

QUEEN'S MATHEMATICAL COMMUNICATOR



SUMMER 1993



Two Heads. John Coleman 1960-1980
and James Williamson 1842-1880

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A FLURRY OF ACTIVITY IN A QUIET CORNER

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In Slater (1966) we read, "...there are many quiet corners of the subject ...which have given much pleasure and intellectual delight to many mathematicians during the past two centuries." Three years ago, in the preface to their excellent new book on the same subject, Gasper and Rahman (1990) spoke of the "flurry of activity" that "has been so infectious that many researchers found themselves hopelessly trapped by this alluring 'q-disease', as it is affectionately called". What is this area of mathematics that has recently "popped up in physics, Lie algebras, transcendental number theory, and statistics, in addition to new developments in ...classical analysis, combinatorics, and additive number theory"? [Andrews (1986)]. Gasper and Rahman used the title "Basic Hypergeometric Series" for their book; this term has often been used, for example by Bailey (1935) and Slater (1966), but Andrews (1991) considers it misleading as "basic" implies neither simple nor fundamental. Andrews (1986) himself used the snappier title "q-series" (the parameter q appears in all the formulas). Here are two of the earliest q -series formulas that were discovered [Euler (1748)]:

$$\prod_{j=0}^{\infty} \frac{1}{(1-xq^j)} = \sum_{r=0}^{\infty} \frac{x^r}{(1-q)\dots(1-q^r)} \quad (1)$$

$$\prod_{j=0}^{\infty} (1-xq^j) = \sum_{r=0}^{\infty} \frac{(-x)^r q^{r(r-1)/2}}{(1-q)\dots(1-q^r)} \quad (2)$$

In both cases we have the striking feature of a relationship between an infinite product and an infinite series.

Before I try to define a q -series, let alone explain my own particular interest in q -series, we need to look at the "wooden plough" of Sawyer (1955, p. 64) and its successor, the generalized hypergeometric series $\sum_{r=0}^{\infty} u_r$, where u_{r+1}/u_r is a rational function of r , usually written as

$$\frac{u_{r+1}}{u_r} = \frac{(a_1 + r)\dots(a_A + r)x}{(c_1 + r)\dots(c_C + r)(r+1)},$$

and $u_0 = 1$. Sawyer's wooden plough, the Gaussian hypergeometric series, is the special case $A = 2$, $C = 1$. Whereas Gauss used the notation $F(a, b, c, x)$, nowadays we write

$${}_2F_1[a, b; c; x] = {}_2F_1\left[\begin{matrix} a, b; x \\ c \end{matrix}\right] = 1 + \frac{abx}{c \cdot 1!} + \frac{a(a+1)b(b+1)x^2}{c(c+1) \cdot 2!} + \dots \quad (3)$$

(to emphasize that $A = 2$, $C = 1$). The majority of the series that one meets as an undergraduate have this form. In particular

$$\begin{aligned} 1 + x + x^2 + \dots &= {}_2F_1[1, b; b; x], \quad |x| < 1, \\ (1-x)^{-k} &= {}_2F_1[k, b; b; x], \quad |x| < 1, \\ e^x &= \lim_{k \rightarrow \infty} {}_2F_1[k, b; b; x/k], \\ \ln(1-x) &= -x {}_2F_1[1, 1; 2; x], \quad |x| < 1, \\ \arcsin(x) &= x {}_2F_1[1/2, 1/2; 3/2; x^2], \quad |x| < 1, \\ \arctan(x) &= x {}_2F_1[1/2, 1; 3/2; -x^2], \quad |x| < 1. \end{aligned} \quad (4)$$

The Chebyshev, Legendre, Gegenbauer, and Jacobi polynomials can also be stated as ${}_2F_1[\cdot]$ functions. Furthermore, the trigonometric functions, $\sin x$ and $\cos x$, and the Bessel functions can be stated as ${}_0F_1[\cdot]$ functions and the Hermite and Laguerre polynomials as ${}_1F_1[\cdot]$ functions, i.e. as limiting forms. No wonder Sawyer described ${}_2F_1[\cdot]$ as the wooden plough when discussing the fertile areas of mathematics that were cultivated up to the middle of the twentieth century. He considered that the boundaries of this area were "but for one or two small clefts explored by pioneers, virgin rock."

Basic hypergeometric series (q -series) are related to generalized hypergeometric functions by analogy rather than by extension or simplification. For a basic hypergeometric series we again have $u_0 = 1$, but now u_{r+1}/u_r is a rational function of q^r for fixed q , giving

$$\frac{u_{r+1}}{u_r} = \frac{(1-a_1q^r)\dots(1-a_Aq^r)x}{(1-c_1q^r)\dots(1-c_Cq^r)(1-q^{r+1})}.$$

Thirty years after Gauss presented his famous Göttingen paper on the ${}_2F_1[\cdot]$ function, Heine (1846, 1847) collated and extended the then known facts about q -series, and in so doing showed that there is a large body of material for them that is analogous to that for ${}_2F_1[\cdot]$ series. Others, such as Cauchy (1843), also obtained q -series results, but it is Heine who is generally considered to be the major researcher in the area at that time. Much is known about Euler who can be regarded as the father of q -series. Heine, however, is today little known. He was born in March 1821, the eighth of the nine children of a Berlin banker; his sister Albertine married the banker Paul MendelssohnBartoldy (the brother of the famous composer), his son Carl was a theatre producer, and one of his four daughters, Anselma, was an authoress who was interested in women's rights. Heine obtained his doctorate in 1842 after studying under Gauss at Göttingen and under Dirichlet at Berlin University. From his subsequent career we can infer that he was one of Gauss's more able students — earlier, in 1810, Gauss had told Bessel, "This winter I am giving two courses of lectures to three students, one of whom is only moderately prepared, another less than moderately, and the third lacks both preparation and ability." In 1848 Heine was appointed professor of mathematics at Halle University where he remained until his death in October 1881.

Bonsall (1982) has commented, "Live mathematics is that body of mathematical theorems that is currently understood by living mathematicians. A substantial trace of this work is left behind in a fossilized form in publications, just as the coral reef is left by the polyps." The coral reef left behind from Heine's work is his (1861, 1878) book "Handbuch der Kugelfunktionen: Theorie und Anwendungen." In the second (1878) edition he chose to include some of his earlier work on q -series. This was set in a smaller typeface than the rest of the book, indicating perhaps that Heine did not consider it to be directly related to his work on spherical harmonics (a close connection was later discovered!). Important results of Heine's are the q -binomial theorem,

$$\begin{aligned} {}_1\Phi_0[a; -; q, x] &= 1 + \frac{(1-a)x}{(1-q)} + \frac{(1-a)(1-aq)x^2}{(1-q)(1-q^2)} + \cdots \\ &= \sum_{r=0}^{\infty} \frac{(1-a) \cdots (1-aq^{r-1})x^r}{(1-q) \cdots (1-q^r)} \\ &= \prod_{j=0}^{\infty} \frac{(1-axq^j)}{(1-xq^j)}, \end{aligned} \quad (5)$$

the q -analogue of a Gauss summation formula for a ${}_2F_1[\cdot]$ series,

$$\begin{aligned} {}_2\Phi_1[a, b; c; q, c/(ab)] &= \sum_{r=0}^{\infty} \frac{(1-a) \cdots (1-aq^{r-1})(1-b) \cdots (1-bq^{r-1})}{(1-c) \cdots (1-cq^{r-1})(1-q) \cdots (1-q^r)} \left(\frac{c}{ab}\right)^r \\ &= \prod_{j=0}^{\infty} \frac{(1-cq^j/a)(1-cq^j/b)}{(1-cq^j)(1-cq^j/(ab))}, \end{aligned} \quad (6)$$

and the q -analogue of Euler's transformation formula for a ${}_2F_1[\cdot]$ series,

$${}_2\Phi_1[a, b; c; q, x] = \prod_{j=0}^{\infty} \frac{(1-abxq^j/c)}{(1-xq^j)} \times {}_2\Phi_1\left[\frac{c}{a}, \frac{c}{b}; c; q, abx/c\right]. \quad (7)$$

