QUEEN'S MATHEMATICAL COMMUNICATOR

SUMMER 1992

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Queen's Mathematical Communicator
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A MATHEMATICAL MODEL FOR THE LOTTERY

M.S. NIKULIN

(Italian *lotteria*, from Hlot - meaning lot or destiny)

According to the American encyclopedia:

"Lotteries are generally schemes for distributing prizes by lot or chance. In their simplest form lotteries consist of the sale of tickets bearing different numbers, duplicate numbers being placed in a receptacle, such as a hat or a drum, from which numbers are drawn to establish the prize winners, being those holding the tickets with those corresponding numbers."

From "Educated Guessing" Samuel Kotz (1983, Marcel Dekker):

"A lottery is a game of chance with low stakes and potentially high winnings, which account for the widespread appeal of this type of gambling. In its simplest form, a player bets on a number and wins if the state also selects that number. While we usually view a lottery as a game, many applications exist in the real world. For example, insurance is a lottery with the premium of a policy playing the role of the value of a lottery ticket".

Gambling in the form of lotteries dates from the earliest times. The Roman emperors Nero and Augustus, used them to distribute slaves. In Europe, one of the first lotteries was in **Florence** in 1530, although little historical information remains. One of the most famous lotteries is one in **Genoa** which has continued since its inception.

In 1528 the great admiral Andrea Doria re-established the city-state of **Genoa** as a republic, with a pronounced aristocratic constitution. The republic was governed by The Great Council. The five members of the Council were to be elected each year from 90 candidates. Naturally, the people of **Genoa** were interested in the results of this election. They forecast and bet on the outcome. The financiers saw in this interest a source of possible gain and accepted stakes in exchange for a promise to pay the winning forecaster a very large sum of money. Of course, an unfortunate forecaster did not receive his/her money back. The financiers soon understood that it wasn't convenient to link their new source of funds solely to the election in the Great Council. Profits were limited as the elections were held but once a year and well informed citizens could accurately predict the results being well aware of the voters' favourites.

The financiers did not need the election itself, but only the model of the election, which might then be repeated often enough (for example, monthly) to thwart the well informed forecasters. It is precisely this model that was used to establish the **famous Genoese Lottery**, which was expanded little by little in many countries of Europe and existed in Austria and Italy until 1914.

The returns from the lotteries were so large that governments became interested and began to take them under their control (a form of "nationalization"). The Genoese Republic itself took control of the Lottery as early as 1620.

The passionate desire for wealth associated with the Genoese lottery was a constant source of misfortune, ruin and crime. In the beginning of the 19th century Laplace spoke out against the organization of lotteries. He underlined the immoral aspect of lotteries as means of robbing the poorest stratum of society. The stake lost by a poor man is equal in form only to the stake lost by a rich man! These protests were supported in several countries and so in the 19th Century the organization of Genoese lotteries was prohibited in England and France.

How was the Genoese lottery organized? Recall that it was the model of the election of the Great Council mentioned earlier. Instead of 90 candidates for five vacant places there were 90 numbers from 1 to 90. Each **drawing** imitated the election of the five members of the Great Council; from 90 numbers five were drawn at random (without replacement). According to the rules of the Lottery one can bet a stake on any one of the numbers from 1 to 90, or on any set of two, three, four or five numbers. In each of these five possible cases a player wins if and only if all numbers bet belong to the set of five drawn. A winning player receives a lot more than his stake. In addition the gain increases abruptly with the number of numbers on which the player bets; see the table below

NOTE: This was the subject of Professor Nikulin's Coleman-Ellis Lecture in March, 1992.

number of numbers on which player bets	gain obtained by winning player (stake is taken equal to 1)
1	15
2	270
3	5 500
4	75 000
5	1 000 000

The organizers of the Genoese lottery wanted their system to maximize profits. It was necessary to construct a mathematical model describing the rules and situations arising in the game and permitting planning through calculation of returns, losses etc. The search for this model continued up to the 18th century. At the beginning much effort was expended on the search for deterministic models. The failures of this approach stimulated a revision of all mathematical models used in the financial transactions and business of organizing lotteries. The most promising approach was connected with "calculating of the possible chances". The search for this model resulted in the creation of the theory of probability in the 17th and 18th centuries.

A probability model of the Genoese lottery can be expressed in terms of drawing balls from an urn. Here is one Model.

An urn contains n balls (n=90), of which m are white (m=5) and n - m are black (n-m=85). From the urn k balls ($k \leq m$) are drawn "at random" (k is the number of numbers on which one bets, k =1,2,3,4,5). If all k drawn balls are white, then a gambler is a winner, otherwise he loses. It is assumed that each individual ball is equally likely to be drawn, and hence there are $\binom{n}{k}$ different ways to choose k balls from the urn. The term "to draw at random" means that all possible ways to draw k balls from the urn are equally likely; it means that the probability that k specified balls will be chosen is

$$\frac{1}{\binom{n}{k}}. \qquad (1)$$

It is evident that the number of favourable cases is $\binom{m}{k}$. Thus the required probability of winning when one bets on k white balls will be

$$P_k = \frac{\binom{m}{k}}{\binom{n}{k}} = \frac{m(m-1)(m-2)\cdots(m-k+1)}{n(n-1)(n-2)\cdots(n-k+1)}, \quad k = 1,2,\ldots,m. \qquad (2)$$

If n=90, m=5 we obtain the probability distribution:

k	1	2	3	4	5
P_k	$\frac{1}{18}$	$\frac{2}{801}$	$\frac{1}{11748}$	$\frac{4}{511038}$	$\frac{1}{43949268}$

(3)

Let X_k be a random variable such that

$$X_k = \begin{cases} 1, & \text{if a player wins (all k drawn balls are white),} \\ 0, & \text{if a player loses.} \end{cases} \qquad (4)$$

Taking the basic bet per trial as the unit and denoting by g_k the gain of the player received in the case of his winning, the "return" of the player is the random variable Y_k:

$$Y_k = g_k X_k - 1 = \begin{cases} -1 & \text{, in the case of losing,} \\ g_k - 1, & \text{in the case of winning.} \end{cases} \tag{5}$$

