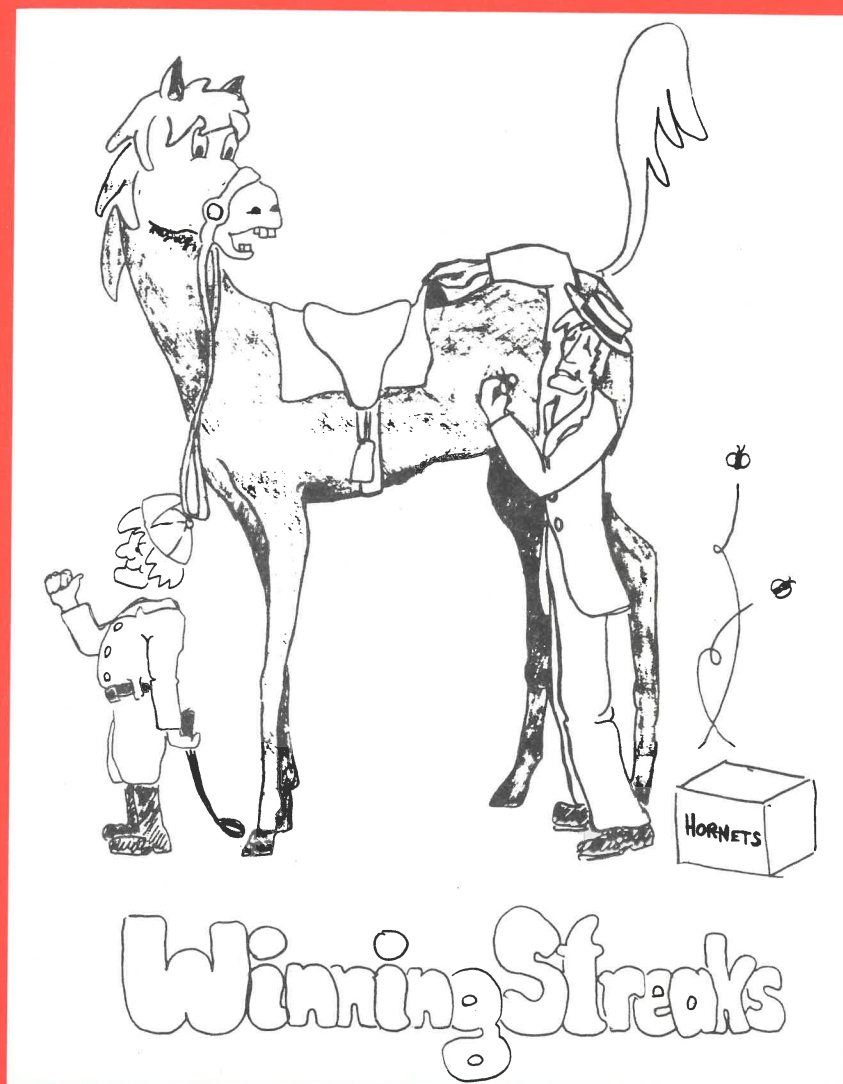


QUEEN'S MATHEMATICAL COMMUNICATOR



Summer 1990



An aperiodical issued at Kingston, Ontario by the
Department of Mathematics and Statistics, Queen's University
Kingston, Ontario K7L 3N6

QUEEN'S MATHEMATICAL COMMUNICATOR
SUMMER 1990

WINNING STREAKS, SHUTOUTS AND THE EXPECTED LENGTH OF THE STANLEY CUP FINAL (W. Woodside)	1
MINICOURSES FOR HIGH-SCHOOL STUDENTS (N. Rice)	10
HUBERT WHITFIELD ELLIS 1918-1990 (L. L. Campbell)	11
PROBLEMS OLD AND NEW	12
NEWS	13
HONORARY GRADUANDS	14

COVER: R. M. Erdahl

ON WINNING STREAKS, SHUTOUTS AND THE EXPECTED LENGTH OF THE STANLEY CUP FINAL

BY WILLIAM WOODSIDE

Prof. Woodside's main interests are in applied mathematics. He has worked at the National Research Council in Ottawa, and at the Gulf Research and Development Co. in Pittsburgh, Pa. Since his arrival at Queen's in 1966 he has been involved with the department's engineering mathematics programs, serving as Chairman for Engineering Mathematics from 1980-86. The following is the subject of his Coleman-Ellis lecture of January 1988.*