Heine made much use of the result $a = \lim_{q \rightarrow 1} \{(1-q^a)/(1-q)\}$ in his derivation of results analogous to many in Gauss (1813). Try setting $x = -y$, $a = q^{-m}$, m a positive integer, in (5), for instance, and then letting $q \rightarrow 1$. This gives

$$\begin{aligned} \lim_{q \rightarrow 1} {}_1\Phi_0[q^{-m}; -; q, -y] &= 1 + \frac{my}{1!} + \frac{m(m-1)y^2}{2!} + \cdots \\ &= (1+y)^m = {}_1F_0[-m; -; -y] \end{aligned}$$

(the usual binomial theorem with positive integer power). The q -binomial theorem seems to have been discovered independently by various people. Often it is called Heine's theorem; sometimes it is called Cauchy's theorem as it can be found in Cauchy's (1843) paper. When $a = 0$, (5) becomes (1); replacing x by x/a in (5) and letting $a \rightarrow \infty$ gives (2).

Two people who were aware of Heine's contribution to the theory of q -series, and similarly quietly obtained "much pleasure and intellectual delight," were Rogers, over the years 1893–1919, and Jackson, who spent from 1904 to 1954 writing a long series

In 1910 the famous Cambridge mathematician G. H. Hardy received a letter from a 23-year old Indian clerk, "I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself" Among the formulas in his letter was

$$\frac{1}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}} = \left\{ \sqrt{\left(\frac{5 + \sqrt{5}}{2} \right)} - \frac{1 + \sqrt{5}}{2} \right\} e^{2\pi/5}. \quad (8)$$

Hardy's reaction was "A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true because if they were not true, no one would have had the imagination to invent them. the writer must be completely honest, because great mathematicians are commoner than thieves or humbugs of such incredible skill." The writer was Ramanujan. The story of how Hardy persuaded him to spend most of the rest of his short life in Cambridge, England, has been told a number of times; see, for instance, Andrews (1986).

One of the undergraduates at Cambridge at the time was W. N. Bailey. Bailey was greatly impressed by Ramanujan's work on q -series, for example by the famous Rogers–Ramanujan identities

$$1 + \frac{q}{(1-q)} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \dots = \frac{1}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})\dots} \quad (9)$$

$$1 + \frac{q^2}{(1-q)} + \frac{q^6}{(1-q)(1-q^2)} + \frac{q^{12}}{(1-q)(1-q^2)(1-q^3)} + \dots = \frac{1}{(1-q^2)(1-q^3)(1-q^7)(1-q^8)(1-q^{12})(1-q^{13})\dots}, \quad (10)$$

obtained independently by the two authors and published in Rogers and Ramanujan (1919). Bailey included material on q -series in his 1935 book. This in turn led Lucy Slater (1966) to do the same. It was these two books and a slim monograph by R. P. Agarwal (1963) that first introduced me to q -series as a graduate student in the sixties.

The recent explosion of interest in the subject is the outcome of the work of three people: George Andrews, Richard Askey, and Rodney Baxter. George Andrews, of Penn State University, is well-known for his researches in number theory where he has obtained many results on partitions involving q -series. Also it is he who recognized the importance of Ramanujan's "Lost Notebook" when he found it in 1976 in the Wren Library in Cambridge where Whittaker had deposited it; see e.g. Andrews (1979). Askey, of the University of Wisconsin, leads a team of people whose researches are predominately about q -gamma and q -beta functions and about orthogonal q -polynomials. The contribution of Baxter of the Australian National University has been to demonstrate the importance of q -series in the solution of one of the most difficult problems that has arisen in theoretical physics in recent years, the hard hexagon model; see e.g. Andrews' (1986) account of the problem and of Baxter's solution.

My own modest interest relates to the use of q -series in discrete distribution theory. Over many years I have been concerned with the important rôle of generalized hypergeometric functions in discrete distribution theory; see Sections 4.1, 4.2, and Chapter 6 in Johnson, Kotz, and Kemp (1992). One of the simplest generalized hypergeometric-type distributions is the binomial. The binomial probability mass function is

$$\Pr[X = r] = \binom{n}{r} \pi^r (1 - \pi)^{n-r}, \quad r = 0, 1, \dots, n,$$

where the random variable X is the number of successes in n trials (e.g. in n throws of a die), π is the probability that a trial is successful (i.e. that a die lands showing a "six"), and n and π are constant. The outcomes of the trials are assumed to be independent. An important tool for dealing with discrete

distributions is the probability generating function (pgf), $\sum_r p_r s^r$. The binomial distribution has the pgf

$$\sum_{r=0}^n \binom{n}{r} \pi^r (1-\pi)^{n-r} s^r = \left(\frac{1+\lambda s}{1+\lambda} \right)^n = \frac{{}_1F_0[-n; -; -\lambda s]}{{}_1F_0[-n; -; -\lambda]}, \quad (11)$$

where $\lambda = \pi/(1-\pi)$. (The binomial pgf is more often stated in the form $(1-\pi+\pi s)^n$, but we shall find (11) an easier form to generalize.)

Consider now a row of n automatic bank tellers located inside a student union building and suppose that exactly $n-1$ of them are outlets for the same bank. The student authorities are interested in the number of tellers that are in use at the busiest time of day, 12:00 P.M. Let the probability that any particular one of the $n-1$ tellers is in use at 12:00 P.M. on a particular day be π , and let the corresponding probability for the n 'th teller be π_n . This corresponds to throwing simultaneously $n-1$ dice each with probability of success π together with an n 'th die with probability of success π_n ; in Kemp and Kemp (1991) we called this a "one-dud-die model." The probability that there are r successes in the n trials is equal to the probability that there are r successes in the first $n-1$ trials and the n 'th trial is a failure, plus the probability that there are $r-1$ successes in the first $n-1$ trials and the n 'th trial is a success, i.e.

$$\binom{n-1}{r} \pi^r (1-\pi)^{n-1-r} \cdot (1-\pi_n) + \binom{n-1}{r-1} \pi^{r-1} (1-\pi)^{n-r} \cdot \pi_n, \quad r = 0, 1, \dots, n.$$

