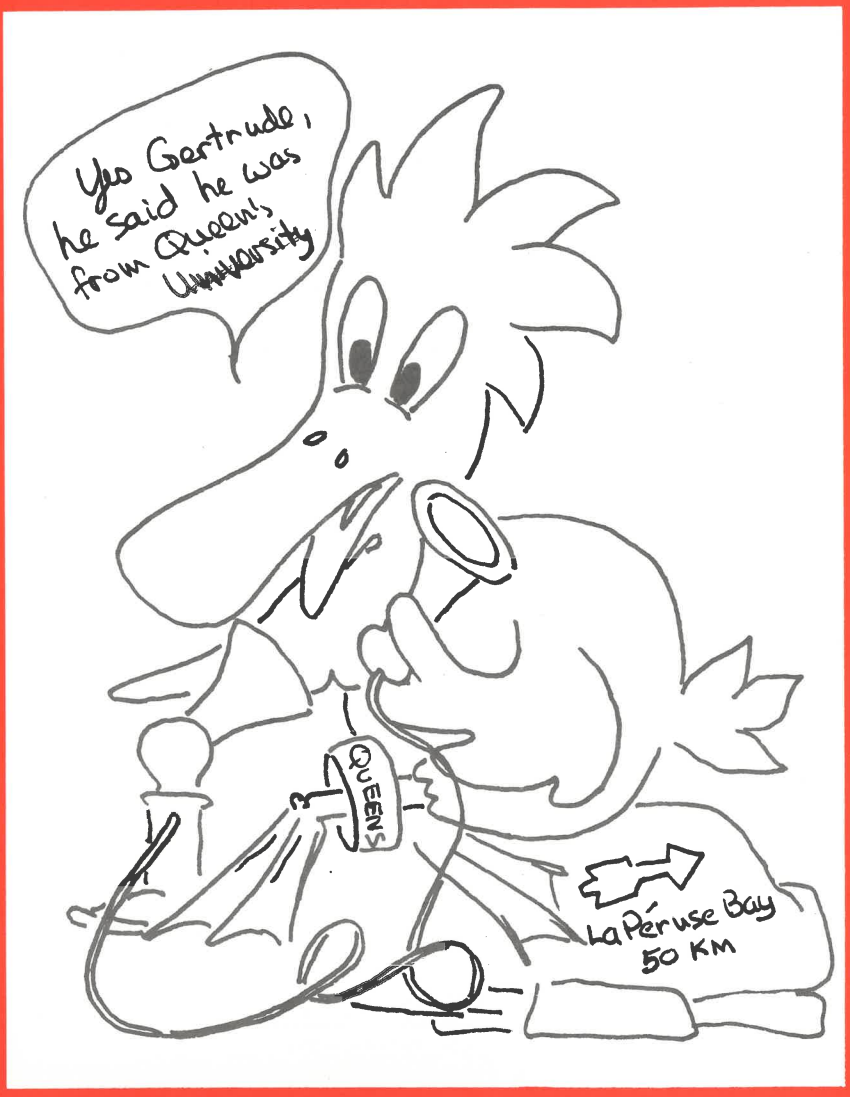
QUEEN'S MATHEMATICAL COMMUNICATOR

Feb. 1979



An aperiodical issued at Kingston, Ontario by the
Department of Mathematics and Statistics, Queen's University

FROM THE HEAD

With this first issue of the Queen's Mathematical Communicator, we are launching an effort to communicate with our former students and with teachers of mathematics.

The Communicator should serve several purposes all related to strengthening the Department of Mathematics and Statistics and making it more effective.

We hope:

- to call forth response from our readers with news about what they are doing and ideas about what we should do. Tell us how your mathematics training helped you in your subsequent activities;
- to bring you up-to-date on developments in the Department and at Queen's;
- to elicit from you the names of teachers and of really bright mathematics students who should hear more about our programme;
- to offer you in each issue one or more brief articles about an important new mathematical discovery or a description of an interesting application of mathematics;
- to pass on news from Queen's mathematics graduates.