The beginnings of probability theory were closely associated with gambling and games of chance. Nowadays it is applied to a wide diversity of scientific and engineering fields; however its application to sports and games retains its appeal to students. Consider a sequence of independent games between two players (or teams) A and B with the probabilities that A or B wins any particular game being respectively p and q where $p + q = 1$, i.e. no ties are allowed. We are interested in strings of consecutive wins by A. Let W_N be the number of games played when A has won N games in succession. For example if the first 11 games result in the sequence B B A B A A B B A A A, then $W_1 = 3$, $W_2 = 6$, $W_3 = 11$. Let μ be the expected value of the random variable W_N , and let L be the number of the game first lost by A. The random variable L has a geometric distribution:

$$P\{L=k\} = p^{k-1}q, \quad k = 1, 2, 3, \dots$$

For $1 \leq k \leq N$, $E[W_N|L=k] = k + E[W_N]$ since a loss by A in the N^{th} game, or earlier, restores the initial situation from the viewpoint of putting together a winning streak of N games.
For $k > N$,

$$E[W_N|L=k] = N.$$

Therefore, conditioning on L ,

$$\begin{aligned} \mu = E[W_N] &= \sum_{k=1}^{\infty} E[W_N|L=k] \cdot P\{L=k\} = \sum_{k=1}^N (k + E[W_N]) p^{k-1}q + \sum_{k=N+1}^{\infty} N p^{k-1}q \\ &= q \sum_{k=1}^N k p^{k-1} + E[W_N]q \sum_{k=1}^N p^{k-1} + Nq \frac{p^N}{1-p}. \end{aligned}$$

*The text is excerpted from a longer paper in the Journal of Undergraduate Mathematics and its Applications (Vol. 10, 1989) and is reprinted here with the permission of the publishers.

Observing that $\sum_{k=1}^N kp^{k-1} = \frac{d}{dp} \sum_{k=1}^N p^k$ and then solving for $E[W_N]$ yields

$$\mu = E[W_N] = \frac{1 - p^N}{(1-p)p^N} \quad (1)$$

Note that

- (i) if $N = 1$, then $\mu = 1/p$ as expected intuitively
- (ii) as $p \rightarrow 1$, $\mu \rightarrow N$ (using L'Hôpital's rule)
- (iii) as $p \rightarrow 0$, $\mu \rightarrow \infty$
- (iv) as $N \rightarrow \infty$, $\mu \rightarrow \infty$
- (v) μ represents the expected number of Bernoulli trials needed to observe a streak of N successes where the probability of success on any given trial is p .

An equivalent interpretation of this result is as follows: suppose a job requires N uninterrupted time units for its completion. If the machine on which the job is being processed breaks down, the job must be restarted from the beginning, after the machine is repaired. If the probability of machine breakdown in any particular time unit is $q = 1 - p$, then the expected total processing time for the job (exclusive of machine repair time) is given by equation (1). Values of μ/N are shown in Table 1.

$N \backslash P$	0.1	0.4	0.5	0.6	0.9
1	10.0	2.50	2.00	1.67	1.11
2	11.0	4.38	3.00	2.22	1.17
5	1.11×10^5	32.2	12.4	5.93	1.39
10	1.11×10^{10}	1.59×10^4	204.6	41.1	1.87

Table 1 . Values of μ/N .

Winning Streaks in Baseball We are interested in the number of games a baseball team must play, on average, before winning N games in succession. The probability of winning a particular game depends of course on many factors including the opposing team and whether the game is played in the home park or 'on the road', but let us assume that it is constant over the course of the season and equal to the team's winning percentage for the season. Then the average number of games a team must play before completing a winning streak of N games is given by equation (1).

The St. Louis Cardinals won the National League East Division in 1982 winning 92 games and losing 70 for a winning 'percentage' of $p = 0.568$. Day by day results show 10 win streaks of exactly 2 games, 4 of exactly 3, 2 of exactly 4, 1 of 5, 1 of 6, 1 of 8 and one phenomenal streak of 12 wins. Of course a streak of exactly 6 wins includes a streak of 5 and two streaks of 3. These data are the observed data in Table 2. The expected frequency of a win streak of N was calculated by dividing the appropriate value of μ into 162, the total number of games played during the regular season.