It is evident that the expectation and the variance of X_k are

$$\mathbf{E}X_k = \mathbf{P}\{X_k = 1\} = P_k \quad \text{and} \quad \mathbf{Var}X_k = P_k(1 - P_k), \tag{6}$$

and hence the expectation and the variance of the return are

$$m_k = \mathbf{E}Y_k = g_k P_k - 1 \tag{7}$$

and

$$v_k = \mathbf{Var}Y_k = g_k^2 P_k(1 - P_k), \quad k = 1,2,...,m. \tag{8}$$

In particular, if n=90 and m=5, then m_k and v_k have the values given in the table: (9)

k	1	2	3	4	5
m_k	$-\frac{1}{6}$	$-\frac{29}{89}$	$-\frac{1562}{2937}$	$-\frac{72673}{85173}$	$-\frac{10737317}{10987317}$
v_k	$\frac{425}{36}$	$\frac{1438200}{7921}$			

It is interesting to note that all m_k are negative!

Suppose that N_k players take part in the lottery independently of each other, and they bet the same stake (equal to 1) on k balls; let X be the random variable

$$X_{ki} = \begin{cases} 1, & \text{if i-th player wins (all k drawn balls are white),} \\ 0, & \text{if i-th player loses, } 1 \le i \le N_k. \end{cases} \tag{10}$$

Let Y_{ki} be the random variable representing the return of the i-th player. Then the statistic

$$G_k = Y_{k1} + Y_{k2} + ...+ Y_{kN} \tag{11}$$

represents the total return of all N players in one trial, bet on k balls. It is evident that

$$G_k = g_k \mu_k - N_k, \tag{12}$$

where the statistic

$$\mu_k = X_{k1} + X_{k2} + ... + X_{kN_k} \tag{13}$$

has the Binomial distribution $B(N_k,P_k)$ with the parameters N_k and P_k. Hence, if $N_k \to \infty$ then according to the Theorem of Bernoulli about the law of large numbers, the **mean return**

$$\frac{G_k}{N_k} = \frac{g_k \mu_k}{N_k} - 1 \tag{14}$$

converges in probability to

$$m_k = g_k P_k - 1 < 0 . \tag{15}$$

Moreover, since the event $\{G \le x\}$ can occur if and only if the event

$$\left\{ \mu_k \le \frac{x + N_k}{g_k} \right\} \tag{16}$$

occurs, then if $N \to \infty$, from the de Moivre-Laplace theorem, it follows that

$$\mathbf{P}\{G_k \le x\} = \Phi\left(\frac{\frac{x + N_k}{g_k} - N_k P_k}{\sqrt{N_k P_k(1 - P_k)}} \right) + o(1), \tag{17}$$

where Φ is the standard normal distribution function. Thus if we choose x so that $(x+N_k)/g_k$

is an integer, we obtain the approximation (with the continuity correction)

$$\mathbf{P}\{G_k \leq x\} \cong \Phi\left(\frac{x + \frac{g_k}{2} - N_k m_k}{\sqrt{N_k v_k}} \right), \tag{18}$$

In particular, if x=0 and N_k/μ_k is an integer, then (18) implies that

$$\mathbf{P}\{G_k \leq 0\} \cong \Phi\left(\frac{g_k - 2N_k m_k}{2\sqrt{N_k v_k}} \right), \ k = 1,2,\ldots,m. \tag{19}$$

For example,

$$\mathbf{P}\{G_1 \leq 0\} \cong \Phi\left(\frac{45 + N_1}{5\sqrt{17 N_1}} \right). \tag{20}$$

From (19) it follows immediately, that the organizers of the lottery will have "a **guaranteed return**" only if there are many players!

Let us consider now the question of a possible "gambling system to win with certainty." Suppose that one player bets the stake $S_{k,t}$ on k balls on the t^{th} trial of the lottery. He wants to win the fortune h and to cover his expenses $E_{k,t}$, accumulated to the t^{th} trial. It is clear that he will achieve his goal, if $S_{k,t}$ satisfies the equation

$$g_k S_{k,t} = S_{k,t} + E_{k,t} + h, \tag{21}$$

i.e., if

$$S_{k,t} = \frac{E_{k,t} + h}{g_k - 1}, \tag{22}$$

and if he wins (!) in this trial. If he loses then it is natural to put

$$\begin{cases} E_{k,t+1} = E_{k,t} + S_{k,t}, \\ E_{k,1} = 0, \end{cases} \tag{23}$$

and to determine the value of the new stake $S_{k,t+1}$ in the next trial by

$$S_{k,t+1} = \frac{E_{k,t+1} + h}{g_k - 1}. \tag{24}$$

It is easy to verify that

$$S_{k,t} = h \frac{g_k^{t-1}}{(g_k - 1)^t} \tag{25}$$

and

$$E_{k,t} = h\left[\left(\frac{g_k}{g_k - 1} \right)^{t-1}\right], \quad t = 1,2,\ldots \tag{26}$$

For example, if k = 1, then from (25) and (26) it follows that

$$S_{1,t} = \frac{h}{14}\left(\frac{15}{14} \right)^{t-1}, \quad E_{1,t} = h\left[\left(\frac{15}{14} \right)^{t-1}\right], \ t = 1,2,\ldots, \tag{27}$$

and one can remark that the variables $S_{1,t}$ and $E_{1,t}$ increase very quickly. Under these tactics the player will receive his return h in the trial with the number T = t (T is a random variable) with probability

$$P\{T = t\} = P_k(1 - P_k)^{t-1}, \; t = 1,2,... \tag{28}$$

(this being the probability of losing the first t-l trials and winning the t^{th}) where P_k is the probability of winning in an individual trial . Hence, as $t \to \infty$

$$P\{T \geq t\} = \sum_{i=t}^{\infty} P_k(1 - P_k)^{i-1} = (1 - P_k)^{t-1} \to 0, \tag{29}$$

from which it follows that

$$P\{T < \infty\} = 1 , \tag{30}$$

i.e., with probability 1 the player will achieve his goal in a finite number of trials; moreover

$$ET = \frac{1}{P_k} < \infty , \tag{31}$$

since $P_k > 0$, and

$$VarT = \frac{1 - P_k}{P_k^2} . \tag{32}$$

In particular, if we put k = 1, n = 90 and m = 5, we obtain

$$P_k = P_1 = \frac{1}{18}, \quad P\{T \geq t\} = \left(1 - \frac{1}{18}\right)^{t-1}, \tag{33}$$

$$ET = 18 \quad \text{and} \quad VarT = 306. \tag{34}$$

We give a table (35) of some values of the probability $P\{T \geq t\}$ and $E_{1,t}$ (the accumulated loss of the player to the t^{th} trial.

