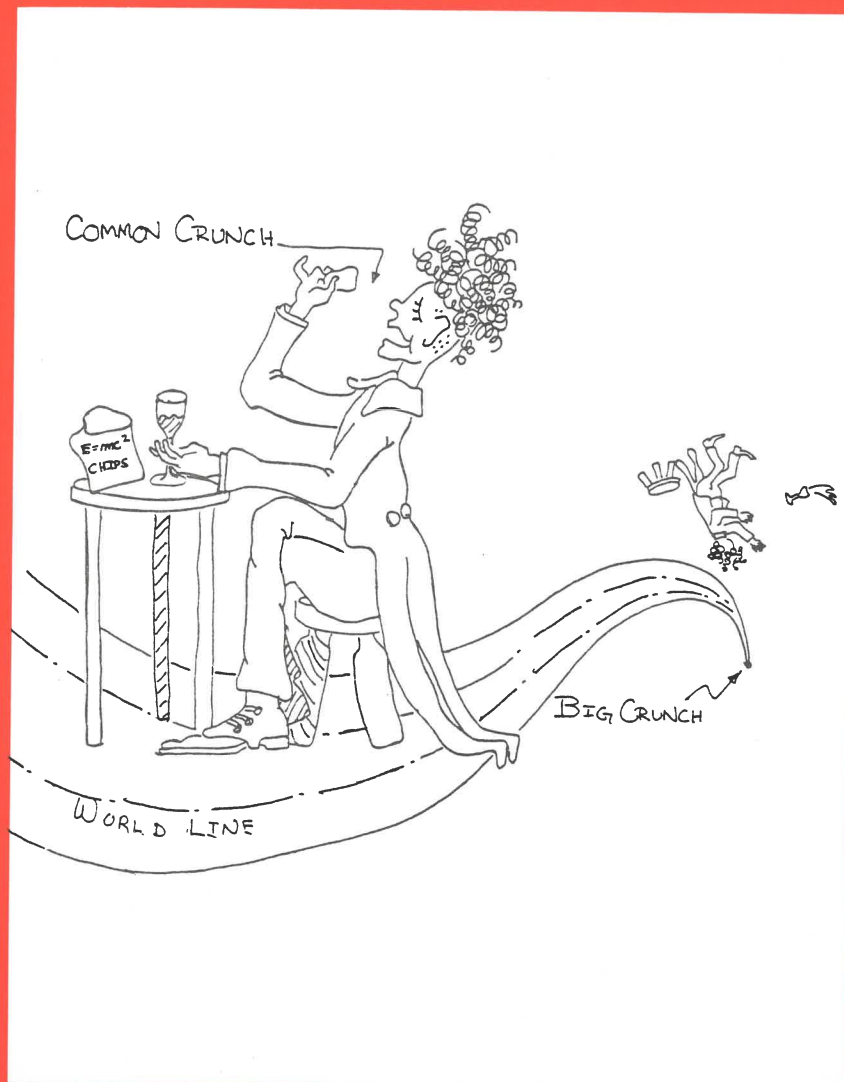


QUEEN'S MATHEMATICAL COMMUNICATOR



SPRING 1989



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ROBERTSON-WALKER MODELS OF THE UNIVERSE AND FIRST YEAR CALCULUS

THE COLEMAN ELLIS LECTURE GIVEN IN MARCH 1989

BY H. J. KUMMER

Hans Kummer came to Queen's in 1965 with a PhD in physical chemistry and later obtained his doctorate in mathematics under Robin Giles. He lives in a house of his own design beside Loughborough Lake, 20 km. north of Kingston, where he devotes his time to philosophy, mathematical physics, skiing and windsurfing.

Most people if they hear the year 1929 mentioned think of the stock market crash. However there was another event that year, which may have a more lasting impact on the history of mankind: The discovery of Edwin Powell Hubble, the country lawyer from Kentucky turned astronomer, that the universe we live in is expanding, a discovery which he couched in the famous formula

$$(1) \quad \boxed{v_x = H_0 R_x} .$$

The speed v_x with which galaxy x recedes from our galaxy is proportional to its distance R_x from our galaxy. The proportionality constant H_0 is nowadays called Hubble's constant and its approximate value in geometric units is:

$$H_0^{-1} \approx 18 \cdot 10^9 \text{ years.}$$

(Geometric units are defined by taking the speed of light c and the gravitational constant G both to be the dimensionless number 1).