The pgf is now

$$\begin{aligned} \sum_{r=0}^{n-1} \binom{n-1}{r} \pi^r (1-\pi)^{n-1-r} (1-\pi_n) s^r + \sum_{r=1}^n \binom{n-1}{r-1} \pi^{r-1} (1-\pi)^{n-r} \pi_n s^r \\ = (1-\pi+\pi s)^{n-1} (1-\pi_n) + (1-\pi+\pi s)^{n-1} \pi_n s \\ = \left(\frac{1+\lambda s}{1+\lambda} \right)^{n-1} \left(\frac{1+\lambda_n s}{1+\lambda_n} \right), \end{aligned} \quad (12)$$

where $\lambda_n = \pi_n/(1-\pi_n)$. By the same form of argument, if all n automatic bank tellers belong to different banks and have different probabilities $\pi_1, \pi_2, \dots, \pi_n$ of being in use at 12:30 P.M. on any particular day, then the pgf for the number of tellers in use at the appointed time is

$$\prod_{j=1}^n \left(\frac{1+\lambda_j s}{1+\lambda_j} \right),$$

where $\lambda_j = \pi_j/(1-\pi_j)$, $j = 1, 2, \dots, n$. Suppose now that there is a log-linear odds relationship between the π_j , i.e. $\ln[\pi_j/(1-\pi_j)] = \ln \lambda_j = \ln c + (j-1) \ln q$, $j = 1, 2, \dots, n$, $0 < q < 1$ (in Kemp and Kemp (1991) we gave reasons for this being a reasonable assumption to make). Then the pgf for the number of tellers in use becomes

$$\prod_{j=0}^{n-1} \frac{1+cq^j s}{1+cq^j} = \prod_{j=0}^{\infty} \frac{1+cq^j s}{1+cq^j} \cdot \frac{1+cq^{j+n}}{1+cq^j} = \frac{{}_1\Phi_0[q^{-n}; -; q, -cq^n s]}{{}_1\Phi_0[q^{-n}; -; q, -cq^n]}, \quad (13)$$

by Heine's theorem, and

$$\Pr[X = r] = \frac{(1-q^n)(1-q^{n-1}) \dots (1-q^{n-r+1}) q^{r(r-1)/2} c^r}{(1-q)(1-q^2) \dots (1-q^r)} \prod_{j=0}^{\infty} \frac{1}{(1+cq^j)}. \quad (14)$$

Numerical computation of these probabilities is very straightforward, as

$$\Pr[X = r+1] = \Pr[X = r] (q^r - q^n) c / (1 - q^{r+1}), \quad r = 0, 1, \dots, n-1. \quad (15)$$

An assumed value can be taken for $\Pr[X = 0]$, the remaining probabilities can be obtained relative to $\Pr[X = 0]$ via this two-term recurrence relation, and finally their true values can be found by forcing them

to add to unity. Because we have a convolution of n Bernoulli distributions (with parameters $\pi_1, \pi_2, \dots, \pi_n$), the mean of the distribution is the sum of the individual Bernoulli means

$$\mu = \sum_{i=0}^{n-1} \{cq^i/(1+cq^i)\}$$

and the variance is similarly

$$\sigma^2 = \sum_{i=0}^{n-1} \{cq^i/(1+cq^i)^2\}.$$

In Kemp (1987) a weapon defense system model led to an alternative log-linear assumption for the π_j , namely $\ln \pi_j = \ln C + (j-1) \ln Q$, $j = 1, 2, \dots, n$, $0 < Q < 1$. This gave a different q -analogue of the binomial distribution with much more complicated expressions for the probabilities.

It is well-known that a binomial distribution tends to a Poisson distribution with parameter μ as $n \rightarrow \infty$, $\pi \rightarrow 0$, with $np = \mu$, μ constant. What happens to our q -analogue (13) of the binomial distribution when $n \rightarrow \infty$? (We need make no assumptions about c or q as $cq^n \rightarrow 0$ when $n \rightarrow \infty$.) Suppose that a very large number of fish in succession approach the vicinity of a fishtrap, but that the lure of the bait diminishes over time such that the j 'th fish to approach the trap enters it with log-linear odds ($\ln c + j \ln q$), i.e. with probability $\pi_j = cq^j/(1+c^j)$, $0 < q < 1$, where $j = 1, 2, \dots$ [Kemp (1992b)]. The pgf for the number of fish caught in the trap is the limiting form of (13)

$$\prod_{j=0}^{\infty} \frac{1+cq^j s}{1+cq^j} = \sum_{r=0}^{\infty} \frac{q^{r(r-1)/2} c^r s^r}{(1-q) \dots (1-q^r)} \prod_{j=0}^{\infty} \frac{1}{(1+cq^j)} \quad (16)$$

(from (2)), and

$$\begin{aligned} \Pr[X = r] &= \frac{q^{r(r-1)/2} c^r}{(1-q)(1-q^2) \dots (1-q^r)} \prod_{j=0}^{\infty} \frac{1}{(1+cq^j)} \\ &= \Pr[X = r-1] q^{r-1} c / (1-q^r), \end{aligned} \quad (17)$$

$r = 0, 1, \dots$ The mean and variance are now

$$\mu = \sum_{i=0}^{\infty} \{cq^i/(1+cq^i)\} \quad \text{and} \quad \sigma^2 = \sum_{i=0}^{\infty} \{cq^i/(1+cq^i)^2\}.$$

This distribution was first derived in an investigation into sequential decisions for oil exploration by Benkherouf and Bather (1988); they named it the Heine distribution. Further models, corresponding to queueing and other stochastic processes, are put forward in Kemp (1992a).

Letting $q \rightarrow 1$ and $c \rightarrow 0$ such that $c/(1-q) = \mu$ in (16), we find that

$$\Pr[X = r] / \Pr[X = 0] \rightarrow \mu^r / r!,$$

and so the Heine distribution can be regarded as a q -analogue of the Poisson distribution with pgf $e^{\lambda s}/e^{\lambda}$. But just as there is more than one q -analogue of the binomial distribution, so there are other q -analogues of the Poisson distribution; some of these are explored in Kemp (1992b). The multiplicity of q -analogues of generalized hypergeometric functions is now widely recognized; Exton (1983, p. 129) commented that "A whole family of basic exponential functions could be conceived and defined in some such manner as

$$E(q, \lambda; x) = \sum_{r=0}^{\infty} \frac{x^r q^{\lambda r(r-1)}}{[r; q]!} \quad (18)$$

where $[r; q]! = (1-q) \dots (1-q^r)/(1-q)^r$. Taking $\lambda = 1/2$ and $x = cs/(1-q)$ shows the connection between (18) and (16).

Sawyer (1955, p. 28) has remarked that "To explore, to discover patterns, to explain the significance of each pattern, to invent new patterns resembling those already known — each of these activities increases the bulk of mathematics. From the practical viewpoint, it becomes extremely difficult to keep track of all the

results that have been discovered; and a vast litter of unconnected theorems hardly constitutes a beautiful subject. Both as a business man and as an artist, the mathematician feels the urge to draw all these separate results together into one. The history of mathematics therefore consists of alternate expansions and contractions."