N	1	2	3	4	5	6	7	8	9	10	11	12
Obs.Freq	92	33	14	9	5	4	2	2	1	1	1	1
Exp.Freq	92	33.3	15.7	8.13	4.40	2.42	1.36	.766	.432	.245	.139	.079

Table 2 Observed and expected frequencies of winning streaks of N games in a 162 game season with $p = 0.568$.

Comparison of observed and expected frequencies shows surprisingly good agreement, considering the assumption of constant p . A χ^2 goodness-of-fit test with the last five classes amalgamated, and also the classes for $N = 6$ and 7 amalgamated, yields the following results: $\chi^2 = 13.00$, the major contribution coming from the last class, number of classes 7 and, since one parameter was estimated from the data, the number of degrees of freedom is 5 ; the 5% point of χ^2 distribution with 5 degrees of freedom is 11.07 and the 2.5% point is 12.83 .

The same analysis can be applied to losing streaks with p replaced by q . The results are shown in Table 3. χ^2 is 1.02 and the 5% point with one degree of freedom is 3.84 .

N	1	2	3	4	5	6	7
Observed Frequency	70	21	9	1	0	0	0
Expected Frequency	70	21.1	8.07	3.32	1.41	.602	.259

Table 3 Observed and expected frequencies of N-game losing streaks in a 162 game season with $p = .432$

Streaks of Wins or Losses The average rate of occurrences of N wins is $1/\mu = qp^N/(1-p^N)$; the average rate of occurrences of N losses is the same expression with p and q interchanged. The average rate of occurrence of both is the sum and so the average number of games needed to produce a streak of N wins or N losses is

$$\left[\frac{qp^N}{1-p^N} + \frac{pq^N}{1-q^N} \right]^{-1} \quad (2)$$

In the special case of $p = q = 0.5$, the average 'time' between N -streaks is $2^N - 1$. Thus if a sequence of equiprobable 1's and 0's (or heads and tails) is randomly generated, a streak of N identical symbols will occur, on average, in every sequence of $2^N - 1$ symbols.

Applying (2) to the baseball data above results in Table 4 which is the 'sum' of tables 2 and 3. The value of χ^2 is 9.86 compared with the 5% significance level of 11.07.

N	1	2	3	4	5	6	7	8	9	10	11	12
Obs.Freq.	162	54	23	10	5	4	2	2	1	1	1	1
Exp.Freq.	162	54.3	23.8	11.45	5.81	3.02	1.62	.877	.480	.266	.148	.083

Table 4 Observed and expected frequencies of N-game streaks in a 162 game season with $p = .568$

Shutouts To win a game of table tennis a player must win 21 points. If A wins before B has scored a single point, then A is said to have scored a shutout. If the probability of A winning any particular rally is p and the rallies are independent, then clearly the probability of A recording a shutout is p^{21} . Players tend to win more points on their own serve however; if p_A and p_B are respectively the probabilities of A winning any particular rally when A serves and when B serves, then the probability of A shutting out B is $p_A^{11} p_B^{10}$, providing A serves first and is $p_A^{10} p_B^{11}$ if B serves first. If the initial serve is determined by the toss of a fair coin, the probability of a shutout by A is $(p_A^{11} p_B^{10} + p_A^{10} p_B^{11})/2$.

In some racquet sports, eg. badminton, racquetball and squash, to win a point it is necessary to have the serve; if the server loses a rally the serve is transferred to the opponent for the next rally but no point is scored. Let E be the event that A, with the serve, wins one point while B wins none. Then, conditioning on the outcome of the first rally, symbolized by W if A wins and L if A loses, we have

$$\begin{aligned} P(E) &= P(E|W) \cdot P(W) + P(E|L) \cdot P(L) \\ &= 1 \cdot p_A + p_B P(E)(1-p_A) \end{aligned}$$

since if A loses the first rally he/she must win back the serve on the next rally to restore the initial situation. Solving for $P(E)$ gives

$$P(E) = \frac{p_A}{1 - p_B + p_A p_B}.$$

To shut out B, A must win N points while B wins none; this happens with probability $(P(E))^N$ if A has the initial serve and $p_B(P(E))^N$ if B has the initial serve. If the initial serve is determined by tossing a fair coin the probability of A shutting out B is

$$\frac{1}{2}(1+p_B) \left(\frac{p_A}{1-p_B+p_A p_B} \right)^N \quad \dots (6)$$

For squash, badminton and racquetball the appropriate values of N are 9, 15 and 21 respectively. Table 4 gives shutout probabilities in a game of squash

$p_A \backslash p_B$.4	.5	.6
.4	.00217	.00487	.0116
.5	.0102	.0195	.0387
.6	.0339	.0563	.0953
.7	.0893	.131	.193
.8	.199	.260	.339

Table 4 Probabilities of a shutout in squash by player A where p_A, p_B are the probabilities of A winning a rally on A's, B's serve respectively.