Hubble's discovery gave a new impetus to Einstein's General Theory of Relativity. In 1922, seven years before Hubble's discovery, the Russian physicist Alexander Friedmann found a non-static solution of Einstein's equation

$$G = 8 \pi T .$$

However his contribution had been largely ignored. Too deeply was the idea of a static universe entrenched in the minds of his contemporaries, in fact so much so that Einstein committed what he later was to call the greatest blunder of his scientific career: He introduced an extra term (involving a "cosmological constant") into his equation in order to force a static solution. Hubble's discovery made it respectable to construct the most general class of solutions of Einstein's equation compatible with the large scale homogeneity and isotropy*) of the universe.

*) Homogeneity means that no two points are physically distinguishable. Isotropy means that no two directions are physically distinguishable. It is evident that homogeneity does not imply isotropy (think for example of a spacial region filled with a constant magnetic field. Its direction is a distinguished direction). On the other hand isotropy at every point implies homogeneity.

This was accomplished independently by the American physicist Howard Robertson in 1935 and by the British Mathematician Arthur Walker in 1936. The underlying space-time manifold of their construction nowadays is called a Robertson-Walker space-time.

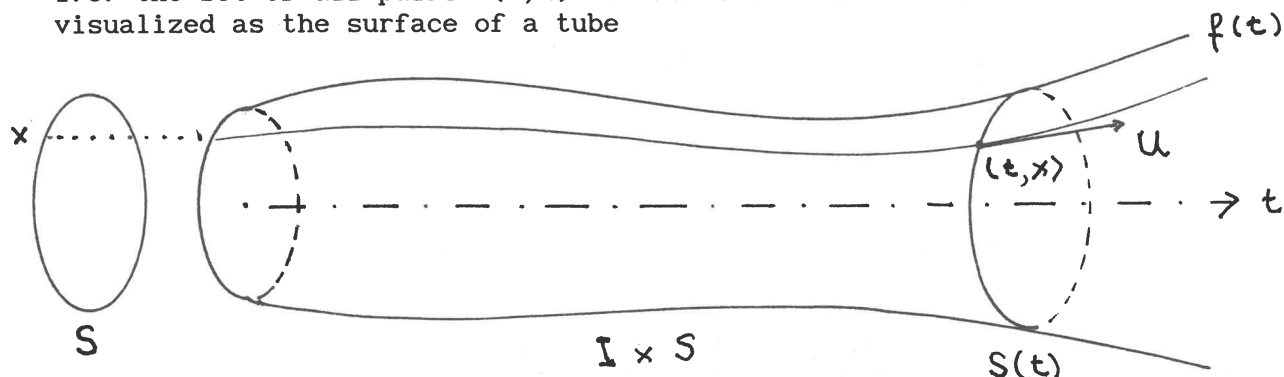
One ingredient of a Robertson-Walker space-time is a 3-dimensional Riemannian manifold S of constant curvature $k \in \{-1, 0, 1\}$. (A three dimensional manifold is the three dimensional analogue of a surface; the adjective "Riemannian" means that each tangent space carries an inner product.) S plays the role of a "standardized" model of the universe to which the actual universe is similar at each instant of cosmological time t . The topologically most simple choices for S are:

	<u>name</u>
$k = -1$ $S = H^3$ (hyperbolic 3-space) - open universe	
$k = 0$ $S = \mathbb{R}^3$ - flat universe	
$k = 1$ $S = S^3$ (3 dim unit sphere) - closed universe.	

The other ingredient of a Robertson-Walker space-time is an open interval $I \subset \mathbb{R}$ of the real line. The underlying manifold of the space-time is the Cartesian product

$$M = I \times S,$$

i.e. the set of all pairs (t, x) where $t \in I$ and $x \in S$. It can be visualized as the surface of a tube



Each point $x \in S$ names a potential galaxy whose world-line is the parameterized curve:

$$\gamma_x(t) = (t, x) \quad t \in I.$$

These world-lines are the flow lines of a vector field U (which is the lift of the unit vector field on I to M).

In order to transform M into a full-fledged space-time we need to endow each tangent space $T_{(t,x)}(M)$ with a scalar product of index 1. The requirement of isotropy implies that this should be done in such a way as to satisfy the following three conditions.