(Let me hear immediately from years '68, '58, '48, '38, '28 ... !)

Launching the Queen's Mathematical Communicator is an experiment. We do not want to increase uselessly the current information overload or the wasteful proliferation of paper. We will aim to produce three issues each calendar year. BUT our persistence will be a function of YOUR RESPONSE.

If you wish to keep in touch, if you wish to help the Department, if you wish to continue to receive the Communicator, DROP US A LINE and tell us what you are doing. In particular, let us know if we do not have your correct address.

Please show the Queen's Mathematical Communicator to at least one other friend whom you think would enjoy it, particularly a mathematics teacher.

READ ON. LET US HAVE YOUR REACTIONS.

John Coleman

BIOMATHEMATICS AT QUEEN'S

(In recent years there has been increasing collaboration between the Biology and Mathematics Departments. Joan Geramita describes below some of the background and consequences of this development, in which both she and Peter Taylor have taken an active part.)

Historically, mathematicians, chemists and physicists have collaborated on problems presented by the "real" world. In fact, in past centuries, there were no physicists or chemists or mathematicians; there were just scientists with interests in one or another area of knowledge. Due to this history of interaction, mathematicians have tended to regard problems generated by biology as being too complicated for meaningful analysis by the mathematics available. Of course, statistics has always been regarded by biologists as a valuable tool; and statisticians have been willing and able to attack problems of data analysis and experimental design. In addition, certain areas of biology such as Mendelian genetics have lent themselves to mathematical treatment. But, on the whole, until the last twenty years or so, biologists have tended to teach and study biology without the use of mathematics and mathematicians have tended to ignore biology as a source of interesting problems which would be amenable to mathematical analysis.

The last twenty years (with roots going back into the 20's and 30's) have seen the growing use of mathematics as a tool not only via statistics in data analysis but in the formulation and presentation of theoretical biology as well. Since most biologists had, until recently, not been trained in mathematics, much of the development of mathematics applied to biology has been the result of collaboration between biologists and mathematicians (or physicists).

Such collaboration has been growing at Queen's in the last five years and has had benefits at the undergraduate and graduate teaching levels as well as in the area of biological research. The Ecology and Evolutionary Biology group in the Biology Department at Queen's has invited interested mathematicians and statisticians

to join them for their weekly colloguium series. A weekly BioMath seminar has been instituted to discuss topics in biology which appear in the literature with a mathematical presentation. This seminar provides biologists with an opportunity to learn about the relevant mathematics and gives everyone a chance to investigate whether the mathematics is indeed relevant.

At the undergraduate level, the calculus course taught to life science students is shaped by the knowledge of those topics and techniques currently in use in the literature. An undergraduate course in the application of mathematics to biology is taught in the biology department by a biologist-mathematician team. Mathematicians and Statisticians serve on supervisory committees for biology graduate students, and participate in graduate courses in the role of resource people.

For a mathematician to work effectively in this setting he (or, in my case, she) must know or have access to a wide range of mathematical information. Today's problem involves differential equations; tomorrow's game theory and the day after that, it is linear algebra. The wide range of interests among mathematicians at Queen's is a valuable resource for the "collaborators". The fact that several papers have been written with two authors, one of whom works with the biologists and one of whom doesn't indicates that the interaction between biology and mathematics ia also yielding the desired result of generating interesting problems for mathematicians.

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Our cover, drawn by Bob Erdahl, was inspired by work that Joan Geramita has been doing on mathematical models for the mating patterns of snow geese in the Canadian Arctic.