Expected Length of the Stanley Cup Finals. The Stanley Cup Final is a 'best-of-seven' series; the team which first wins 4 games wins the series. Such a series may last 4, 5, 6 or 7 games. Consider the probability of team A winning the series in 6 games. To do so A must win the 6th game; of the first 5 games A must win any 3 and B the remaining 2, and so the required probability is $\binom{5}{3} p^3 q^2 p$. The other entries in Table 5 are calculated similarly.

	Team A	Team B
Probability of winning any game	p	q
Probability of winning series in 4 games	p^4	q^4
in 5 games	$\binom{4}{3} p^4 q$	$\binom{4}{3} p q^4$
in 6 games	$\binom{5}{3} p^4 q^2$	$\binom{5}{3} p^2 q^4$
in 7 games	$\binom{6}{3} p^4 q^3$	$\binom{6}{3} p^3 q^4$

Table 5 Probabilities of winning a best-of-seven series in 4, 5, 6 or 7 games

The probability that A wins the series is then

$$\begin{aligned} W(p) &= p^4 \left(1 + 4q + 10q^2 + 20q^3 \right) \\ &= p^4 \left(35 - 84p + 70p^2 - 20p^3 \right) \quad \text{since } q = 1-p \quad \dots (7) \end{aligned}$$

and the expected length of the series is

$$\begin{aligned}
 E(p) &= 4(p^4 + q^4) + 5 \binom{4}{3} (p^4 q + p q^4) + 6 \binom{5}{3} (p^4 q^2 + p^2 q^4) + 7 \binom{6}{3} (p^4 q^3 + p^3 q^4) \\
 &= 4(1+p+p^2+p^3) - 52p^4 + 60p^5 - 20p^6 \quad \text{games} \quad \dots (8)
 \end{aligned}$$

$E(0.5) = 5.81$ and $E(p)$ is symmetric about $p = 0.5$. I do not have data for the Stanley Cup Finals. However, the observed average length of the World Series, which is also a best-of-seven affair, is 5.85 games through 1985.

Generalization Consider now a best of $(2N+1)$ game series; the winning team is the first to win $N+1$ games. Let ℓ be the number of games won by the losing team; the possible values for ℓ are $0, 1, \dots, N$, and the possible lengths of the series are $N+1, N+2, \dots, 2N+1$.

Clearly the probability that A wins the series is

$$W_N(p) = \sum_{\ell=0}^N \binom{N+\ell}{N} p^{N+1} (1-p)^\ell \quad \dots (9)$$

and the expected number of games in the series is

$$E_N(p) = \sum_{\ell=0}^N (N+\ell+1) \binom{N+\ell}{N} (p^{N+1} q^\ell + p^\ell q^{N+1}) \quad \dots (10)$$

Equations (7) and (8) above are special cases of these polynomials with $N = 3$.

Properties of the 'Winning Polynomials' $W_N(p)$ The first few polynomials are

$$\begin{aligned}
 W_0(p) &= p \\
 W_1(p) &= p^2(3-2p) \\
 W_2(p) &= p^3(10-15p+6p^2) \\
 W_3(p) &= p^4(35-84p+70p^2-20p^3)
 \end{aligned}$$

Properties:

- (i) $W_N(p)$ is a polynomial of degree $2N+1$ with integer coefficients
- (ii) $W_N(p)$ is an increasing function of p with $0 \leq W_N(p) \leq 1$ for $0 \leq p \leq 1$.
- (iii) $W_N(0) = 0$, $W_N(1) = 1$ for $N = 0, 1, 2, \dots$, and $W'_N(0) = 0$, $W'_N(1) = 0$ for $N = 1, 2, 3, \dots$
- (iv) $W_N\left(\frac{1}{2}\right) = \frac{1}{2}$, since if both teams are evenly matched, the probability that A wins the series is $1/2$. This proves the identity