- For each $t \in I$ the slice $S(t) = t \times S$ (which models the actual universe at cosmological time t) should be a Riemannian submanifold of M which is similar to S : $S(t) \sim S$.
- For each $t \in I$ the vector field U should be perpendicular to $S(t)$: $U \perp S(t)$.
- U representing the galactic flow should be a time-like vector: $\langle U, U \rangle = -1$.

Now each "four-vector" $v \in T_{(t,x)}(M)$ can be decomposed in a unique way along U and along $S(t)$:

$$(2) \quad v = \mu U + \vec{v}, \quad \mu \in \mathbb{R}, \quad \vec{v} \in T_{(t,x)}(S(t)).$$

(Here $T_{(t,x)}(S(t))$ denotes the subspace of $T_{t,x}(M)$ consisting of all vectors tangential to $S(t)$). Thus by conditions (b), (c) we obtain

$$(3) \quad \langle v, v \rangle = -\mu^2 + \langle \vec{v}, \vec{v} \rangle.$$

Moreover condition (a) implies that

$$(4) \quad \langle \vec{v}, \vec{v} \rangle = f(t)^2 (d\sigma(\vec{v}), d\sigma(\vec{v})),$$

where (\cdot, \cdot) denotes the inner product on S and $\sigma : I \times S \rightarrow S$ is the projection of M onto S . Here $f : I \rightarrow (0, \infty)$ is a positive valued smooth function called the scale function. Its value $f(t)$ at cosmological time t is the amount the standardized universe S has to be scaled up (or down) in order to obtain the actual universe $S(t)$. Therefore, intuitively its graph gives the shape of the tube depicted above. Thus we arrive at the following definition of a Robertson-Walker space-time.

Definition: A space-time of the form $M = I \times S$, where $I \subset \mathbb{R}$ is an open interval and S is a 3-dimensional Riemannian manifold of constant curvature $k \in \{-1, 0, 1\}$ whereby each tangent space $T_{(t,x)}(M)$ is endowed with the scalar product

$$(5) \quad \langle v, v \rangle = -\mu^2 + f(t)^2 (d\sigma(\vec{v}), d\sigma(\vec{v}))$$

is called a Robertson-Walker space-time. [For the meaning of μ , v , σ and $f(t)$ consult the preceding paragraph in particular formulas (2), (3) & (4).]

What can we do with a Robertson Walker space-time? Well, quite a number of interesting things!

1. Reconstruction of Hubble's formula:

Let $o, x \in S$ be our and some other galaxy. Let $d(o, x)$ be their distance in the standardized universe S . Then their actual distance R_x at time t is given by:

$$(6) \quad R_x(t) = f(t)d(o, x).$$

Differentiating (6) we obtain:

$$v_x = R'_x(t) = f'(t)d(o, x) = \frac{f'(t)}{f(t)} f(t)d(o, x) = \frac{f'(t)}{f(t)} R_x.$$

Thus our model suggests that the Hubble "constant" is the present value of the function

$$H(t) = f'(t)/f(t)$$

called the Hubble function, i.e. that $H_0 = f'(t_0)/f(t_0)$ where t_0 is the present instant of cosmological time.

2. A Robertson-Walker space-time carries a solution of Einstein's equation which identifies the vector field U as the flow of a perfect fluid of density

$$(8) \quad \rho = \frac{3}{8\pi} \left[\left(\frac{f'}{f} \right)^2 + \frac{k}{f^2} \right] \text{ yr}^{-2}$$

and pressure

$$(9) \quad p = -\frac{1}{8\pi} \left[\frac{2f''}{f} + \left(\frac{f'}{f} \right)^2 + \frac{k}{f^2} \right] \text{ yr}^{-2}.$$

where $k \in \{-1, 0, 1\}$ is the curvature. (Both formulas are in geometric units: to obtain cgs-units, multiply ρ by $1.5 \cdot 10^{-8}$ and p by $1.35 \cdot 10^{13}$). Formula (8) shows that it is the present density of matter $\rho_0 = \rho(t_0)$ which determines whether our universe is open, flat or closed. Indeed let

$$\rho_{\text{crit}} := \frac{3}{8\pi} H_0^2 = 3.68 \cdot 10^{-22} \text{ yr}^{-2} = 5.5 \cdot 10^{-30} \text{ g/cm}^3.$$

