the table above, there is also a unique noncommutative pair. Is this significant?

(Bill Lorimer is a graduate student in our department, with a B.Sc. (Hons.) in Maths and Computing Science from Dalhousie University. He is interested in Graph Theory and Combinatorics; his thesis, supervised jointly by David Gregory of this department and Henk Meijer of Computing and Information Science, is on Cryptographic Pseudo-Random Number Generators. His coffee cup features a representation, in purple nail polish, of the non-planar graph $K_{4,4}$.)

JOHN COLEMAN RETURNS TO CHINA

Readers of the Communicator will recall that John Coleman was invited to Jilin University in the north of China in 1982 to give a series of lectures on Reduced Density Matrices, a topic which has applications in quantum physics and chemistry. Coleman is regarded as a world authority in this field, and in the summer of 1985 a Symposium on Density Matrices and Density Functionals was held here in his honour. The Proceedings of that Symposium, edited by R. M. Erdahl of our department and Vendene Smith of Chemistry, has just been published in a handsome volume of more than 700 pages.

The papers included in the volume are by theoretical chemists, physicists and mathematicians specializing in the theory of interacting quantum particles. Many of the papers deal with ideas pioneered by John Coleman, who formulated and studied some of the difficult mathematical problems associated with density matrix theory. The symposium itself was interdisciplinary and the seventy participants came from over 15 different countries. There were three theoretical chemists from mainland China and many solid state and molecular physicists from Europe.

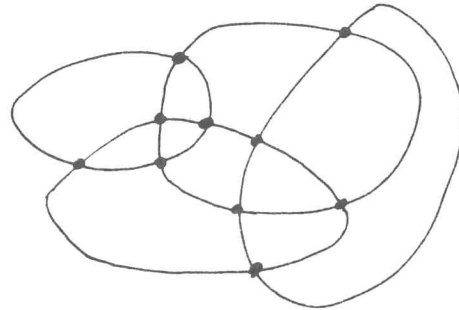
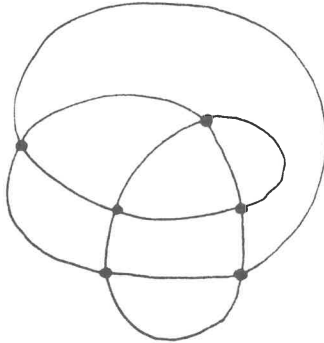
One of the participants, Prof. Sun Chia Chung, arranged for an invitation to Prof. Coleman to return to China, from April 15 to May 15, 1987 to lecture in Shanghai, Nanjing, Jinan and Beijing. During twenty working days he gave 16 two-hour lectures: six on Kac-Moody algebras and ten on Reduced Density Matrices with emphasis on their application to the explanation of long-range order, for example in superconductivity or ferromagnetism. He was especially pleased that several outstanding Chinese theoretical physicists manifested considerable interest in his ideas which are currently very topical because of the excitement occasioned by the recent discovery of high-temperature superconductors.

While John interacted with professors and graduate students in mathematics, physics and chemistry, his wife Marie-Jeanne lectured on Biblical Studies to Protestant theological students and pastors in Nanjing and Shanghai. The communist government of China having forced the Chinese churches to break off all dependence upon foreign countries, the Church has now managed to become self-governing and self-supporting. It has emerged remarkably strong after its suppression during the Cultural Revolution. John remarked that whereas in 1982 there were six Protestant churches functioning in Shanghai there are now twenty-three and that the number of theological colleges has jumped from one to eight. The growth among Roman Catholics is equally impressive, from 20 theological students in 1982 to 600 in 1987.