$$\sum_{\ell=0}^N \binom{N+\ell}{N} \left(\frac{1}{2}\right)^\ell = 2^N \quad (\text{which can be verified by induction})$$
- (v) $W_N(p) + W_N(1-p) = 1$, since the terms on the left are respectively the probabilities that A wins the series and that B wins the series. This implies
- (vi) $W'_N(p) = W'_N(1-p)$.
 The graphs of $W_N(p)$ in Fig. 1 indicate, as expected, that the larger N the "fairer" the contest in the sense that the better team is more likely to win a longer series.
- (vii) The sequence of polynomials $W_N(p)$ converges pointwise on $[0, 1]$ to the discontinuous limit function:

$$\lim_{N \rightarrow \infty} W_N(p) = \begin{cases} 0 & \text{if } p < \frac{1}{2} \\ \frac{1}{2} & \text{if } p = \frac{1}{2} \\ 1 & \text{if } p > \frac{1}{2} \end{cases}$$

Thus, in the limit, the contest is perfectly fair; the better team is certain to win. However, since $E_N(p) \rightarrow \infty$ as $N \rightarrow \infty$, this happy state of affairs involves the cost of a rather long series, a cost which not even the most ardent fan or player would be willing, or able, to bear. We must therefore be satisfied with less than perfection; in any finite series there is always a chance that the better team will lose. A more complete version of Fig. 1 allows us to calculate, for a given $p > \frac{1}{2}$, how large N must be in order to be, say, 95% certain that the better team wins. Of course, the closer p is to $1/2$, the larger will this critical value of N be.

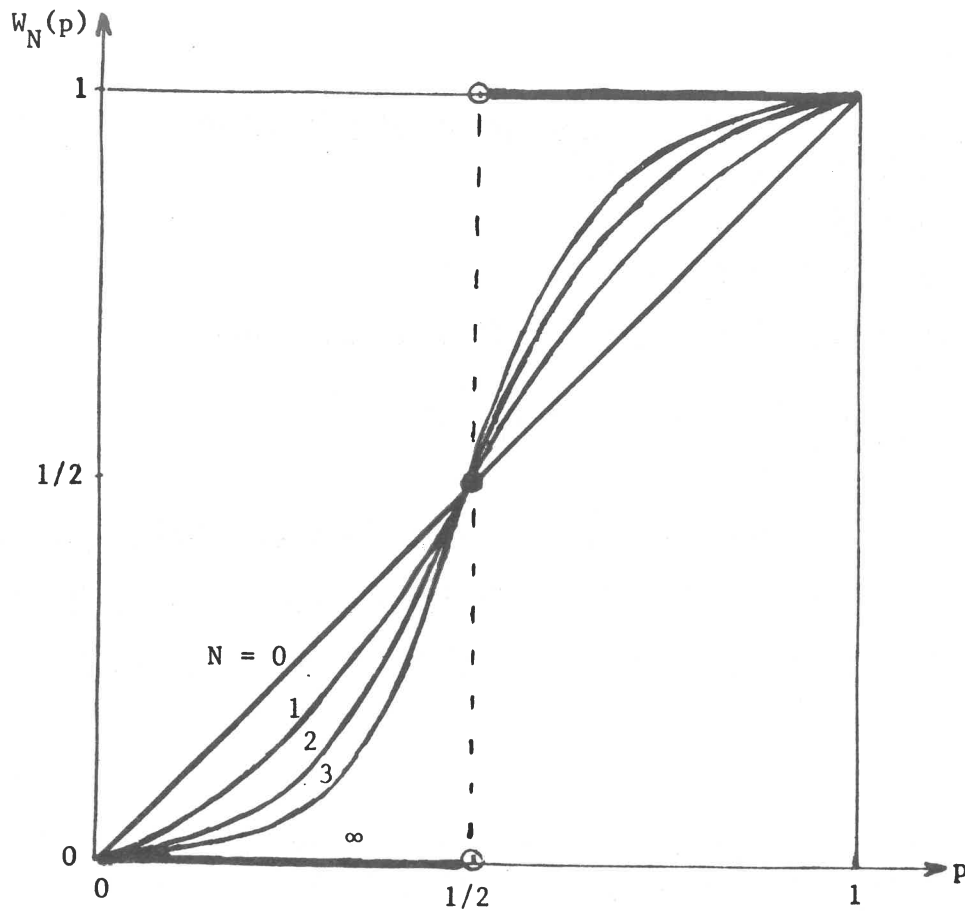


Fig. 1 Graphs of $W_N(p)$, the probability that team A wins a 'best-of- $2N+1$ -game series' versus p , the probability that A wins any particular game.