Then evaluating (8) at t_0 we obtain:

$$(10) \quad \rho_0 = \rho_{\text{crit}} + \frac{3}{8\pi} \frac{k}{f(t_0)^2}$$

whence

$$k = \text{sign}(\rho_0 - \rho_{\text{crit}}).$$

Thus we conclude that

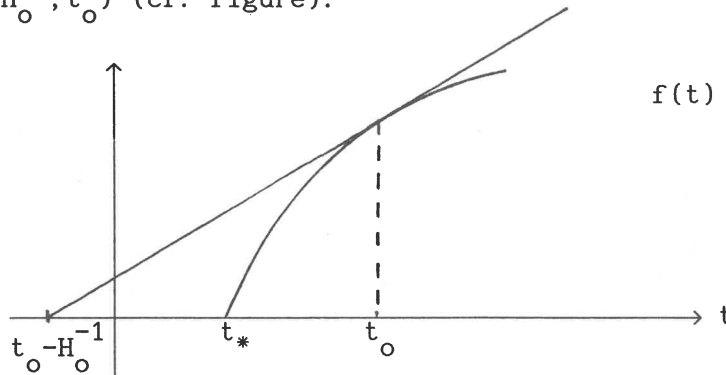
- (a) If $\rho_0 > \rho_{\text{crit}}$ the universe is closed.
- (b) If $\rho_0 \approx \rho_{\text{crit}}$ the universe is flat.
- (c) If $\rho_0 < \rho_{\text{crit}}$ the universe is open.

Because of the possible existence of a vast amount of non-luminous matter the present density ρ_0 is very difficult to estimate. Nevertheless most cosmologists today seem to favour inequality (c) which implies that we live in an open universe.

3. Next observe that adding three times equation (8) to equation (9) yields:

$$\rho + 3p = -\frac{3}{4\pi} \frac{f''}{f}.$$

Thus under the physically reasonable hypothesis that $\rho + 3p > 0$ we obtain that $f'' < 0$, i.e. that f' (and therefore H) is decreasing: The graph of f is concave downwards. Together with the observation that $f'(t_0) = H_0 f(t_0) > 0$ this implies that f must have had a singularity at a time $t_* \in (t_0 - H_0^{-1}, t_0)$ (cf. figure).



Under additional conditions it can be shown that $\lim_{t \rightarrow t_*+} f(t) = 0$ and

$\lim_{t \rightarrow t_*+} f'(t) = \infty$ i.e. that the singularity is a "big bang" (cf. Barrett

O'Neill [1]). Furthermore since $f'(t_0) > 0$ and f' is decreasing we can envisage two possibilities

- (a) $f'(t) > 0$ for all $t \in I$: The universe continues to expand.
- (b) f' changes sign: The universe reaches a maximum size and then begins to contract ending eventually in a big crunch, i.e. in a singularity t^* such that

$$\lim_{t \rightarrow t^*-} f(t) = 0 \quad \lim_{t \rightarrow t^*-} f'(t) = -\infty .$$

It turns out that which of these possibilities is realized depends on whether the universe is open, flat or closed. This can be seen most easily by studying the case where $p = 0$.

4. Differentiating (8) we obtain after some algebraic manipulation

$$\rho' = -3(p+\rho)f'/f .$$

If we put $p = 0$ in this equation we obtain

$$\rho'f + 3\rho f' = 0$$

and hence after multiplication by f^2

$$\frac{d}{dt}(\rho f^3) = 0 .$$

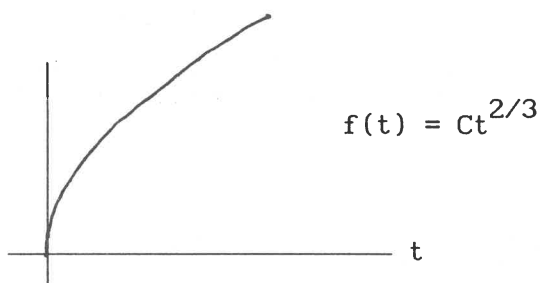
Thus the expression ρf^3 is independent of time: $\rho f^3 = B$. Combining the last equation with (8) we obtain the famous Friedmann differential equation

$$(11) \quad \boxed{f'^2 + k = A/f} , \text{ where } A = \frac{8\pi}{3} B .$$

5. Let us solve Friedmann's equation in case $k = 0$. In this case the equation takes the form

$$f'^2 f = A ,$$

admitting the simple solution $f = Ct^{2/3}$ where C is a constant satisfying $4C^3 = 9A$.