t	10	20	30	40	50	60
$P(T \geq t)$	0.598	0.337	0.191	0.107	0.061	0.034
$\frac{E_{1,t}}{h}$	0.9	2.7	6.4	13.8	28.4	57.6

(35)

Let us change the rules of the game a little. We suppose that

$$S_{1,1} = 1, \quad E_{1,1} = 0, \quad h = 0. \tag{36}$$

On the one hand it is clear that if h = 0 , it is not interesting to play. But we consider the situation where a player was talked into buying a ticket, and let

$$\begin{cases} g_1 S_{1,t} = S_{1,t} + E_{1,t} , & t = 1,2,... \\ E_{1,t+1} = E_{1,t} + S_{1,t} , & t = 1,2,... \end{cases} , \tag{37}$$

i.e., we consider the tactics where the player wishes simply to recover his expenses. In this case $E_{1,2} = 1$ and

$$E_{1,t+1} = E_{1,t} \frac{g_1}{g_1 - 1} , \; t = 2,3,... \tag{38}$$

Therefore

$$E_{1,t} = \left(\frac{g_1}{g_1 - 1}\right)^{t-2} , \; t = 2,3,... \; (E_{1,1} = 0), \tag{39}$$

and

$$S_{1,t} = \frac{g_1^{t-2}}{(g_1 - 1)^{t-1}}, \quad t = 2,3,... \qquad (40)$$

Now we consider the lottery Lotto 6-49. There are n = 49 numbers, m = 6 and k = 0,1,2,3,4,5,6. In this case the probability that exactly k of a player's 6 pre-selected numbers are among the six numbers drawn is

$$p_k = \frac{\binom{6}{k}\binom{43}{6-k}}{\binom{49}{6}}, \quad k = 0,1,2,3,4,5,6. \qquad (41)$$

One can verify that

$$p_3 = 0.0176904039; \quad p_4 = 0.0009686197; \quad p_5 = 0.0000184499; \quad p_6 = 0.0000000715,$$

and hence the probability of winning is equal to

$$P = p_3 + p_4 + p_5 + p_6 = 0.018637545 \qquad (42)$$

and the probability of losing is

$$Q = 1 - P = 0.981, \qquad (43)$$

From (40), (42) and (43) it follows that

$$ET = \frac{1}{P} \approx \frac{1000}{19} \approx 53, \qquad (44)$$

$$ES_T = P + \sum_{t=2}^{\infty} \frac{g^{t-2}}{(g-1)^{t-1}} P(1-P)^{t-1} = \qquad (45)$$

$$= P\left(1 + \frac{1-P}{g-1} \sum_{t=1}^{\infty} \left[\frac{g(1-P)}{g-1}\right]^{t-2}\right). \qquad (46)$$

This series is divergent, since its general term is greater than 1, (as it was in the case of the Genoese lottery $\frac{15\cdot 17}{14\cdot 18} > 1$) , i.e., the mathematical expectation of expenses is infinite; the "duration of the game" is finite, but one needs to have infinite capital to win. There is the paradox!

Acknowledgements

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AUTHOR'S NOTE: This is a version of a lecture given by L. Bol'shev in Moscow and by me in Leningrad in 1974. In particular, I gave it in the high school where my daughter, Lenochka was a student at that time. She and all her classmates immediately ran out to buy lottery tickets. Quite naturally when not one of them won anything, they blamed me. Not only that, they wanted nothing to do with mathematics in their subsequent careers. My daughter became a pediatrician and still denies the role of lotteries in medicine. So, "no gambling with the State, especially the Socialist State" as Bol'shev used to say. I dedicate this article to the memory of my teacher Login Bol'shev.

* Readers interested in the history of the Genoese lottery and early attempts at probability calculations may wish to consult a recent paper: D.R. Bellhouse "The Genoese Lottery" STATISTICAL SCIENCE 6, 141-168b 1991.

THE YEAR OF THE RUSSIANS - THE QUEEN'S-STEKLOV EXCHANGE

ROBERT ERDAHL, Chair of Graduate Studies

These days it is commonplace for mathematics departments to have one or two visiting scholars from Russia or some other Eastern European country. This past year our department had eight such scholars: six Russian mathematicians, one Ukrainian and one Bulgarian. This is certainly a record number for any Canadian university, and possibly a world record (if such universities as the University of Moscow are properly disqualified). In addition, this coming year we will have two new Ph.D. students from the University of Kiev and one from the University of Moscow.

Many of these visitors are the most recent participants of a scientific exchange program between our department and the V. A. Steklov Institute, Moscow. This program was initiated by the visits of Arkady Malsev and Sergei Ryshkov in the fall of 1979. Since that time there has been a steady stream of mathematicians travelling between Kingston and Moscow. The Russian participants have all been members of the Steklov Institute, with branches in both Moscow and St. Petersburg. Many of the Canadian participants have been from Queen's; other Canadian universities have also participated.

The opportunity for mathematicians at Queen's to visit Steklov and work with researchers there has been of considerable scientific advantage. The Institute is unlike almost any other scientific establishment in the world. Founded in the 1930's as part of the USSR's Academy of Sciences, it has grown to become the world's largest research organization for pure mathematics, consisting of some 300 researchers with no responsibilities other than the investigation of fundamental questions in mathematics.