Exton's book was the first to be published which dealt exclusively with basic hypergeometric functions; it was written only ten years ago at a time when the subject of q -series was expanding very rapidly indeed. There is the possibility now though, see Andrews' (1991) discussion of an apparently very complicated q -series formula, that the subject is moving from an era of expansion into an era of contraction when "some exceptional genius will say, 'All that we know can be seen as almost obvious if you look at it from this viewpoint, and bear this principle in mind'" [Sawyer (1955, p. 29)].

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TOPICS IN PROBABILITY AND STATISTICS

Andrew McKellips

This past year, participants in a new course, *Topics in Probability and Statistics*, played hosts to several distinguished guests. Professor Agnes Herzberg scheduled a number of department visitors to meet with students in an informal setting to discuss various aspects of probability, statistics, graduate studies and general scientific research.

Speakers included Sir David Cox of Nuffield College, Oxford, who discussed survival analysis and graduate studies at Oxford; Dr. Alexander Tsukanov, who was a visitor at Queen's for a period of three months, spoke about his experiences at the Instrument Making Institute in Sevastopol, Ukraine, various facets of expert data systems, and the shape of academics and research past and present in the former U.S.S.R.; Dr. Christopher Field of Dalhousie University recounted his duties as an expert witness; Dr. David Thomson of Bell Laboratories compared the working environment in a research institute with that in academia; Dr. Jamie Myles of Queen's University discussed clinical trials along with various aspects of graduate studies; Ms. Christine Anderson of the University of Waterloo also talked with the class about graduate studies; Dr. Douglas Dale of Carleton University discussed sampling theory and the earlier days of surveying at the Dominion Bureau of Statistics; Dr. Peter Donnelly of Royal Holloway and Westfield College, University of London, spent time with the class over breakfast introducing coupling theory and several powerful results (including profitable wagers with a deck of playing cards); Dr. Adrienne Kemp and Professor David Kemp of the University of St. Andrews presented several probability distributions, modelling problems and q-series, and spoke about graduate studies in Scotland; most recently, Professors Erich Lehmann and Juliet Shaffer, of the University of California at Berkeley, spent breakfast with the class discussing their experiences in graduate school and academia.

Needless to say, class participants found the experience to be very informative and even inspiring, as new ideas for research topics presented themselves frequently. Each student in turn worked on and presented an independent research project, with topics ranging from censored data analysis to wavelets. Dr. Kemp was kind enough to present some of her work with q-series in this issue.

Editor's Note: Andrew graduated from Queen's in Mathematics and Engineering in 1992. He is currently a member of our first class in the new M.Sc. (Engineering) program in Mathematics, working under the supervision of Dr. Lorne Campbell. This fall he will begin a doctoral program at Princeton University. The editor gratefully acknowledges Prof. Herzberg's powers of persuasion in connection with Dr. Kemp's article.

MATHEMATICS AND ENGINEERING AT QUEEN'S A BRIEF HISTORY OF THE EARLY YEARS OF THE DEPARTMENT

William Woodside

This year marks the Centennial of the Faculty of Applied Science; last year the university celebrated its Sesquicentennial, so it seems appropriate that we look back at the early days of the department and in particular the close ties it has had with engineering.

The Royal Charter establishing Queen's University was signed by the young Queen Victoria on October 16 in 1841, the year in which Kingston became the capital city. The founding of Queen's by the Presbyterian Church in Canada, assisted by the Church of Scotland, was a move designed to counterbalance the founding of King's College in Toronto in 1827 under the aegis of Bishop Strachan and the Church of England. King's College eventually became the University of Toronto; the rivalry between the two institutions continues to this day.

The first Principal was the Reverend Dr. Thomas Liddell, a minister from Scotland. He chose as the first professor of mathematics and natural philosophy the Reverend James Williamson of the University of Edinburgh, the university on which Queen's was to be modeled. It must have been difficult to attract scholars from the relative comfort of an established Scottish university to an insecure and uncertain future in an institution which had opened with only a dozen students, no building of its own and little or no financial support. Many of the first professors who came out from Scotland returned after a few years. Liddell himself endured for only four years. Williamson however, who had been 36 on his appointment in 1842, served as Head of mathematics until 1880 and from 1876 to his death in 1895 as Vice-Principal. A bronze bust, made in 1892 to celebrate his fifty years of service to the university, stands in the Mathematics Library. A plaque with the engraving "In loving memory of Professor James Williamson D. D., long known as the students'

friend" hangs in Grant hall. In 1852 he married Margaret Macdonald, sister of Sir John A. Macdonald, our first Prime Minister and another hardy Scot. Much credit goes to Williamson for helping to keep the young university alive during difficult times.

Williamson was succeeded in the Chair of Mathematics by Nathan F. Dupuis, son of a Frontenac County farmer. Dupuis had served for four years as a clockmaker's apprentice in Kingston and had taught public school for six years, before graduating from Queen's with a B.A. in 1866 and an MA in 1868. He had already served as Head of Chemistry and had taught physics, geology and biology when he assumed the Chair of Mathematics in 1880. A further three years were to elapse before he was able to relinquish his responsibilities in chemistry to W. L. Goodwin. Dupuis strengthened the program in mathematics which, not surprisingly, had been quite elementary in Williamson's time. He wrote textbooks in Geometrical Optics (1868), Plane Geometry (1889), Solid Geometry (1893), Algebra (1893), Astronomy (1902), Spherical Trigonometry (1910) and the Measurement of Time (1911). The clock in Grant Hall was designed by Dupuis and built by his students in the Mechanics Laboratory. He was active in nurturing the newly established Faculty of Medicine. Among the first at Queen's to recognize the importance to Canada of applied science, he was the major mover in the founding of the School of Mining which developed into the Faculty of Applied Science. He was the first Dean of Applied Science, serving in that capacity from 1894 to his retirement in 1911, all the while continuing as Head of Mathematics. He served as President of the Royal Society of Canada in 1897. On his retirement his students endowed three scholarships in his name, one for each of the Faculties of Applied Science, Arts and Science and Medicine. A remarkable man. His portrait hangs in Dupuis Hall, the home of chemical engineering. Some of his textbooks and a collection of mathematical models which he made over a hundred years ago are on permanent display on the second floor of Jeffery Hall.

The third head of mathematics was John Matheson (M.A. (Queen's)), considered by Dupuis to have been his outstanding pupil. Associated with Queen's for forty five years as student, instructor, professor and finally Dean of Arts, he served as head from 1911 to 1943. He had several years' experience as a high school teacher, maintained a close connection with the Ontario Educational Association and his advice was often sought by the Ontario Department of Education. An inspiring teacher and sympathetic adviser, he was the second recipient of the medal given by Montreal alumni to honour the Builders of Queen's. There were three other members of the department during the Matheson era. C. F. Gummer was appointed in 1911, the year of Dupuis' retirement, after taking a rare triple first at Oxford. He brought with him an appreciation of the spirit of modern mathematics and an insistence on mathematical rigour. Earning a doctorate at Chicago, the first Queen's mathematician with a Ph.D., he served as an associate editor of the American Mathematical Monthly and a member of Council of the Mathematical Association of America. He was also a talented and versatile musician. He taught at Queen's, mainly in the honours program in the Arts Faculty, until his death in 1946. K. P. Johnston graduated from Queen's with a B.A. in mathematics and then studied for three years in the Faculty of Applied Science for a B.Sc. degree in civil engineering. After a brief period of professional work he joined the mathematics department in 1916 teaching engineering students and acting as liaison between the department and the Applied Science Faculty until 1946. Norman Miller joined the department in 1919 after graduating from Queen's with an M.A., two years of high school teaching, a Ph.D. at Harvard and active service in France during World War I. Through his clear lectures, his encouragement of able students to take up mathematics teaching in the high schools, his participation in teachers' organizations and on Boards of Examiners and his authorship of school textbooks he made a great contribution to mathematics teaching in Ontario during the forty year span from 1919 to 1959.