Thus (in case of $p = 0$) the flat universe starts with a big bang (at $t = 0$) and is forever expanding. The Hubble function $H(t) = f'(t)/f(t)$ is easily calculated to be

$$H(t) = \frac{2}{3t} .$$

Inserting the present instant gives an estimate for the age of the universe of:

$$t_0 = \frac{2}{3} H_0^{-1} = 12 \cdot 10^9 \text{ years} .$$

6. Suppose a photon left galaxy x at some instant t_x in the past and reaches us today. What is the distance R_x of x from our galaxy o ? Well, if we scale the speed of light $c = 1$ to the standardized universe S we obtain $\frac{1}{f(t)}$ for the rescaled speed of light. Thus the distance between galaxy x and our galaxy as measured in the standardized universe is given by $d(x, o) = \int_{t_x}^{t_o} \frac{dt}{f(t)}$. In order to obtain the actual present day distance we have to multiply this expression by the scale factor $f(t_o)$ (cf. formula (6))

$$R_x(t_o) = f(t_o) \int_{t_x}^{t_o} \frac{dt}{f(t)}.$$

Assuming $f(t) = Ct^{2/3}$ we obtain

$$R_x(t_o) = t_o^{2/3} \int_{t_x}^{t_o} \frac{dt}{t^{2/3}} = 3(t_o^{1/3} - t_x^{1/3}).$$

If the photon left the galaxy at the time $t_x = t_* = 0$ of the big bang, $R_x(t_o) = 3t_o^{1/3} = 36$ billion light years. Thus we arrive at the conclusion:

If the universe is flat we cannot observe galaxies which are outside a ball of radius 36 billion light years because the light emanating from these galaxies did not have time to reach us.

7. Finally let us solve the Friedmann equation in the case where $k = 1$:

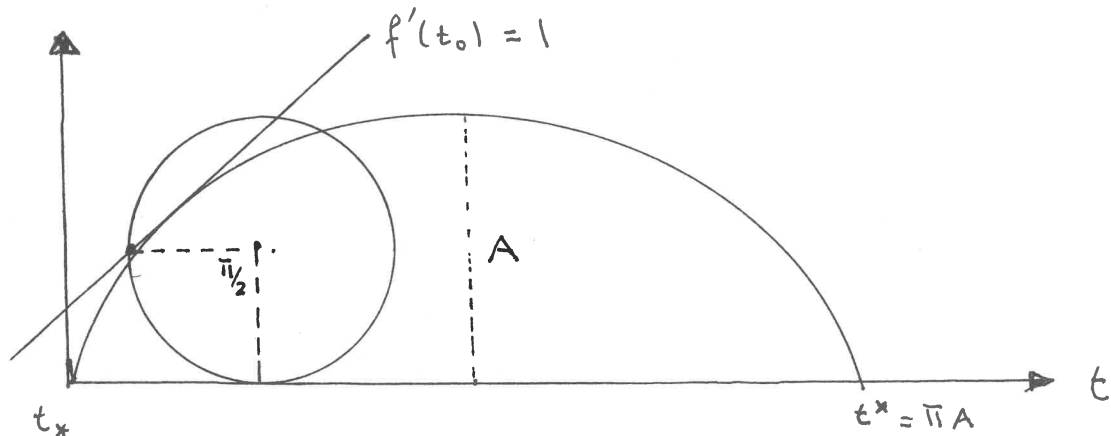
$$f'^2 + 1 = A/f.$$

The solution can be expressed in parametric form

$$(12a) \quad f(t) = A/2(1 - \cos\theta)$$

$$(12b) \quad t = A/2(\theta - \sin\theta).$$

This is the equation of a hypocycloid i.e. of a curve traced out by a point on the rim of a wheel of radius $A/2$ which rolls along the t -axis.



[That the equations (12) solve the Friedmann equation can easily be checked.

Indeed we obtain $f' = \frac{df/dt}{d\theta/dt} = \frac{\sin\theta}{1 - \cos\theta}$ and therefore

$f'^2 + 1 = \frac{2}{1-\cos\theta} = A/f]$. Thus the closed universe also starts in a big bang at $t_* = 0$ and reaches a maximum radius A at the time when $\theta = \pi$ i.e. at $t = (\pi/2)A$. It ends in a big crunch at the time when $\theta = 2\pi$, i.e. at $t^* = \pi A$. In order to obtain an estimate for A as well as for the age of the universe within this model of the universe, let's assume that $\rho_0 = 2\rho_{\text{crit}}$. Then using formula (10) we see that the present radius equals $r_0 = H_0^{-1} = 18 \cdot 10^9$ light years. Therefore

$$\frac{\sin\theta_0}{1-\cos\theta_0} = f'(t_0) = H_0 r_0 = 1, \text{ an equality}$$

which implies that the present value of the parameter θ is $\theta_0 = \pi/2$.