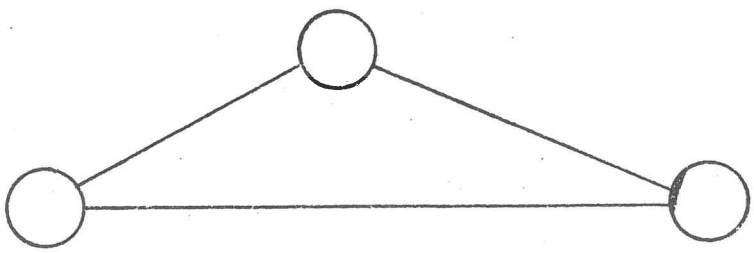
SOME MATHEMATICS OF THE TELEPHONE NETWORK

by
Ron Horn

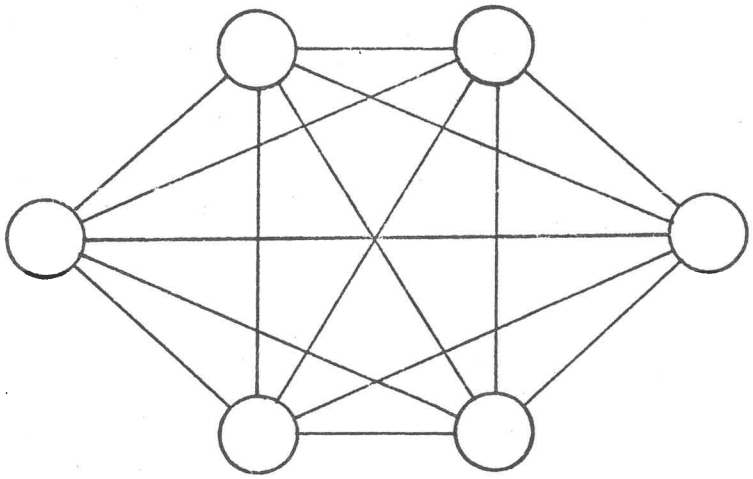
(Ron Horn was born in London, Ontario and attended Adam Scott CVI in Peterborough. He graduate in 1967 in mathematics from Queen's and completed his M.Sc. and Ph.D. degrees there. His thesis, entitled "Convex Programming: the introduction of some numerical methods and a comparison of these new methods with existing methods", was done under the supervision of Bruce Kirby).

When Alexander Graham Bell invented the telephone, he built two of them, connected them together with a piece of wire, and voila, telecommunication!

However, two telephones were not enough to satisfy everyone. The third one was constructed, and was connected to each of the other two by pieces of wire.

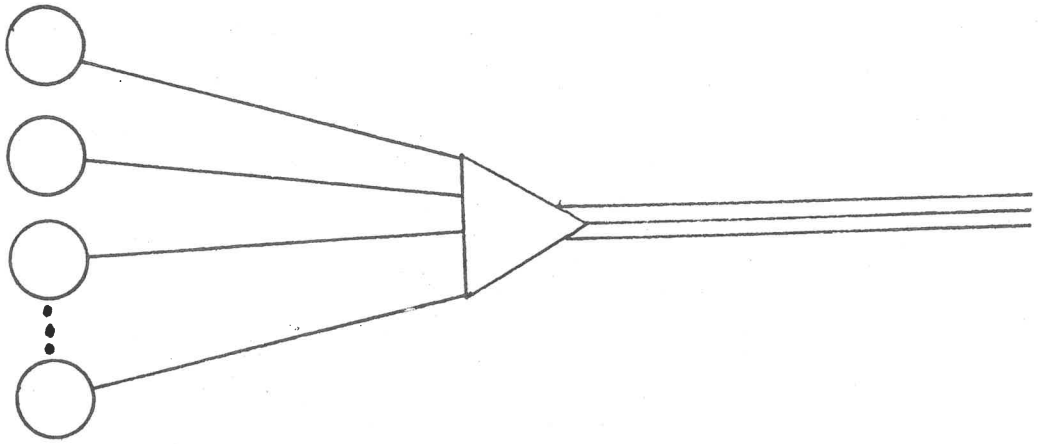


And soon the fourth, fifth, sixth telephones were added, and a problem was identified. How should all of these telephones be connected to allow any one of them to communicate with any other one?