Properties of the $E_N(p)$ polynomials

The expected length of a best of $(2N+1)$ game series is

$$E_N(p) = \sum_{\ell=0}^N (N+\ell+1) \binom{N+\ell}{N} \left(p^{N+1} q^{\ell} + p^{\ell} q^{N+1} \right)$$

games where $q = 1 - p$. The first few of these polynomials are

$$E_0(p) = 1$$

$$E_1(p) = 2 + 2p - 2p^2$$

$$E_2(p) = 3 + 3p + 3p^2 - 12p^3 + 6p^4$$

$$E_3(p) = 4 + 4p + 4p^2 + 4p^3 - 52p^4 + 60p^5 - 20p^6$$

Properties:

- (i) $E_N(p)$ is a polynomial of degree $2N$ with integer coefficients
- (ii) $E_N(0) = E_N(1) = N + 1$
- (iii) $E_N(p) \geq N + 1$
- (iv) $E_N(1-p) = E_N(p)$
- (v) $E_N(p)$ attains its maximum value when $p = \frac{1}{2}$
- (vi)
$$E_N\left(\frac{1}{2}\right) = \sum_{\ell=0}^N (N+\ell+1) \binom{N+\ell}{N} \left(\frac{1}{2}\right)^{N+\ell}$$

$$\approx (2N+2) - 2\sqrt{\frac{N+1}{\pi}} \text{ using Stirling's approximation.}$$

For example, the World Snooker Championship is a best of 35 game contest; $E_{17}\left(\frac{1}{2}\right) = 31.25$, whereas the above approximation gives $E_{17}\left(\frac{1}{2}\right) \approx 31.21$.

Why is it that so many best of seven series seem to last 6 or 7 games?

Given equally matched teams, the probability that a best of $(2N+1)$ game series lasts $N + \ell + 1$ games is $\binom{N+\ell}{N} \left(\frac{1}{2}\right)^{N+\ell}$. For $N = 3$, the probabilities are:

Number of games	4	5	6	7
Probability	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{5}{16}$

Thus the two most likely series lengths are 6 and 7 when the teams are equally matched. This phenomenon is true in general, since it is easy to verify that

$$\max_{\ell=0,1,\dots,N} \binom{N+\ell}{N} \left(\frac{1}{2}\right)^{N+\ell} = \binom{2N}{N} \left(\frac{1}{2}\right)^{2N}$$

and occurs when $\ell = N$ and when $\ell = N - 1$. If $p \neq \frac{1}{2}$, the situation is different; for example with $N = 3$ the probabilities are

Number of games	4	5	6	7
$p = 0.60$.155	.269	.300	.276
$p = 0.65$.194	.289	.282	.235
$p = 0.70$.248	.311	.256	.185

The probability that the series goes to the limit (ie. lasts $2N + 1$ games) equals the probability that the first $2N$ games are split, which is $\binom{2N}{N} \left(\frac{1}{2}\right)^{2N}$ for equally matched teams. Using Stirling's approximation, this is approximately $\frac{1}{\sqrt{N\pi}}$ for large N . (The approximation is quite good for N as low as 3 (.3257, compared with the exact value of .3125)). In a best of 35 series, which may last 18, 19, ..., 34, or 35 games, the probability that the series will last 34 or 35 games exceeds 0.27 with equally matched opponents.

MINI-COURSES FOR HIGH SCHOOL STUDENTS

NORMAN RICE

Each May, approximately 600 high school students selected from all the school boards in the Kingston area descend on Queen's to participate in a week of "Enrichment Mini-Courses".

About 30 different week-long courses are offered by the university, including one entitled "*Mathematical Explorations*". This course is designed to expose the students to some new mathematical ideas, and to engage them actively in exploring these ideas. Some of the more successful topics were offered again this year: **Morris Orzech** talked about "*Secret Codes in Everyday Life*" (really a discussion of some of the mathematical aspects of unraveling the genetic code); **Joan Geramita** talked about "*Statistics for Science Fair Projects*"; **Leo Jonker** discussed "*Numbers of Numbers*" (different kinds of infinity) and the interrelated geometry and analysis involved in finding "*Straight Lines in a Cornfield*". Two new topics were also introduced. **Norman Rice** described how to use mathematics to find "*Winning Strategies for Everyday Games*"; **Jaap Top**, who is visiting the Department this year from Utrecht, talked about "*Big Numbers, Really Big Numbers, Prime Numbers, and Factoring*".