(The wheel has turned a quarter of a revolution since the big bang!) Hence the maximum radius of the universe is

$$A = 2r_0 = 36 \cdot 10^9 \text{ light years} .$$

It will be reached when $\theta = \pi$ i.e. at

$$t_{\text{max}} = \frac{\pi}{2} \cdot 36 = 56.5 \cdot 10^9 \text{ years} .$$

The closed universe will end in a big crunch at

$$t^* = \pi \cdot 36 = 113 \cdot 10^9 \text{ years} .$$

Since the age t_0 of the universe within this model is given by

$$t_0 = r_0(\pi/2 - 1) = 18(\pi/2 - 1) = 10.27 \cdot 10^9 \text{ years}$$

we still have $\sim 103 \cdot 10^9$ years to go!

Literature:

Barrett O'Neill: Semi-Riemannian Manifolds with Application to Relativity
AP 1984.

Some endorsements by prominent colleagues on the announcement for a Mathematics Banquet (Ramada Inn, April 1989):

Laplace: "It will transform me."
 Lebesgue: "Nothing can measure up to it."
 Poisson: "I'll probably go."
 Rouché: "You can't argue with this principle."
 Jordan: "I'll be there in good form."
 Taylor: "I'm very serious about this."

MATHEMATICS AND ENGINEERING - A PROGRESS REPORT - MAY 1989

DAN NORMAN

Last fall, Lorne Campbell and I sent a letter to graduates of our Mathematics and Engineering program describing some concerns raised by the Canadian Engineering Accreditation Board (CEAB). Accreditation was extended until 1991 but the Board questioned whether there was enough engineering content in the curriculum, and whether the program was sufficiently controlled by professional engineers. We need to respond to their concerns before their next visit in 1990.

To those 63 graduates who replied to the questionnaire, thank you for your assistance. We appreciated the detailed and thoughtful comments. Some of you had also responded to a more general survey we conducted in 1985, so we now have responses from over 100 graduates. This has given us a picture of what our graduates have been doing, and a useful range of opinions on what we should do. Here is a summary of the responses and a progress report on our curriculum revision.

We were very heartened by the generally positive attitude to the program: many of you indicated that it had been an excellent preparation for your careers. We continue to be impressed (but not surprised) by the fact that most of our graduates have had successful careers in engineering. A few have moved into useful and satisfying careers in high school teaching, or business, or actuarial work, but the great majority of those who responded are doing work which is clearly engineering. This is of particular interest to us because our accreditation visitor wondered if our graduates really were oriented to engineering.

On the issue of where we might reduce the mathematics content of the program to make room for more engineering, there was some variety of opinion, with most courses having both critics and defenders. The balance of opinion was that we should give up some fourth year mathematics. This agreed with the advice from the program's founder, Professor Hogarth. Some supported reduction of time spent on Complex Analysis, but more defended the existing course.

We have decided to drop both MATH 495 (Methods of Applied Math II) and the fourth year Mathematics elective, and replace them by specified or elective courses from the appropriate engineering department. (Students may still take a fourth year mathematics course as an extra to the required program; among this year's graduates, several took extra fourth year courses.) We have also revised the third year course: there will now be 18 weeks of Complex Analysis taken with the Honours Mathematics students, followed by an introduction to classical control systems (Nyquist stability, etc.). Instructors who have taught MATH 320 believe that the 18 weeks will give sufficient time to do a thorough job on the complex analysis, and that it may work better for all students to have the class split into engineers and pure mathematicians for the last 6 weeks. The introduction to these control ideas should be useful for students in all options.

The Control and Communications students will no longer take the Electrical Engineering introduction to Control (ELEC 341); instead they will take the Engineering Physics course on electromagnetic waves. They will also add a course on semi-conductors. In this option, the second year Electrical laboratory is still unavailable because of space limitations. To

provide more experience in measurement and analysis of data, we are going to require students in this option to take the Spring survey school. (Students in our other options are required to take 3-week spring courses in the appropriate engineering departments.)