D. S. Ellis graduated from Queen's first in mathematics (M.A. 1908) and then in civil engineering (B.Sc. 1910), lectured in mathematics and physics from 1911 to 1919 as well as serving with distinction in the army during World War I, before joining the civil engineering department eventually becoming head and Dean of Applied Science (1943-1955). Likewise, D. M. Jemmett took degrees in mathematics and electrical engineering at Queen's, taught in the mathematics department for a brief period before joining the electrical engineering department and serving as head for many years.

George L. Edgett graduated from Mount Allison before teaching high school for four years and studying at the University of Illinois for his Ph.D. He joined the department in 1930 remaining until his retirement in 1969. He appears to have been the first in Canada to offer a university course in statistics. Many prominent statisticians were introduced to the subject by Professor Edgett, including Colin Blyth, Agnes Herzberg and Harold Still, all of who became professors of statistics at Queen's. The Statistical Society of Canada honoured his contribution "to the emergence of statistics as both a science and an applied discipline" by electing him as its first honorary member. STATLAB, the statistical consulting service at Queen's, was

renamed the George Edgett Statistical Laboratory in 1979, and the department's name was changed to the Department of Mathematics and Statistics in 1978.

From 1842 to 1943 the department had had only three heads, surely a record for longevity, and one not likely to be broken in the future. The fourth head was Ralph Jeffery, a one-time Nova Scotia fisherman who graduated from Acadia University and then went on to take a master's degree and a doctorate at Cornell. He came to Queen's as head in 1943 serving until his retirement in 1960. During his tenure there was a marked increase in activity at the research and graduate level both in mathematics and statistics with Professors I. Halperin (1939-1966), H. W. Ellis (1947-1983), G. Edgett and Jeffery himself all involved.

Israel Halperin, having earned bachelor's and master's degree from the University of Toronto and a Ph.D. from Princeton, taught at Yale and Harvard before coming to Queen's in 1939. During the war he served in the Canadian Army conducting research in explosives and artillery; he was promoted to the rank of major before returning to Queen's. On 15 February 1946 he was arrested by the RCMP and taken to Ottawa where he was held incommunicado for several weeks. On April 25 he was charged under the Official Secrets Act with having communicated secret information to Soviet agents. He was acquitted for lack of evidence in March 1947 but it was not until May 1948 that he was fully reinstated at Queen's by the Board of Trustees. He worked at Queen's until 1966 when he returned to his alma mater in Toronto. In 1989 Queen's awarded him an honorary doctorate. In parallel with his distinguished academic career, Prof. Halperin has worked long and hard to enlist international support for the release of Soviet dissidents such as Orlov and Scharansky, for the improvement of prison conditions and for the advancement of human rights.

Hu Ellis graduated from Acadia in 1940 but remained for two years to study under Jeffery. From 1942 to 1945 he served in the Naval Research Establishment working on the protection of ships from magnetic mines both in Halifax and Vancouver. After the war he continued his graduate work at the University of Toronto earning his MA in 1946 and Ph.D. in 1947. He worked at Queen's from 1947 until his retirement in 1983. He was Chair for Undergraduate Studies from 1962 to 1978. Working with Halperin and Jeffery he made Queen's an important centre for the study of analysis during the 1950's and 60's.

Jeffery was the first Dean of the Graduate School at Queen's. One development which contributed much to the progress of mathematics in Canada and at Queen's was the Summer Research Institute of the Canadian Mathematical Congress. Jeffery, as Chairman of the Research committee of the Congress, with the support of the National Research Council, established these Research Institutes whereby mathematicians from across the country gathered for three months during the summer for research and study and mutual interaction. For many years they were held at Queen's.

Another highlight of Jeffery's years was the performance of the Queen's Putnam team in 1952. The Putnam competition is an annual mathematics contest with teams consisting of three undergraduates entered from the leading American and Canadian universities. Usually dominated by Harvard, M.I.T., CalTech, Stanford and other prestigious institutions, the first contest in 1939 was won by the team from the University of Toronto. One of the members of that team was John Coleman, later to succeed Jeffery as head at Queen's. In fact Toronto repeated as winners in 1940, 1942 and 1946. The winning Queen's team of 1952, consisted of Richard Cowper, Allan Reddoch and Hale Trotter and was selected and encouraged by Norman Miller. The first two were both Applied Science students, in engineering physics and engineering chemistry respectively! Trotter was a mathematics student who taught at Queen's from 1958 to 1970, and later became head of the mathematics department at Princeton.

During this time the liaison between Applied Science and the department, begun by K. P. Johnston, was carried on by Professor F. Morris Wood. Continuing in the tradition of Johnston, D. S. Ellis and Jemmett, Wood earned the M.A. degree from Queen's with honours in mathematics before entering Applied Science and graduating with an honours B.Sc. in civil engineering. After experience as an engineer in Alberta and the Dominion Engineering Works involving design of hydraulic machinery he taught mathematics at Queen's from 1920 to 1922 and then joined the faculty at McGill University. In 1946 Jeffery persuaded him to return to Queen's where he stayed until his retirement in 1960. His training as an engineer and his experience in practical work won the confidence of his engineering students. In 1992 as Queen's oldest living Applied Science graduate Morris Wood celebrated his 100th birthday with family and friends and colleagues from both this department and Civil Engineering.

The Mathematics Department has always enjoyed the support of the Engineering Departments and the Departments of Physics and Chemistry. This was due in no small measure to men like Johnston and Wood. D. S. Ellis and Jemmett all of whom were mathematics students at Queen's in the days of Dupuis and Matheson and later took degrees in engineering. The possibility of an honours program in mathematics with

a strong minor in engineering subjects was raised by Jeffery in 1952. Such a program would not produce engineers on first graduation, but many graduates would probably complete the work for an engineering degree and even those who would not would have an excellent background for fundamental engineering research and for the teaching of engineering mathematics. Nothing was to come of this idea until ten years later.

Jeffery's successor as head of the department was John Coleman who graduated from the University of Toronto in 1939, and, as mentioned earlier was a member of the team which won the first Putnam contest. He took an M.A. degree from Princeton followed by a Ph.D. in relativistic quantum mechanics at Toronto in 1943. He taught at Queen's from 1943 to 1945 and then served as secretary of the World's Student Christian Federation in Geneva for four years, returning to the University of Toronto where he remained until his appointment as head at Queen's in 1960. The twenty years of Coleman's headship was a period of spectacular growth for the department. When he arrived in 1960 the nine professors of mathematics had just settled into a newly renovated Carruthers Hall, formerly the Science Hall. John Deutsch, then Vice-Principal, predicted this would be the home of the department for all time. Though seldom wrong he had not reckoned with the great upsurge in mathematics and science which was to occur throughout North America following the Soviet launching of Sputnik in 1957 and the revolution in computing technology. The university system as a whole was to experience rapid growth during the 1960's but enrolments in mathematics courses grew even more rapidly. By 1967 the department had grown to a staff of 32 housed in Carruthers Hall, Summerhill, Watson Hall and Nickle House (now the Grey House). The department was reunited in a splendid new building, Jeffery Hall, in the spring of 1969 with a faculty complement of 41.