Over the past twelve years about 25 Canadians and an equal number of Russians have participated in the exchange. This activity has resulted in a considerable number of scientific discoveries and mathematical papers. The exchange was initiated in 1978 by John Coleman, and Bob Erdahl has acted as secretary and run the exchange since that time. Many members of the department have participated including Dan Offin, Ernst Kani, Eddie Campbell, Madan Wasan, Agnes Herzberg, Terry Smith, Tony Geramita and Paulo Ribenboim. But only a portion of Canadians visiting Moscow are from Queen's. In fact, the exchange functions as a national exchange. The number of Canadian mathematicians that have used our exchange over the past twelve years has far exceeded that for the official national exchange.

Our visitors have added a great deal to the mathematical life of the department. Several have taught courses, all gave stimulating lectures on their research topics, many collaborated with members of the department on various research programs. Our visitors this year were:

Konstantine Oskolkov: This past year Kostya was a popular lecturer, teaching several sections of MATH 227 to engineers. Our graduate students enjoyed his course on functional analysis. He gave a series of research level seminars on a variety of topics and collaborated with Dan Offin. Kostya is an eminent researcher in approximation theory and harmonic analysis. He is a member of Steklov, Moscow.

Michail Nikulin: A frequent visitor to Queen's, Misha spent the year collaborating with Terry Smith in STATLAB, and lecturing undergrads. He was so appreciated by his STAT 360 class that they nominated him for the prestigious Frank Knox prize for teaching. Misha is a prolific researcher in statistics, the author of several books, and a member of Steklov, St. Petersburg. As an added bonus, Misha visited Kingston with his wife Valya.

Oleg Bogoyavlenski: Dan Offin and Peter Henriksen (Physics) invited Oleg to Kingston in November 1991, to collaborate on research. This fall he will be joining the faculty, and our dynamical systems group will increase to three members (Dan Offin and Leo Jonker being the other two). Oleg is a prolific researcher, author of several books and a member of Steklov, Moscow.

Slava Futorney: A Lie algebraist, Slava won a prestigious NSERC International Fellowship to work with John Coleman on Lie and Kac-Moody algebras. He has joined us for two years. He was a popular teacher in MATH 121 this year, and an active participant in our Lie Algebras seminar. Slava came with his wife Julia, and two daughters Alona and Dasha. Slava is a member of the

Mathematics Faculty of the University of Kiev.

Nikolai Lyashenko: Part algebraist, algorithm theorist, statistician, probabilist, Nikolai joined Terry Smith in STATLAB a year ago. He has participated actively in the work of STATLAB, and helped in the direction of several graduate students in statistics. Nikolai is visiting Queen's with his wife Tanya and daughter Anna. Formerly Nikolai was a leading researcher at the St. Petersburg Institute of Computer Science and Automation.

Anatole Andrianov: Tolya was invited by Ernst Kani to give a series of lectures on his specialty, modular forms. Tolya is one of the world's leading experts on this topic. He came to Kingston for two months with his wife Olga.

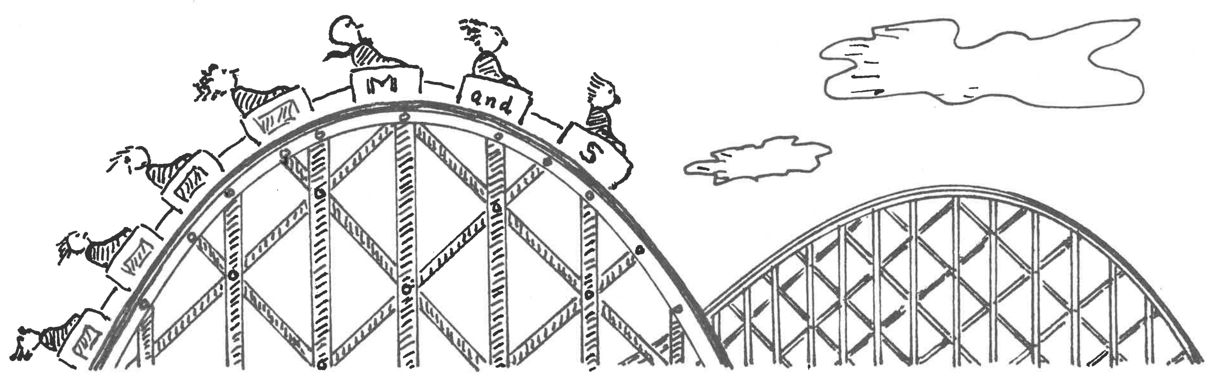
Anatole Vitushkin: Tolya made a three day visit to Queen's last fall. He gave an interesting lecture on the applications of the theory of functions of several complex variables to electrical engineering. Tolya is an eminent member of Steklov, Moscow and the mathematics faculty of the University of Moscow.

Jordan Stoyanov: Last Fall Jordan's graduate course "Topics in Probability Theory" was so popular the students asked that a sequel be given in the Winter term. Much to their delight he agreed, and gave a second set of lectures. Jordan is a probabilist from the Mathematics Institute of the Academy of Sciences, Sofia, Bulgaria. Although a Bulgarian, Jordan did all his post-graduate studies in Moscow at the Steklov Institute.

The Department of Mathematics and Statistics will never be the same again.

ROLLER COASTERS AND DREAMS

LEO JONKER, Head



Now that I have been department head for two years, people have stopped asking how it is going. I am not sure that I could give a much clearer answer than I did during the first few months, when my reaction was mainly one of bewilderment. It still feels like a roller coaster ride most of the time, at once frightening and exhilarating, and tolerable only because it will not go on forever. Also, as in the case of a roller coaster, you do not have much control even when you sit at the front. Everything seems to be in flux, government funding for universities (down), student numbers (up), numbers of faculty in this department (down), research expectations on the faculty (up), salaries (down this year if measured in constant dollars), class size (up), and so on. We have lost a number of faculty positions over the years. While we used to have 48 positions in the department, the number of long term tenure track positions is now down to 40. At a time when there are more outstanding young mathematicians and statisticians looking for positions than ever before, we do not have the funds to hire them.