Some of you expressed concern about the lack of design courses: one such course is now required in the fourth year of the Process Control option, and design courses are now required in the Structures option (students have always taken them anyway). Two design courses are also required in the fourth year of the Applied Mechanics option. Another change will be that Process Control students will now be required to take the third year Chemical Engineering laboratory.

The Computer Science option has its supporters among you and not only among people who took it. Your argument that there is a need for graduates with such a blend of skills is persuasive, but we do not think we could possibly win an argument with the CEAB that this is truly an engineering program and that we are the right home for it. The CEAB was most concerned about this option, and if we do not respond in a realistic way the whole program will be in jeopardy. With real regret, we have decided that we must withdraw the option.

There were also expressions of regret about the end of the Thermosciences option. I should point out that the Applied Mechanics option includes (or can include, by appropriate choice of elective) all the courses of that option except for two Chemical Engineering courses on Transport Phenomena and on Heat Transfer Operations. Some of this material is available in some of the Mechanical Engineering Courses. The Mechanical Laboratory was sometimes a problem for students in Thermosciences, but the Applied Mechanics students are well prepared for this lab.

We are optimistic that these changes respond sufficiently to the stated concerns of the Accreditation Board about curriculum. We think that we now meet their criteria for engineering content, and that we still have a sound and attractive program for students who want to become engineers yet also want to include a much stronger mathematical foundation than is available in other engineering programs.

We have also made progress on the Board's concern about sufficient control of the program by professional engineers. We have a year's experience with our new curriculum procedure, which involves members from the appropriate engineering departments. From July 1, the chairman for the program will be a professional engineer, **Jon Davis**, who is well known to many of you. We have also recently appointed two engineer-mathematicians: **Marc Maes**, an applied probabilist (July '88) and **Wenceslao Cebuhar**, a control theorist (beginning July '89). (See "Four New Staff Members", this Communicator).

As a footnote, it is pleasing to be able to report that our graduating class this year is up to previous high standards. Of 24 graduates, 9 will graduate with first class honours; this 37.5% for our program compares with 16% for all of Applied Science. Seven of our graduates were awarded NSERC post-graduate scholarships. Most of the graduates have either accepted jobs or have definite plans for further study.

FOUR NEW STAFF MEMBERS

The Department is pleased to introduce four talented new colleagues.

Marc Maes is an applied probabilist with a strong background in engineering. Born in Belgium, Dr. Maes received his Engineering Diploma from the Catholic University of Louvain in 1977, with Great Distinction. He then alternated between industry and university, receiving his MSc and PhD degrees in Civil Engineering from Calgary in 1980 and 1985, respectively. His current fields of interest are structural reliability, risk analysis, and environmental processes.

Now 8 of the 48 full-time staff members in the department are Queen's alumni. **Agnes Herzberg** graduated from Queen's in 1961 and went on to obtain her MA and PhD degrees at the University of Saskatchewan in 1963 and 1966, respectively. Since then, in addition to her duties at Imperial College, University of London, Dr. Herzberg has served each year as editor or associate editor for one of seven journals and has worked in various capacities for the NSERC, the Amer. Assoc. for the Advancement of Science, the Amer. Stat. Assoc., the Royal Stat. Soc. and the Stat. Soc. of Canada. Her current interests include exploratory data analysis and the statistical design of experiments.

Dan Offin obtained his graduate degrees in applied mathematics from U.B.C. (1979) and the Univ. of Calgary (1984). Between degrees, he worked on systems design for Dow Chemical of Canada. After graduation he accepted a position at the University of Missouri (1984-88). Dr. Offin specializes in dynamical systems: his research interests include mathematical chaos, non-linear oscillations, and classical mechanics.

Wenceslao Cebuhar is a specialist in control theory with degrees from Rosario National Univ. in Argentina (Elec. Eng. '77) and from the Division of Applied Sciences at Harvard University (PhD '88). Between degrees Dr. Cebuhar was a Control Engineer at INTEC (the Institute of Technological Development for the Chemical Industry, Argentina), and also taught at Rosario. He works mainly in Nonlinear Control but is also interested in Optimal Control and Dynamical Systems and Computer Control.

QUEEN'S ENRICHMENT MINI-COURSES FOR HIGH SCHOOL STUDENTS

NORMAN RICE

During the week of May 8-12 approximately 600 high school students from the nine school boards in the Kingston area descended on Queen's to participate in a new program of "enrichment mini-courses".