Year	1842	1860	1880	1900	1920	1940	1950	1960	1970	1980	1990
No. of Professors	1	1	1	2	4	6	6	9	41	48	47

1860	1	Williamson
1880	1	Dupuis
1900	2	Dupuis, Matheson
1920	4	Matheson, Gummer, Johnston, Miller
1940	6	Matheson, Gummer, Johnston, Miller, Edgett, Halperin
1950	6	Jeffery, Miller, Edgett, Halperin, Wood, Ellis
1960	9	Coleman, Edgett, Halperin, Ellis, Kirby, Kemp, Hogarth, Wasan, Obreanu

Table I. The Growth of the Department

Jeffery Hall was built on the last remaining central site on University Avenue facing Grant Hall and adjacent to Ellis Hall. The latter building, named for D. S. Ellis, is the home of Civil Engineering and the Applied Science Office and stands on the former site of Jeffery's house. Almost two thirds of Jeffery Hall is below ground, not because of the depth of mathematical thought nor any desire on the part of the university administration to keep mathematicians submerged, but in order not to dominate the University Avenue skyline. The centrepiece of the building is a fine library with sunken courtyards on the east and west. Most of the classrooms and all three lecture theatres are below ground; most of the staff offices are above.

In 1967 Coleman applied to the National Research Council for a Negotiated Development Grant. Such grants were provided at that time to selected universities and selected disciplines to help raise the level of research and graduate study, with the university agreeing to continue financing the department at an increased level at the expiry of the three year granting period. It may well be that the department's success in being almost the first to be awarded a development grant was due, in no small measure, to the fact that the President of N.R.C. was a Queen's engineering graduate from whom Coleman learned about the program before it was publicly announced! The department was awarded \$300,000 in 1968. These funds were wisely and conservatively expended over the next five years. Nowadays an average of \$60,000 per year does not seem much, but then it made a great difference. During 1969/70 ten research associates and four post-doctoral fellows were supported, with the university providing almost half the cost, much more than the minimum 25% required by the agreement with N.R.C. As a result research in algebra, analysis, control theory, statistics and the foundations of quantum mechanics was given a great stimulus. Many of the research associates later became regular members of the department. It was at this time that the Queen's Papers in Pure and Applied Mathematics were inaugurated. In 1972/73 the department had 43 positions, 48 graduate

students and a total budget of just over a million dollars. The number of subscriptions to research journals doubled and library acquisitions grew apace.

Coleman was Chairman of the Ontario Mathematics Commission for four years during the period when the "new math" was introduced to the province's schools. From 1973 to 1975 he was President of the Canadian Mathematical Congress and at the same time was appointed Director of the Study of the Mathematical Sciences in Canada, sponsored by the Science Council of Canada, the Canadian Mathematical Congress, the Canadian Operational Research Society, the Canadian Institute of Actuaries, the Canadian Information Processing Society and the Statistical Science Association of Canada. He argued cogently with the granting bodies that mathematics is a subject especially suitable for development in Canada because it does not require the massive equipment expenditures necessary in other scientific endeavours such as high-energy physics and some branches of engineering. He deplored the policy favoured by some that only applied mathematics was worthy of increased support; he also opposed the splitting of departments into pure and applied departments insisting that further interaction between pure and applied mathematicians must be fostered and encouraged. Following Jeffery and Matheson he continued to promote good relations between the department and the Applied Science Faculty. Evidence of the health of the department was provided by the report of the ACAP assessors in 1975 who concluded that we were unusually strong in both pure and applied mathematics and in statistics. The programs in control theory and statistics were held up as models for other Ontario universities.

As the department grew its administration became more complex. Not all issues could be settled during a coffee break. The important work of liaison with Applied Science and responsibility for service courses for the engineering students and the emerging program in Mathematics and Engineering was formalized with creation of the position of Chairman for Engineering Mathematics. The first occupant was J. E. Hogarth (1959-86). See Table II.

K. P. Johnston	1916 – 1946
F. M. Wood	1946 – 1960
J. E. Hogarth	1960 – 1970
B. J. Kirby	1970 – 1980
W. Woodside	1980 – 1986
R. D. Norman	1986 – 1989
J. H. Davis	1989 – 1991
R. Hirschorn	1991–

Table II. Professors Responsible for Engineering Mathematics

James Williamson	1842 – 1880
Nathan Dupuis	1880 – 1911
John Matheson	1911 – 1943
Ralph Jeffery	1943 – 1960
John Coleman	1960 – 1980
Lorne Campbell	1980 – 1990
Leo Jonker	1990–

Table III. Department Heads

Coleman stepped down as Head in 1980 and formally retired in 1983. However, he remains very active, continuing to teach regular courses until 1991 and still working in Lie algebras and Kac-Moody algebras. A conference on Modern Trends in Lie Algebra Representation Theory was held last month at Queen's to honour his 75th birthday. In October he will be one of ten recipients of honorary doctorates, one from each of the engineering disciplines at Queen's, at the Special Centennial Convocation of the Faculty of Applied Science.

Acknowledgements: The author is indebted to Norman Miller for several articles which appeared in the Queen's Review between 1944 and 1965 and to Ralph Jeffery and John Coleman.

Note: An article on the origins of the program in Mathematics and Engineering appeared in the summer 1987 issue of the Communicator.

LESS SOPHISTICATION - MORE PLAY

Peter Taylor

Here I focus attention on the math we teach to our “general” undergraduate arts and science students – not the math and physics graduates, and not the engineers – but the students in economics, biology, psychology etc, though some of what I say might apply more generally. I hold to the argument that we are failing to give these students very much that actually makes a positive impact on their future lives, so that much of what we do is wasted. In these hard financial times, if this is true, it is worth a close look.

My thesis is that we teach them too much material, at too high a level of sophistication. For both those reasons, they never manage to do any serious playing with the ideas and techniques, and their learning is sterile and of little importance to their lives. These difficulties are not new, but they are not easy to resolve, and I find I have to keep struggling to get hold of them.

One thing that keeps me at this struggle is the conviction that I am better placed than most of my colleagues to make progress because in my professional work, as a theoretical biologist, I come into daily contact with the best of what were once general math students – graduate students in biology and scientists writing for journals of population biology and evolutionary ecology. Here I find a wealth of simple mathematical modelling, typically using the basic ideas of calculus and linear algebra which are part of the background of all the individuals involved. But for the most part, the methods of the calculus are inaccessible to them. They can make the analogous discrete arguments, but the conceptual and computational power of the continuous model is unavailable to them. My conclusion is that the root of their alienation is a sophisticated and powerful notation that serves mathematicians well, but only enslaves those who are not able to make it their own.