Sometimes, when I am going down a particularly steep section of the roller coaster journey, I close my eyes and dream what one might hope for the future of this department. Elsewhere in this issue you can read about the recent opening of the Fields Institute, surely the most exciting development in the mathematical sciences in Ontario in a long time. When the

Fields Institute looks for a permanent location, through a process to take place this fall, could Queen's University put in a bid to become that location? It would imply significant benefits for the research activity in this department as well as for the training of our graduate students, and a boost for the reputation of Queen's internationally. What would be required? There will be competition from other universities more centrally located. We would have to present the site selection panel with plans for an institute building (about 30,000 square feet) on or near the campus, and housing for the visitors. The space would be rented by the Fields Institute, but a significant amount of seed money from Queen's would sweeten any proposal we could make. Five million would buy a very suitable facility, I suspect, and one million would build or buy some visitor housing. Does anyone out there want to dream with me?

On a more modest scale, there are several other projects I dream about. If the department had access to $200,000 or so, we could use that money to create a new position for a junior faculty member. The money would be used to provide salary and benefits for about four years, at which time the newcomer would replace a retiree. The number of outstanding position candidates is enormous these days. With cutbacks in Canada and abroad it is dishearteningly difficult for these young mathematicians and statisticians to get work. A rich lode of talent is going to waste.

Another dream of mine concerns the foreign graduate students who want to come to Queen's University. It would be of benefit to the department to have more of them. They enhance the intellectual environment, enrich our cultural life, and will benefit their home countries with their training when they return there. Unfortunately, tuition for foreign students in Ontario is prohibitively high. What is needed is a set of further scholarships to offset all or some of these high tuition fees. Anything from $2000 up can be used for this purpose. Sometimes I dream of a named scholarship for this purpose.

Recently we received a gift of $25,000 from a generous former student. We are planning to use the money to set up a system of tutorials named after him. These will serve the purpose of providing tutorial experience as well as funds to upper year students, and give the first and second year honours students an opportunity to have regular contact with senior honour students in mathematics or statistics. Once the details of the donation have been worked out, we will announce the donor.

Even if we do not find the Fields Institute settling in Kingston, I dream of a large house near the University Campus, owned by the department of Mathematics and Statistics, and used to house our many regular visitors. It would reduce our time spent helping visitors find a temporary home, it would allow us to invite more visitors because we could occasionally provide subsidised accomodation, and it would provide a continuous research environment for those visitors who are housed together in that location. $500,000 would probably be enough to buy such a facility. It could be named after a benefactor or after a Queen's person.

Of course, I also have little dreams. Every year we receive various smaller donations to the department, for which we are extremely grateful. We use these to build up our department trust fund. This fund allows us to cover various expenses that are not covered by University money. It allows us to bring in visitors to give talks on experiments in education at other post secondary institutions, it allows us to buy the occasional extra piece of computing equipment, and it allows us to fund the conference travel of a faculty member who has no other funds for it.

Well, it doesn't cost anything to dream, and the roller coaster keeps moving.

LIST OF DONORS

Mr. H.N. Beiles, Mr. James Bout, Mr. Robert Butcher, Dr. Hing Chang, Mr. F. Donato, Mr. Kevin Holloway, Miss Sandra Matsui, Dr. Joseph McPhail, Mr. B.R. Osborne, Mr. Howard S. Patch, Mr. C.L. George, Mr. D.B. Milligan, Miss H.D.H. Palmer, Mr. W. Soohoo, Mr. David Tanner, Mr. U.Vilmansen

M.SC. (ENGINEERING) IN MATHEMATICS

ROBERT ERDAHL, Chair of Graduate Studies

In September five new graduate students will be joining our department, all with undergraduate engineering degrees. They will form the first class in our new Mathematics and Engineering graduate programme leading to a Masters of Science in engineering. This new programme will complement our undergraduate Mathematics and Engineering programme which will enter its 28th year starting this fall.

One of the characteristics of modern engineering practice is the use of ever more sophisticated methods of design and analysis which directly depend upon difficult mathematical theories. It frequently happens that research or development engineers lack the necessary mathematical tools and insights to effectively understand and keep up with the innovations in their particular area. It is the theoretical side of an issue which deals directly with fundamental questions, and is often the most intriguing. We look at the new masters programme as a way of building more bridges between mathematics and engineering. By pursuing fundamental questions graduates of the programme will become more versatile, and better prepared for research careers in engineering.

Mathematical research has moved much closer to applications over the last twenty or so years. Many of the more famous recent discoveries are already attracting the interest of engineers: the theory of dynamical systems, mathematical chaos, nonlinear differential equations and modern control theory, the theory of turbulence. These are hot topics for both engineers and mathematicians. It is worth noting that besides being glamorous these topics are technically very complicated from both the mathematical and engineering points of view. It takes a great deal of mathematical preparation to penetrate and master any of these fascinating research areas.

To be eligible for the new master's programme a student must have an undergraduate engineering degree. We are looking for either recent B.Sc.(Eng.) students who are well prepared in mathematics and want to further explore advanced mathematics, or practicing engineers who have involved themselves in engineering mathematics and want to further explore the foundations of their topic. (In the latter case students are eligible for NSERC scholarships.)

The engineering master's programme will emphasize a thesis on a topic of engineering interest. Initially we are looking for students in the following areas:

- Control theory and robotics
- Communication theory and systems
- Optimization
- Applied mechanics
- Numerical analysis
- Stochastic processes
- Quality Control
- Engineering probability

The faculty most closely associated with the engineering programme are: L. L. Campbell, W. A. Cebuhar, J. H. Davis, R. M. Erdahl, R. M. Hirschorn, M. A. Maes, R. D. Norman, D. C. Offin, N. M. Rice, J. H. Verner, D. G. Watts, W. Woodside.

MATHEMATICS AND ENGINEERING

MATHEMATICS AND ENGINEERING

One of the characteristics of modern engineering practice is the use of ever more sophisticated methods of design and analysis which rely on high level mathematical tools and insights. At Queen's this trend resulted in the formation of the Mathematics and Engineering program. This program is unique in Canada, and it has been fully accredited and producing engineers eligible for professional registration since 1967.

IS THIS PROGRAM FOR YOU?