If all telephones were to be directly connected to each other, a result from COMBINATORICS says that the number of pieces of wire would have to be $n(n-1)/2$, where n is the number of telephones. There are about 15,000,000 telephones in Canada alone in 1979; this means that about 112,000,000,000,000 pieces of wire would be needed if the original connection process had been continued.

Fortunately, it wasn't. Someone was quick to realize that every telephone would not be used continuously. In fact, in 1978, STATISTICS showed that the average residential telephone was only used about 5 minutes, and a business telephone about 10 minutes, during the busiest hour. It was thus decided to establish several stages of concentration.



MANY TELEPHONES CONCENTRATION FEWER LINES

Initially, this concentration was provided by operators; it is now done by sophisticated switching machines.

A problem then arose in PROBABILITY; how many lines are required to serve a certain number of telphones, making sure that all telephone users can have access to a line almost every time they so desire?

Today's worldwide telecommunication network is designed using tens of thousand concentration points. Certain portions of this massive network are operated by government agencies, other sections are controlled by private industry. In either case, the objective is to provide excellent service at minimum cost. This goal can be restated as a mathematical OPTIMIZATION problem of minimizing cost, subject to numerous constraints.

Since leaving Queen's in 1970 with a Ph.D. in Mathematics, I have worked at Bell-Northern Research in Ottawa. BNR is a wholly-owned subsidiary of Bell Canada, and of Northern Telecom. Bell Canada is the largest telecommunications carrier in Canada, serving over 8,000,000 telephones as well as data and video customers, and Northern Telecom is a world-wide manufacturer of telecommunications equipment. With laboratories in Ottawa, Montreal and Toronto, BNR employs about 2100 persons of whom about 1070 hold university degrees. Research and development at BNR is goal-oriented in support of, primarily, Bell Canada and Northern Telecom.

In mentioning COMBINATORICS, STATISTICS, PROBABILITY, and OPTIMIZATION, I have just scratched the surface of the application of Mathematics at BNR. It is interesting to note that much of the Mathematics which I studied during my post-graduate years at Queen's, which did not appear to have many real-world applications then, is now being used on a daily basis by scientific organizations such as Bell-Northern Research.

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The February 1979 Bulletin of the Can. Assoc. of Univ. Teachers is running an article announcing to the world (what we all knew) that Queen's students and alumni believe that Queen's is the only university.

All of the students in the Class of '79 in Course J - Mathematics and Engineering already have jobs. Several of them were offered three or four. This programme, created by Jacke Hogarth, was and is unique. It produces a very sophisticated type of engineer, much in demand and increasingly so.

A CURRENT PROBLEM

Submitted by Ron Horn, BNR, Ottawa

(In order to visualize the problem, one would be advised to consider the process of customers arriving at a bank at random, and requiring service from any of the servers. The only difference is that, if enough servers are not available, the customer leaves instead of waiting.)

Consider a group of M identical servers.

There are N types of customers, each with their own individual characteristics.

For each type i, $i \in N$,

(a) customers are assumed to arrive at random
(b) the average arrival rate is λ_i customers per unit time
(c) service time for each customer is assumed to follow a negative exponential distribution
(d) the average service time is $\frac{1}{\mu_i}$ units of time
(e) m_i servers must be available simultaneously to serve 1 customer.

The problem is to find, for each type of customer, the probability that a customer will be denied service due to a shortage of servers.