A few do manage to work successfully with continuous models, but usually they have abandoned the sophisticated notation in favour of a “bare hands” approach (see the solution below) that sustains them through their analysis. All they need from me is some help with the write-up; their mathematical work has been a pleasure for them, and has invariably strengthened their analysis of the original problem.

Let me try to illustrate my thesis with an example from the differential calculus. I consider that circle of related ideas around the chain rule, implicit differentiation, related rates, and linear approximation – all close to the heart of the subject, but somehow shrouded in such dense undergrowth, that the forest is typically lost to view.

PROBLEM: Consider the following function F : for any t , $F(t)$ is defined as the largest root z of the equation:

$$z^3 - 2tz - 4 = 0.$$

At $t = 1$, you can verify that $z = 2$. Indeed, at $t = 1$, the equation factors as

$$(z - 2)(z^2 + 2z + 2) = 0$$

and $z = 2$ is the only real root.

Now small “input” variations around $t = 1$ will cause corresponding “output” variations in $z = F(t)$, which we also expect to be small. How are the z -variations related to the t -variations?

We all know how to solve this problem; indeed the standard notationally slick approach is one of the wonderful offerings of the calculus and we hold that it is our duty and our pleasure, as responsible teachers, to share it with our students.

The trouble is that, in a deep sense, most of these students never really “get” it, and they certainly can’t reconstruct it (and are even afraid to try) years later. In fact, a lot of them can’t even solve this problem at final exam time. [Go ahead, if you don’t believe me – try it out.]

But it’s a very important type of problem for the scientists I work with: the roots of the polynomial might be the eigenvalues of a linearized system of equations and it is required to know how they change in response to small variations in the parameter t . I would go so far as to suggest that if there is one thing that a graduate of introductory calculus should be able to do it is to solve that problem, not necessarily quickly or elegantly, but to fight their way analytically through to an answer. So I conclude that, in a very real sense, we are currently failing our students.

My proposal is that we refrain from teaching all of that beautiful notational manipulation, but instead adopt an elementary approach such as the following.

SOLUTION: Denote the input variation by δ , and the output variation by ε so that $t = 1 + \delta$ and $z = 2 + \varepsilon$, for δ and ε both close to 0. We are after the relation between δ and ε , and to discover this, we plug everything into the defining equation:

$$(2 + \varepsilon)^3 - 2(1 + \delta)(2 + \varepsilon) - 4 = 0.$$

Ignoring 2nd and higher powers of ε and δ , this becomes:

$$8 + 12\varepsilon - 2(2 + \delta + \varepsilon) - 4 \simeq 0$$

which gives

$$\varepsilon \simeq (0.4)\delta$$

and we discover that small input variations δ result in output variations ε that are approximately proportional to δ and 0.4 times as large. This proportionality idea is important (in fact central) but is also largely lost on most students.

The main advantage of this elementary approach, is that it offers the students an opportunity to play with the ideas and constructs of the course, and thereby to discover new results and make old results their own. That kind of play is of course a crucial part of the learning process, but most students are able to do very little of it in the traditional approach to the calculus.

Exactly what am I suggesting here? Well one of the problems of the much heralded calculus revolution is that, for all the wonderful new types of problems that have come along, it is still overburdened with material. In fact, at a calculus reform workshop in San Antonio a while ago, each working group was asked to come up with a list of topics in their assigned area that could be omitted. The exercise was a revealing failure; there was almost no agreement on what could be cut. Well, my current idea is, in a sense, to throw everything out. And then take one or two ideas, like the above $\delta - \varepsilon$ idea, and work a few examples which demonstrate the essential local linearity of most functions, and then solve a bunch of interesting and useful problems, and then you don't have to do limits or Newton quotients or chain rule or implicit differentiation or related rates, because you've given the student the idea that's at the heart of all this, and an idea that can actually be carried into the world. And as far as the rest of the stuff is concerned, well the good students will discover what they need to know when they need to know it; they always do.

CHANGES IN THE DEPARTMENT

Leo Jonker

Once again the year has been eventful. Drs. Bruce Kirby and Robin Giles both retired, having reached normal retirement age. We will miss them. Many of our readers will remember them as fine lecturers who could make difficult concepts look simple. Bruce Kirby served as chair of the Mathematics and Engineering program from 1970-1980. Unfortunately, Dr. Paulo Ribenboim was also forced into retirement because of his deteriorating eyesight. We will miss his lively seminars. He continues to work on his books and papers in number theory. As it turns out, Paulo and his favorite problem are retiring together, for just a few weeks ago, after more than 300 years, a proof was announced for Fermat's Last Theorem. Dr. Don Watts is also taking early retirement. From now on he will devote his time to statistical consulting and, no doubt, to cycling and hiking.

In last year's issue we wrote of the expected arrival of Dr. Duncan Murdoch, a statistician, and Dr. Oleg Bogoyavlenskij, an applied mathematician. Both have now settled in at Jeffery Hall and are teaching courses and supervising students. This fall we will be joined by the young statistician Glen Takahara, a fresh Ph.D. graduate from Carnegie Mellon University.

As we become increasingly aware of the important role of mathematics and statistics in other science departments, we have begun to recognize this by means of cross-appointments. Three statisticians in the Queen's University Clinical Trials Group, Yuk-Miu Lam, James Myles and Benny Zee, have joined the Department in this fashion.

David Bacon, former Dean of the Faculty of Applied Science and a member of the Department of Chemical Engineering, accepted a cross appointment some years ago.

Just a few weeks ago we were informed by Marc Maes that he is leaving Queen's and returning to the Civil Engineering Department at the University of Calgary. We wish him well.

THE FIELDS INSTITUTE

Leo Jonker

In last year's issue of the Mathematical Communicator we wrote of the ongoing effort to find a location at Queen's University for the Fields Institute for Research in the Mathematical Sciences. We were gratified by the enthusiastic support generated for the plan within the Queen's University administration and within the Queen's research community. The proposal that came out of it eventually stands as a testimony to the energy and the vision of this University and the centrality of mathematics and statistics in all scientific endeavour. Clearly, the Fields Institute gave very serious consideration to locating at Queen's. In the end, however, the Institute opted to locate more centrally, at the University of Toronto. We congratulate the University of Toronto on its selection, and we wish the Fields Institute well in its striving to promote the Mathematical Sciences throughout the country.

DR. ERNEST C. GILL

Leo Jonker

In January, 1992, we mourned the death of Ernest Gill, one of our most distinguished graduates. Ernest graduated with a BA in 1923. He was class president that year and earned the gold medal in mathematics. In 1957, he was awarded an honorary doctorate in recognition of his many contributions to Queen's and to the community generally.

Dr. Gill spent his career with the Canada Life Assurance Company, becoming president and vice-chair of the board in 1951. He served on many committees and boards, including the Queen's University Board of Trustees. He was its chair from 1958 to 1962.

Dr. Gill never forgot his student days at Queen's University. In his will he left the Department of Mathematics and Statistics \$25,000 — to found the "Ernest C. Gill Memorial Fund". We are grateful for his generosity. As a first project to be funded by this fund, we plan "Ernest C. Gill Mathematics and Statistic undergraduate Assistantships". These assistantships will be awarded to deserving upper year students who will then be given the task of providing tutorial assistance to students in their first and second year.