The program is designed to provide students with the Engineering background required to operate in their chosen field, together with a level of mathematical expertise which allows them to understand the "why" of the currently used methods of modelling and analysis. They are also able to create new methods as required and follow the evolution of the field as it develops.

To be successful in the program, you first of all must want to solve practical problems using mathematical and scientific ideas in a career as a practicing engineer. You must enjoy mathematics, and have a desire to know why some methods or formulas work (and where they stop working). You should enjoy discovering your own ways of using and adapting ideas to solve new problems. You should be motivated to acquire the skills that will allow you to keep up with progress in your field after you graduate.

CURRICULUM STRUCTURE

The students in the program take engineering courses offered from the traditional programs in common with the students from those departments. The course selection is designed to provide the fundamental background and insight into the ideas and methods of the chosen area. At the present time the following are available options:

- Control and Communication
- Process Control
- Applied Mechanics
- Structures

In addition to these courses the students in Mathematics and Engineering take special mathematics courses designed for their needs and to complement their engineering studies. These mathematics courses in the second and third years include Algebraic Structures, Linear Algebra, Advanced Calculus, Numerical Methods, Differential Equations, Methods of Applied Mathematics, Complex Analysis and Introduction to Control Systems, Probability Theory, Statistics. In addition there are courses in the fourth year offered specific to the separate options such as Linear Systems (modern control systems), Filtering and Estimation (stochastic control), and Continuum Mechanics.

CAREER PREPARATION

The graduates of this program have been successful in finding a variety of challenging engineering jobs in fields such as:

- Planning and developing communications systems
- Aerospace systems
- Manufacturing processes
- Design and analysis of structures
- Controlled fusion research
- Biomedical engineering

In their jobs (or in Graduate school if they choose to pursue further studies) the graduates find that their strong mathematical foundations make them versatile, adaptable, and confident in tackling new problems. A recent review of graduates from the years 1982-1989 show that the great majority are now either employed as engineers or doing graduate studies in engineering departments. The breakdown of results was as follows:

- 60% Employed as engineers
- 19% Engineering graduate study
- 21% Teaching, M.B.A., law and so on

FOR FURTHER INFORMATION

Chair for Mathematics and Engineering, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6.

THE THIRD CONFERENCE OF THE CANADIAN NUMBER THEORY ASSOCIATION AUGUST 18-24, 1991

NORIKO YUI

The Canadian Number Theory Association was founded in 1987 with the purpose of enhancing scientific communication and exchange among number theorists in Canada.

The first conference was held at Banff Conference Center in 1988, and the second at the University of British Columbia in 1989.

The third conference was hosted by Queen's University in the period August 18-24, 1991. It was part of the celebration of Queen's sesquicentennial anniversary, and was partially dedicated to Paulo Ribenboim on the occasion of his retirement from Queen's.

There were 177 registered participants: 50 from Canada and 127 from abroad. The emphases of the third conference were Analytic Number Theory, Arithmetical Algebraic Geometry and Diophantine Approximation. A special session was organized in honour of Paulo Ribenboim.

The Conference Banquet was emceed by A.J. Coleman. The four after dinner speakers, A. Granville, M. Griffin, R. Mollin and T. Viswanathan are all former students of Ribenboim. They were followed by a musical presentation consisting of a duet by J. Neukirch and Ute Jannsen accompanied by D. Zagier on the piano, and some classical guitar pieces played by M. Klassen.

There were 33 invited addresses: 10 for the plenary sessions and 23 for the special sessions. These speakers are listed below. In addition, 37 contributed papers were read at the conference.

The scientific activities were overseen by the Scientific and Organizing Committee, whose members were A. Granville (Georgia), F. Gouvea (Queen's and Colby), R Gupta (UBC), E. Kani (Queen's), H. Kisilevsky (Concordia), R. Mollin, (Calgary), C. Stewart (Waterloo) and N. Yui (Queen's).

The conference was supported by in part by a conference grant from the Natural Sciences and Engineering Research Council of Canada (NSERC). It also received financial support for young participants from the National Science Foundation and the National Security Agency of the United States. Invited speakers for the special session in honour of P. Ribenboim received partial support from Queen's University.

Invited Speakers to Plenary Sessions

There were 10 plenary addresses to the general number theory audiences of the conference. These were one hour talks and discussed the major problems, as well as current trends and advances in the fields in question.

The following mathematicians spoke at plenary sessions.

F. Beukers (Utrecht)
J. Coates (Cambridge)
J. Oesterlé (Paris VI)
R. Heath-Brown (Oxford)
R. Murty (McGill)
J. Neukirch (Regensburg)
A. Granville (Georgia)
D. Zagier (MPI Bonn)
J. Coates (Cambridge)
J.B. Friedlander (Toronto)
M. Waldschmidt (Paris VII)

Invited Speakers to Special Sessions

There were four special sessions. Three were organized by topic, focusing on Analytic Number Theory, Arithmetical Algebraic Geometry, and Diophantine Approximation. The fourth special session was devoted to Paulo Ribenboim: Former students, close research associates, friends and colleagues presented papers. These sessions consisted of thirty minute talks.

The following mathematicians spoke at the special sessions:

Analytic Number Theory
B. Conrey (Oklahoma State
J. Hoffstein (Brown)
M. Jutila (Turku)
H. Montgomery (Michigan)
H. Stark (San Diego)

Arithmetical Algebraic Geometry
U. Jannsen (MPI, Bonn)
K. Murty (Toronto)
N. Nygaard (Chicago)
T. Oda (RIMS, Kyoto)
L. Szpiro (ENS, Paris)
Yu. G. Zarhin (Pushchino)

Diophantine Approximation
D. Boyd (UBC)
D. Brownawell (PennState)
J.H. Evertse (Leiden)
J. Mueller (Fordham)
J. Valler (Texas)
P. Vojta (Berkeley)

'Ribenboim session'
K. Dilcher (Dalhousie), M. Filaseta (South Carolina), J. Minac (Western Ontario)
R. Mollin (Calgary)