HUBERT WHITFIELD ELLIS, 1918-1990

L. LORNE CAMPBELL

I regret to report the death of Professor Emeritus Hubert Whitfield Ellis of the Department of Mathematics and Statistics, on January 1, 1990, in Ormond Beach, Florida.

Dr. Ellis was awarded Bachelor's and Master's degrees by Acadia University in 1940 and 1942. After wartime service, he went to the University of Toronto where he received the M.A. and Ph.D. in 1946 and 1947, respectively. From 1947 until his retirement in 1983 he was a member of the faculty at Queen's University.

While at Queen's he was among the most respected and helpful members of the Department of Mathematics and Statistics. Along with Ralph Jeffery and Israel Halperin, he made the department an important centre for the study of analysis in Canada in the 1950's and 1960's. He guided six students to the Ph.D. and he taught many of the undergraduate courses in mathematics. He served as Chairman for Undergraduate Studies in the department for the period 1962-1978. It was during this period that our instructional program expanded substantially and assumed its present form. His tact and good humour helped significantly in carrying out these duties as Undergraduate Chairman.

Although he was heavily committed to departmental duties, he served the University as a member of Senate and he served the mathematical community as a member of a Grant Selection Committee at the National Research Council and as a member of the Council of the Canadian Mathematical Congress.

Hu Ellis will be fondly remembered by colleagues who worked with him as a teacher and a scholar who did more than his share to make sure that the university worked in a human way.

He is survived by his wife Dorothy, his daughter Diane, and by two grandchildren. We wish to express our sympathy to them.

OLD PROBLEMS

A Non-Calculus solution to Jim Whitley's problem. (N. Rice)

Minimize the total area A enclosed by N regular polygons with pre-specified side numbers n_1, n_2, \dots, n_N and total perimeter L .

For regular polygons with n_i sides, the area A_i and perimeter L_i are related by $A_i = k_i L_i^2$ for some constant k_i depending only on n_i . Recall Cauchy's inequality: $(\sum a_i b_i)^2 \leq (\sum a_i^2)(\sum b_i^2)$, with equality holding if and only if there is some constant α for which $a_i = \alpha b_i$, all i . Applying this with $a_i = \sqrt{k_i} L_i$, $b_i = 1/\sqrt{k_i}$ gives:

$$L^2 = (\sum L_i)^2 = (\sum a_i b_i)^2 \leq (\sum a_i^2)(\sum b_i^2) = (\sum A_i)(\sum 1/k_i) = A \sum 1/k_i = AK$$

where $K = \sum 1/k_i$. Thus $A \geq \frac{1}{K} L^2$, and this minimum is achieved iff

there is some α for which $a_i = \alpha b_i$ for all i . Since $\frac{a_i}{b_i} = \frac{k_i L_i^2}{L_i} = A_i/L_i$, the minimum is achieved if and only if A_i/L_i is constant for all i .

(Solutions using Lagrange multipliers were also given by Jim Hodder and the proposer).

NEW PROBLEMS

Martin Kreuzer, a postdoctoral fellow at Queen's from the Universität Regensburg, F.R.G., proposes the following problems:

Suppose that each point in the plane is coloured either black or white. Prove that no matter how this 2-colouring of the points of the plane is done, there will always be a rectangle whose four corner vertices all have the same colour.

Dr. Kreuzer first encountered this problem in the 1979 German federal high-school competition in mathematics and actually proved a bit more:

No matter how the points of the plane are 2-coloured, there will always be a square whose four corners all have the same colour.

He was later able to show that even if the points of the plane are coloured with n colours, there will still always have to be a rectangle whose corners all have the same colour. However, he does not know if there always has to be a square with this property if n colours are used. This is the final problem:

If the points of the plane are coloured with n colours, must there always be a square whose four corners all have the same colour?

NEWS

In January, **L. Lorne Campbell** was made a fellow of the Institute of Electrical and Electronics Engineers (IEEE) for his work on signals and noise in nonlinear devices. This is a distinction made even more remarkable by his status as a mathematician, his dedication to teaching and research, and ten demanding years as head of department. In addition to this honour, he has recently become a licensed member of the Association of Professional Engineers of Ontario. This will be helpful to our program in Mathematics and Engineering. This summer he completes his tenure as head and begins a sabbatical year.