For $N = 1$, the problem has been solved by A.K. Erlang (among others). The solution procedure is as follows (assume $m_1 = 1$ with no loss of generality):

1) Consider extremely small time intervals.
2) Assume that j servers are busy with Probability $P(j)$. (The answer that we are looking for is $P(M)$.
3) The system may be considered a Makov process, in that
   - the probability of starting in state j and moving to state $(j+1)$ is $\lambda_1 P(j)$

3) (Cont'd)

- the probability of starting in state j and moving to state $(j-1)$ is $j\mu_1 P(j)$
- the probability of starting in state j and remaining in state j is

$$(1-\lambda_1-j\mu_1)P(j) \ .$$

4) The expressions above may be restructured to derive the probability of being in state k at the end of a small interval, as

$$P(K) = \lambda_1 P(K-1) + (K+1)\mu_1 P(K+1)$$

$$+ (1-\lambda_1-K\mu_1)P(K)$$

5) The above system yields M equations (nontrivial) in $M+1$ unknowns, ie. $P(0), P(1),\ldots,P(M)$.

6) The $(M+1)$th equation can be added as

$$\sum_{j=1}^{M} P(j) = 1 \ .$$

7) The solution to the equations yields

$$P(M) = \frac{A^M/M!}{\sum_{j=0}^{M} A^j/j!} \qquad (A = \frac{\lambda}{\mu})$$

which is the Erlang B formula.

The problem for more than 1 type of customer is unsolved.

THE ACAP ASSESSMENT

The independent assessment of the Mathematical Sciences in Ontario by the ACAP Committee - (eleven out-of-province internationally known mathematicians, statisticians and computer scientists) gave very high marks to our group of statisticians:

"The faculty in statistics at Queen's are of good quality; they are proceeding in a thoughtful, steady manner.

The STATLAB provides a valuable statistical service that might well be imitated by other universities. It is an integrating force for the statisticians and it provides educational opportunities for the students and research motivation for the faculty."

The ACAP Committee noted that the activities of our algebra group "have attracted many visitors from Canada and abroad". They noted our "wide-ranging strength in functional analysis ... with applications to physics and the mathematical foundations of quantum mechanics". They concluded that the Queen's "staff could capably handle a larger number of qualified graduate students".

About Jeffery Hall the ACAP Committee expressed the opinion "it is the most functional building housing a mathematics department of this size that we have ever seen".

The ACAP Committee was particularly enthusiastic about the Queen's programme in Control Theory, stating "It is the only programme of its kind in Ontario. Control theory is an important branch of modern applied mathematics having direct applications to technology and significant implications for pure mathematics."

PEOPLE IN THE NEWS

Jon Davis has served for three years as editor of IEEE Transactions on Control. It is a great honour for anyone so young to be asked to take on this important task. It means that he is absolutely up-to-date (even a year ahead of published research) on the most sophisticated aspects of Control Theory.

John Coleman, the Head of the Department for the past eighteen years and editor of the successful W. J. Gage series of mathematics textbooks, hopes to win the Liberal nomination in Kingston and the Islands and to beat Flora MacDonald. Quite a task! He claims that Ottawa needs someone who knows both how to add and subtract!

Ron Dimock, who graduate in Mathematics and Engineering in 1971 and who later studied law at Queen's, has recently been named Big Brother of the Year in Toronto.

This has been a big year for Donald Watts. In addition to being elected Fellow of the American Statistical Association (ASA), president of the Statistical Society of Canada and president-elect of ASA's section on education, Donald has just been elected a member of the International Statistical Institute. The institute has 1,014 members, of whom 40 are Canadian.

PROBLEM

Submitted by Peter Taylor

Suppose we have p standard coins, all identical except that one has a different weight from the others. We wish to use a simple balance to find the non-standard coin. Assume we have an inexhaustible supply of standard coins which we can use in the weighing.

Let $Q(n)$ be the maximum value of p for which the problem can be solved in n weighings.

To get a feeling for the problem discover that

$$Q(1) = 2$$

$$Q(2) = 5$$

$$Q(3) = 14 \quad .$$

Find $Q(n)$ and prove that it holds.

Can you think of any other, more difficult versions of this problem?