GRADUATE DEGREES RECENTLY AWARDED BY THE DEPARTMENT

Student	Supervisor	Title of Thesis
DOCTORAL DEGREES:		
Kechagias, Epaminondas	E. Campbell	Homology Operations & Modular Invariant Theory
Tautz, Walter	N. Yui	Reduction of Abelian Varieties over Number Fields & Super singular primes
Wong, Cecilia	R. Giles	A Non-Classical First Order Logical System for Deductive Reasoning
Grinnell, Raymond	O.A. Nielsen	Lorentz Improving Measures on Compact Abelain Groups
Jaworski, Wojciech	E.J. Woods	Poisson & Furstenberg Boundaries of Random Walks
Ngai, Hung Man	T. Stroud	Simultaneous Estimation of Poisson Means--A Hierarchical Bayes Approach
Dillon, Douglas	P.D. Taylor	A Proposed Curriculum & its Implementation for OAC Algebra and Geometry
Dzieciolowsi, Krzysztof	W.H. Ross	Methods of Inference & Analysis of Influence in Multi-response Nonlinear Regression
Huang, Jie Jay	L.L. Campbell	Tail Probability of a Noncentral Indefinite Gaussian Quadratic form with Applications to Trellis Coded MDPSK Studies.
MASTERS DEGREES:		
Berard, Helene	L. Broekhoven	Acidification process in Lakewater of the Adirondack Mountains: A study of the relationships between chemical and watershed characteristics with diatom & crysophyte inferred values of ph & dissolved organic carbon.
Franklin, Sarah	L. Broekhoven	Estimating variability in blood pressure time series
Lorimer, William	D.A. Gregory	Cryptographically Secure Pseudo-Random Number Generators
Molson, Charles	L. Broekhoven	A Report on the Experiments of the Letgoes Team Summer 1990, Morrisburg Plant
Dupuis, Debbie	M.A. Maes	Asymptotics-Based Importance Sampling for Determining Multivariate Failure Probability
Guhit, Marjie	L. Broekhoven/ M.P. Griffin	Analysis of Metabolite Behaviour of Soybean Nodules
Lee, Perry	M.A. Maes	The Analysis of Indentation Pressures Generated by the Extrusion of Crushed Ice
Miller, Gary	M. Orzech	Clifford Algebra via Exterior Algebra
Slamet, Surjadi	D. DeCaen	Maximum Independent Sets in a Graph
Vander Meulen, Kevin	D.A. Gregory	Induced Subgraphs of Kneser Groups
Kreuzer, Bettina	A.V. Geramita	On the number of extremal points in a finite subset of the projective plane
Edriss, Abdi	M.P. Griffin	Improving Quality: Analysis of Control & Correlated Process Characteristics through Time Series Models
MacDonald, Larry	W.H.Ross/ M.P. Griffin	Parameter Invariant Methods of Inference for Parameters in Quasi-Likelihood Models.

Changes In The Department Since The Preceding Issue Of The Communicator

LEO JONKER, Head

There have been three new appointments since the spring of 1991. Ed Chow, a young statistician from Berkeley joined the faculty in the summer of 1991. He comes with expertise in statistical computing and something called the bootstrap method and a reputation (since confirmed) of being an excellent teacher. Already Ed finds himself supervising a graduate student. This summer we will be joined by another statistician, Duncan Murdoch. Duncan has several years' experience as a research statistician at Health and Welfare as well as at Carleton University and at the University of Waterloo, with teaching experience to boot. He comes with a substantial research record in several areas of statistics. Both Ed and Duncan are welcome additions to our Statistics group. They will help us in our quest to make the research environment in this department as lively as possible.

This past year we had an especially large number of mathematicians and statisticians visiting from abroad. These include a German, an Italian, an Argentinian, someone from India, a Bulgarian, several Americans, and no less than seven from the former Soviet Union. In addition there were many shorter term visitors. Several of these long term visitors did extensive teaching for us. One of the Russian visitors, Oleg Bogoyavlenskij, has accepted a position as professor in this department, and so constitutes the third appointment since the spring of 1991. Oleg has the position of leading researcher at the prestigious Steklov Institute, as do two of our other Russian visitors. He is a mathematician of the highest international reputation, working in the area of dynamical systems and mathematical physics. We are very excited to have him come aboard as well.

At the same time we lost some of our faculty through retirement or resignation. Jim Woods, Harold Still and Jim Whitley retired from their positions at Queen's. Many of the readers will no doubt remember them. They will be missed. We also lost the services of Bill Ross, who after a few years on our staff decided to take a position with the civil service in Ottawa.

ANOTHER TEACHING AWARD FOR PETER TAYLOR

Professor P.D. Taylor has been selected as the winner of the first distinguished teaching award of the Seaway Section (Ontario, Quebec and upstate New York) of the Mathematical Association of America. The award was presented at the recent Spring Meeting of the Section held at Queen's. Peter has long been active in developing innovative teaching methods and programs not only for Queen's students but also for local high school students and teachers. In 1986 he shared the ASUS Teaching Excellence Award with Professor Bill Barnes of the English Department for their new Mathematics and Poetry course. Besides, being keenly interested in teaching he is an active and productive researcher in mathematical biology.

GOVERNOR GENERAL'S ACADEMIC GOLD MEDAL

ROBERT ERDAHL, Chair of Graduate Studies

At the Spring Convocation this year, **Dr. Wojciech Jaworski** was awarded the Governor General's Gold Medal. This medal is awarded yearly to the best graduating masters or doctoral student in all areas. Wojciech entered the doctoral program in Mathematics and Statistics in September 1988 and emerged less than three years later with one of the best doctoral dissertations ever written in the department. He defended his thesis on July 15, 1991.

This is a very high distinction for Wojciech, for his thesis advisor, Jim Woods and for the Department. The competition for the medal is intense, involving all of the 600 or so graduate students receiving either their masters or doctoral degrees from Queen's in either the Spring or previous Fall Convocation.

The Governor General's Gold Medal is one of the most prestigious awards that a graduate student can receive. Each Canadian university with a graduate program awards one such medal annually. There is also a Silver Medal awarded at the baccalaureate level to the student graduating with the highest academic standing. The bronze medal is awarded in secondary schools and in post-secondary diploma programmes. It was the Earl of Dufferin, the third Governor General after Confederation who inaugurated this system of awards for academic excellence. However, it should be noted that our present standards for academic excellence are somewhat different than those applied in 1873 when the first eighteen medals were awarded. Some of those first citations were for marksmanship, skating, cricket and aquatics.