Leo Jonker has been appointed as the new head of the department for a five year term.

In February, **Agnes Herzberg** was elected a Fellow of the American Association for the Advancement of Science for her work in statistics. This honour was recently succeeded by another: Prof. Herzberg was elected President-Elect by the Statistical Society of Canada. She will become president in July 1991.

Bob Erdahl shared with **Sergei Ryshkov** a 1000 ruble prize awarded by the Steklov Institute for their joint paper, *Dual Systems of Integer Vectors and their Applications to Geometry*.

Noriko Yui has been promoted to Full Professor; **Jamie Mingo** has been promoted to the rank of Associate Professor.

Terry Smith has been reappointed as Director of the George L. Edgett Statistical Laboratory. STATLAB has just been awarded a N.S.E.R.C. infrastructure grant of \$34,000 yearly, for three years. This is nearly twice the amount received previously. The award is mainly intended to help meet some of the more complex demands made on the consulting centre and to support activity that leads to advances in statistical methodology. In addition to its services to the Queen's community, STATLAB provides a working environment for graduate training in statistical consulting.

HONORARY GRADUANDS

Two of the honorary doctorates granted by Queen's this year were to scholars in mathematics or statistics.

Israel Halperin was honoured by Queen's last fall with the degree Doctor of Laws. A graduate of Toronto and Princeton, and a Fellow of the Royal Society of Canada, Dr. Halperin was an influential member of Queen's faculty from 1939 to 1966. He is widely respected not only as a scholar and researcher but also as a human rights activist. He taught and inspired many mathematicians, including more than 10% of the current staff. Here are some of their comments:

(Jim Woods, Sci. '57)

A superb teacher, his treatment of mathematics brought it fully alive. He presented the subject as if he and the students were redeveloping these great ideas together in the classroom.

(Peter Taylor, Arts '64)

What Israel did was to let us see that what we thought we understood we didn't really understand at all, and what we thought we had read was not read carefully or completely enough, and what we thought we had written well had still some polishing to do. I discovered how incredibly far it is possible to go in terms of getting inside an idea or a structure, even after you think you have plumbed its depths. He was relentless: there was no going on until the job at hand had been properly done, no matter how humble that job at first (but rarely at second!) seemed. He always said just enough: never more. It was more than mathematics he was teaching: it was economy, and care, and something close to integrity.

(Agnes Herzberg, Arts '61)

Mathematics 21 was a whole year course. In February, Dr. Halperin told us that he would stop lecturing at the end of the month and for the rest of the year we would have a review. It was not to be the type of review we had thought it would be when we all brought our lists of questions to the next lecture hour. It was "_____, up to the board." "_____, can you prove that the rational numbers are enumerable?" "I think so." "Do it." At the end, "If you had said you could have, I would not have asked you to do it. One must be precise."

The reason Dr. Halperin gave for the long time spent "reviewing" in the class was that without it we would probably learn only 50% well. Then for anything that was built on that in later years, we would only learn 50% of 50% etc.

We learned many mathematical concepts from Dr. Halperin; we learned also what an excellent professor and teacher were. We always had the impression that in some way he was learning from us, not the other way around. We learned that no matter how many topics were listed in the calendar for the course or the topics on the outline he handed out, it did not matter if we got only to the first one as long as we were all interested, something which is worth remembering.

Dr. Halperin was the best mathematics professor I had.

Sir David Roxbee Cox, currently the Warden of Nuffield College, Oxford is regarded as one of the world's leading statisticians. He has over 200 publications to his credit, including 13 books. For almost 25 years, he has been editor of Biometrika, one of the leading statistical journals. He has been unstinting of his time on both British and international committees; most recently he has been chairman of the United Kingdom working party on AIDS.

He has been recognized through his election to the Royal Society of London in 1973, his knighthood in 1985 and his election as a foreign member to the United States National Academy in 1988.

This spring, Queen's honoured him for his outstanding contributions to the theory and practice of statistics over a wide range of topics.

THANK YOU

Our thanks to the many people who have sent donations in the past year to help keep the Communicator coming. Our cost now is approximately \$2000 for each issue. If you would also like to help, please send your cheque to the address below, payable to the Communicator, Queen's University.

Address for all correspondence

Editor, Queen's Mathematical Communicator
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario, K7L 3N6