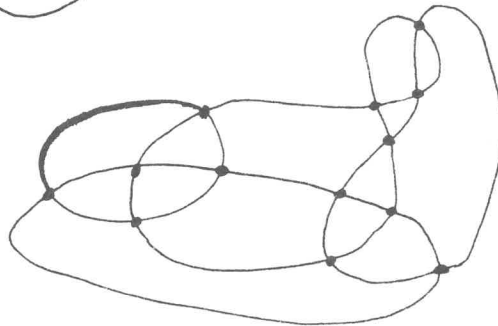
A CURIOUS COLOURING PROBLEM

BY HERB SHANK

We consider 4-valent graphs G that are embedded in the plane. 4-valent means that every vertex is on exactly 4 edges. Embedded in the plane means that G is drawn in the plane, without any intersections of edges except at their end vertices. Thus, each of



and



is the sort of graph

under consideration.

We raise the question: when does an embedded 4-valent planar graph G have the property that the faces of G can be four-coloured in such a way that, among the four faces which share a vertex in their boundaries, all four colours occur? (Notice that the "background" counts as a face.)

The answer is surprisingly simple. The field $GF(4)$ with four elements $A, B, C, 0$ plays a useful role; its additive structure is described by: $A + 0 = A$, $A + A = 0$, $A + B = C$, and all equations arising from these by permutations of A, B, C . (Note that $A + B + C = 0$.) It's also useful to consider partitioning the edges of G into one or more closed paths

(curves) by walking along the graph like this:



-- that

is, always walking straight ahead, turning neither left nor right.

The reader is invited to use these hints to solve the problem without reading further!

It turns out that the faces of G can be four-coloured in the desired way if, and only if:

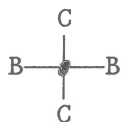
- (1) Each path is simple (i.e., does not intersect itself), and
- (2) The set of paths can be three-coloured so that only paths of different colours intersect.

To see the equivalence, suppose first that the faces are suitably coloured; call the colours $A, B, C, 0$, so a typical vertex looks like this:



. Attach to each edge the sum of the colours of the faces of

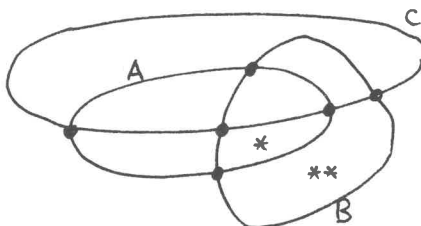
which it is the common boundary (so the example above yields



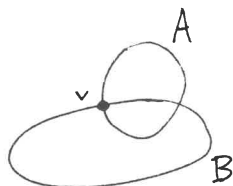
) . The result is a colouring of closed paths, partitioning

the set of edges of G , and satisfying (1) and (2) (verify!)

Conversely, suppose the set of edges of G is partitioned into simple closed paths coloured A, B, C , satisfying (1) and (2). Colour each face with the sum of the colours of the closed paths enclosing it. Example: the face labeled $*$ gets coloured $A + B$, i.e. C , and the face labeled $**$ gets coloured B :



We claim that this colouring does the job. To see this, suppose first that a vertex v is the intersection of an A curve and a B curve, say like this:



Then the faces around v are coloured $0, A, B$, and $A + B = C$, in some order. Adding in the contribution of any other curve just permutes these; done!

For a bit more information, see the paper by K. A. Berman and H. Shank in Journal of Graph Theory 3 (1979), pp. 291-294.

Notice that exactly one of the three initial examples can be coloured in the desired way.

[Herb Shank is a combinatorialist who has been visiting at Queen's for the past few years. He has also taught at the University of Massachusetts (Boston) and Waterloo.]

QUEEN'S MATHEMATICAL COMMUNICATOR

SUMMER 1987

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An Appeal: "COMMUNICATOR" in our title is intended to suggest a two-way process; the aim is not only to keep alumni/alumnae and friends of Maths and Stats at Queen's in touch with developments in our department, but also to keep us in touch with your ideas and doings, and to help keep you in touch with each other. Reader input is essential to our health. Send us your news, views, problems/solutions, and we'll put you in the next issue. Let others know what you're up to! You are also invited to send a donation from time to time to help keep the Communicator coming (address below; cheques payable to The Communicator, Queen's University). Our production cost is approximately a dollar per copy.

Address for all correspondence:

Editor
Queen's Mathematical Communicator
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario
K7L 3N6