About 30 different week-long courses were offered, including one by the Department of Mathematics and Statistics entitled "Mathematical Explorations". The intent in this course was to expose the students to some new mathematical ideas, and to engage them actively in exploring the ideas. Four Department members met with the students for two or three half-day

sessions each: **Morris Orzech** talked about "Secret Codes in Everyday Life" (really a discussion of some of the mathematical aspects of unraveling the genetic code); **Joan Geramita** talked about "Statistics for Science Fair Projects"; **Leo Jonker** discussed "Numbers of Numbers" and the geometry involved in finding "Straight Lines in a Cornfield"; **Norman Rice** talked about "Graph Theory" and "How to Analyze Games".

On Friday afternoon, as a wind-up to the week, three faculty members delivered mini-colloquium talks: **Tony Geramita** described some of his work and some open problems related to Hadamard matrices; **Marc Maes** described his modeling of iceberg flows in the North Atlantic (including why it's very expensive and why the oil companies are keen to have it done); **Joan Geramita** explained how some mathematics created a hundred years ago for a completely different purpose is now being put to use in a project she is involved with in studying goose populations in northern Canada.

The students involved claimed to have enjoyed all aspects of the week: a chance to be in a university environment for a while, to learn some mathematics, to hear about some mathematics, and to do some mathematics.

STATPAD - SOFTWARE FOR STATISTICS STUDENTS

MALCOLM GRIFFIN

StatPad, which joins PC based programs CalculusPad, MatrixPad, DEPad and PolyPad, is a program to help with learning statistics. Rather than perform statistical calculations on data (as do a number of PC based programs of various degrees of sophistication, at a variety of prices), it illustrates various fundamental ideas in statistics.

The concepts dealt with are: distributions, sampling distributions (including the Central Limit Theorem), confidence intervals and simple linear regression. Topics are presented graphically and dynamically, but numerical output can also be obtained. Calculations can be left running to answer computationally heavy questions.

The program can be used by beginning students: for example to demonstrate that averages of ten observations from a rectangular distribution are indeed indistinguishable from normal. More advanced students can find, for example, the difference (in success rate and in interval length) between normal, nonparametric, and exact (based on chi-square) confidence intervals for twenty observations from the exponential distribution.

StatPad is available in 3.5" or 5.25" format for \$8.00 from the department and also from the Microcomputer Store in computing services.

NEWS

Peter Taylor has been elected to the Board of Governors of the Mathematical Association of America, for a three year term. He will be Governor-at-Large for Canada. The MAA exists to promote the interest of the mathematical sciences in America, and to monitor and improve the teaching of mathematics.

The **George L. Edgett Memorial Scholarship in Statistics** was announced in Senate on 30 March 1989. Dr. Edgett began teaching at Queen's in 1933; his courses marked the beginning of formal instruction of statistics in Canada. Valued at \$500.00, the Scholarship is awarded each fall to the student in fourth year Honours statistics with the highest weighted average in STATS 251*, 261* and 360.

This is the first scholarship at Queen's designated specifically for statistics students.

Israel Halperin, a former member of the department who is now Emeritus Professor of Mathematics at the University of Toronto, will be granted an honorary doctorate by Queen's on Saturday 28 October 1989. Prof. Halperin will be addressing the departmental Colloquium on Friday 27 Oct. at 2:30 P.M. There will be a departmental reception for him that Friday evening. Friends, students and colleagues of Prof. Halperin are cordially invited to attend.

Ed Campbell has been promoted to Associate Professor, effective July 1, 1989.

THANK YOU

The department is very grateful to the former friends, colleagues and students of Dr. Edgett whose contributions made the Edgett Memorial Scholarship in Statistics possible.

Our thanks also to the many people who have sent donations to support the Communicator.

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SPRING 1989

ROBERTSON-WALKER MODELS OF THE UNIVERSE AND FIRST YEAR CALCULUS (H. Kummer)	1
MATH AND ENGINEERING - A PROGRESS REPORT (D. Norman)	8
FOUR NEW STAFF MEMBERS	10
QUEEN'S ENRICHMENT MINI-COURSES FOR HIGH-SCHOOL STUDENTS (N. Rice)	10
STATPAD - SOFTWARE FOR STATISTICS STUDENTS (M. Griffin)	11
NEWS	12

(COVER: R. M. Erdahl, "The Restaurant at the End of the Universe").

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