Wojciech's doctoral thesis was titled *Poisson and Furstenberg Boundaries of Random Walks* and was given the highest praise by his examiners. One enthused that "Jaworski's thesis is among the best that has been written in operator algebras in Canada, and, quite probably among the best that has been written in mathematics in Canada". Dr. Jaworski's thesis is in the area of mathematics called operator algebras. This theory has been in centre stage in abstract functional analysis since its inception in the late thirties. The theory was initiated by F.J. Murray and J. von Neumann in the three famous "Rings of Operators" papers and has engaged the ablest minds in functional analysis since that time. The theory has played a significant role in the study of the foundation of quantum theory. Wojciech has now published three papers relating to the material of his dissertation.

There is one remarkable fact that distinguishes Dr. Jaworski from any other Ph.D. student we have had, in fact, from any Ph.D. student we have ever heard of. During the period he was preparing his thesis he was also collaborating with Professor David Wardlaw of the Chemistry Department. This collaboration resulted in a total of eight papers on quantum mechanical scattering theory, all published in prestigious international journals. Their collaboration started two years prior to entering our Ph.D. program, but the intense part of writing and much of the research was done simultaneously with his research activities in mathematics (scattering theory and operator algebras have little in common).

Wojciech came to mathematics by a rather circuitous route. He first studied physics at Nikolas Copernicus University in Poland receiving a Ph.D. in 1986. He then worked as a post-doctoral fellow for Professor Wardlaw in the Chemistry Department at Queen's for two years before entering our Ph.D. program. He presently is a post-doctoral fellow in Mathematics at Carleton University.

HIGHEST HONOURS TO MATHEMATICS STUDENT

GRACE ORZECH, Chair of Undergraduate Studies

IMIN CHEN, who graduated with a B.Sc. (Honours) in Mathematics and Computing and Information Science in May, was the highest ranking student on the degree list for the Faculty of Arts and Science this year. For this achievement, he received the Governor General's Silver Medal and the Prince of Wales Prize as well as the medal in Mathematics.
Mr. Chen has also been awarded the prestigious NSERC Science 1967 Scholarship which will pay him $21,000 per year for four years. He plans to use this to begin graduate studies at Oxford University in the fall.

OLD PROBLEMS

SOLUTION TO THE DARTBOARD PROBLEM 1 - BY THE EDITOR (Bill Woodside)

Problem I: Find the probability that n darts thrown at random at a circular dartboard leave at least half (any half) of the board dart-free.

Solution: Let P_n be the required probability. Clearly $P_1 = P_2 = 1$ and $P_n < 1$ for $n \geq 3$. Taking the origin at the centre of the board, let (r_i, ϑ_i) be the polar co-ordinates of the ith dart, where ϑ_i is measured as a fraction of 2π relative to the radius through the first dart ie. $\vartheta_1 = 0$. Let ϑ_n = Max $\{\vartheta_2,\ldots,\vartheta_n\}$ and $\underline{\vartheta}_n$ = Min $\{\vartheta_2,\ldots,\vartheta_n\}$. Then P_n = Probability that $\vartheta_n - \underline{\vartheta}_n < 1/2$. But $R_n = \vartheta_n - \underline{\vartheta}_n$ is the range of $n - 1$ observations taken from a uniform distribution on the interval $[0,1)$. It is well-known* that R_n has a Beta distribution with mean $(n-2)/n$ and distribution function

$$F_n(\vartheta) = \Pr\{R_n < \vartheta\} = (n-1)(n-2)\int_0 x^{n-3}(1-x)\,dx$$

$$= \vartheta^{n-2}\,[n - 1 - (n-2)\,\vartheta]$$

$$\therefore\ P_n = \Pr\{R_n < \tfrac{1}{2}\} = F_n(\tfrac{1}{2}) = \frac{n}{2^{n-1}}\ .$$

From this we get the following result: suppose the first n darts 'succeed'; the probability that the $(n+1)^{th}$ dart also succeeds is the conditional probability $P_{n+1}/P_n = \frac{1}{2}\,(1 + \frac{1}{n})$. Can this be proved directly?

*See eg. de Groot 'Probability and Statistics' or Parzen 'Modern Probability Theory and its Applications.'

Dartboard Problem II Find the probability P_n that n darts thrown at random at the surface of a sphere leave at least half (any half) of the surface dart-free. This is still an open problem; however there is a rumour of a conjecture by M.A. Maes that

$$Pn = \frac{n^2 - n + 2}{2^n}$$

NEW PROBLEMS

Taylor's Jogging Problem I am in the forest at A . There is a road through the forest from B to C and I want to get to C . My jogging speed through the forest is w and along the road is v , where $v > w$. My strategy is to jog in a straight line to some point P on the road and then follow the road to C . Amazingly it doesn't matter at what point P I meet the road; my total time to C is the same. What is the shape of the road?

NOTE: This problem appears in Peter Taylor's recently published textbook, Calculus: The Analysis of Functions, Wall and Emerson 1992 which is aimed largely at the OAC market.

THE FIELDS INSTITUTE

Undoubtedly the most exciting recent development in the mathematical sciences in Ontario is the founding of the Fields Institute. This research institute joins the ranks of the Institute of Advanced Studies in Princeton, MSRI in Berkeley, IMA in Minnestoa, the Newton Institute in Cambridge England, IMPA in Rio de Janeiro, and IHES in France, as one of the great research centres in the world of mathematical science. It is named after the former Ontario mathematician Fields, who also gave his name to the Fields medal, the mathematical analogue of the Nobel prize. The Fields Institute is temporarily located in Waterloo, where it was officially opened on June 12, although its activities started as early as January. Our own Oleg Bogoyavlenskij is currently on a six month visit to the Fields Institute, and several of our graduate students, along with faculty member Wen Cebuhar have taken part in workshops at the Fields Institute. We look forward to close and fruitful involvement with this new venture.