We hope that these assistantships will bring suitable honour to the name of one of our distinguished graduates even as they serve to expose others to the beauty and power of mathematics.

RALFE J. CLENCH JR

Ralfe Clench died at his home in Kingston on August 4th in his 58th year. He had taken early retirement in 1983 after serving Queen's for many years as a mathematics instructor and chief organizer of the examination system. An unusual character, he will be fondly remembered by a host of Queen's alumni who conquered the complexities of introductory calculus with the help of his unorthodox but effective teaching methods.

NEWS OF GRADUATES

Mark Green (Math. & Eng. 1987) has been appointed Assistant Professor of Civil Engineering at Queen's having completed his Ph.D. in structural dynamics at Cambridge University (1991) and worked at Queen's as a post-doctoral fellow since then.

Doug Milligan (Math. & Eng. 1983) works for SEL (Canada) in Toronto on the development of software for the control of trains. Doug and his wife Doreen have been in London for the past year in connection with the Docklands Light Railway link to the Isle of Dogs, the site of the Canary Wharf development.

Stephen Norman (Math. & Eng. 1985), son of Prof. Dan Norman, has completed his Ph.D. at Stanford University and is now Assistant Professor of Electrical Engineering at the University of Calgary.

Karen Rudie (Math. & Eng. 1985) has completed her Ph.D. in Electrical Engineering at the University of Toronto and will begin an appointment as Assistant Professor in the Electrical Engineering Department at Queen's this summer.

Greg Wilson (Math. & Eng. 1984). Contrary to popular rumours Greg did not enter a monastic order after his departure from Queen's. For the past five years he has been at the University of Edinburgh in Scotland

in the Parallel Computing Centre, supervising projects and some Master's students, and working part-time on a Ph.D. program. He has also been writing articles on popular science and computing. He looks forward to returning to Canada soon.

OLD PROBLEMS

Solution To Taylor's Jogging Problem (by the editor)

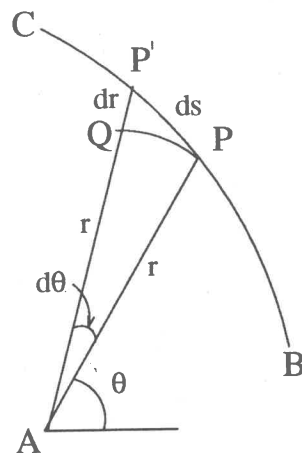
Problem: I am in the forest at the point A . There is a road through the forest from B to C and I want to get to C . My jogging speed through the forest is w and along the road is v , where $v > w$. My strategy is to jog in a straight line to some point P on the road and then follow the road to C . Amazingly it doesn't matter at what point P I meet the road; my total time to C is the same. What shape is the road?

Solution: Choose polar co-ordinates with the origin at A . Let $P(r, \theta)$ and $P'(r + dr, \theta + d\theta)$ be neighbouring points on the road. The time required to traverse the path $APP'C$ is the same as that for $AP'C$. Removing $P'C$ which is common to both paths, and replacing AP' by $AQ + QP'$, where $AQ = AP$, we have

$$\frac{r}{w} + \frac{ds}{v} = \frac{r + dr}{w}, \text{ or } \frac{ds}{v} = \frac{dr}{w}.$$

But $ds^2 = (r d\theta)^2 + dr^2$. Therefore $r^2 d\theta^2 + dr^2 = \frac{v^2}{w^2} dr^2$, and so $(\frac{dr}{d\theta})^2 = \frac{r^2}{k^2}$, where $k = \sqrt{\frac{v^2}{w^2} - 1}$. Taking the square root and remembering that we want r to increase as θ increases, we obtain $\frac{dr}{d\theta} = \frac{r}{k}$. Solving this simple separable differential equation we find

$$r = R \exp\left(\frac{\theta - \Theta}{k}\right)$$



to be the polar equation of the road, where (R, Θ) are the polar coordinates of C .

The curve BC is characterized by the fact that the angle between the tangent at P' and the radial line AP' is constant since $\frac{ds}{dr} = \frac{v}{w}$.

Solution to the Dartboard Problem II, received from John Holbrook, Department of Mathematics and Statistics, University of Guelph.

Problem: Find the probability P_n that n darts thrown at random at the surface of a sphere leave at least half (any half) of the surface dart-free.

Solution: Let the unit vector u_k denote the position of the k -th dart on the surface of the unit sphere S^2 in \mathbb{R}^3 . Let s denote a sign-vector of $n \pm 1$'s and define the random variable X_s to be 1 if the set R_s is nonempty, where $R_s = \{u \in S^2 : u \cdot s_k u_k > 0\}$. Let $X_s = 0$ otherwise. Now the required probability P_n is just the expectation $E(X_{\vec{1}})$ where $\vec{1}$ is the sign-vector with $n + 1$'s. This is because $u \in R_{\vec{1}}$ iff all the darts are in the half-sphere centered at u . But by symmetry $E(X_s)$ is the same for all 2^n choices of s , so that

$$P_n = \frac{1}{2^n} \sum_s E(X_s) = \frac{E(\sum_s X_s)}{2^n}.$$

But $\sum_s X_s$ is the number Q_n of regions into which S^2 is divided by the n great circles $\{u \in S^2 : u \cdot u_k = 0\}$. Generically, ie ignoring special coincidences among the great circles, the value Q_n is the same for any set of n great circles, namely $Q_n = n^2 - n + 2$. This is clear by induction, since $Q_1 = 2$ and $Q_{n+1} = Q_n + 2n$; to see the last relation, think of adding one more great circle to a system of n : it will intersect these n circles at $2n$ points and will therefore create $2n$ new regions. Finally, then,

$$P_n = \frac{E(\sum_s X_s)}{2^n} = \frac{Q_n}{2^n} = \frac{n^2 - n + 2}{2^n}.$$

This method may be adapted to solve various related problems. In particular it solves the Dartboard Problem I as

$$p_n = \frac{q_n}{2^n},$$

where q_n is the number of arcs into which the unit circle is divided (generically) by n lines through its centre. Evidently $q_n = 2n$, so we have an alternative proof that $p_n = \frac{n}{2^{n-1}}$.

I don't know whether this (relatively painless) approach to such problems is new, but it turns out that the formulas are known as standard results in geometric probability. See, for example, formula (18.45) in Santaló, "Integral Geometry and Geometric Probability", Addison-Wesley 1976.

(**Editor's Note:** This is an elegant solution. It appears to me to be the same as Wendel's proof (1962) of the equivalent problem in \mathbb{R}^m , cited in Santaló (reference 716), and the earlier proof due to Schläfli (1950).)

NEW PROBLEMS

Taylor's Divisibility Problem: The number 374,625 has a remarkable property. Not only is it divisible by 37, but if I permute the first three digits in any way, and permute the last three digits in the same way, the resulting number is also divisible by 37. For example, 734,265 is divisible by 37. (a) Find all 6-digit numbers with this property. (b) What is the corresponding problem for 8-digit numbers?

ERRATUM In the last issue Dr. Wojciech Jaworski, the recipient of the Governor General's Gold Medal as the best graduating masters or doctoral student in all disciplines, was mistakenly listed as currently being a post-doctoral fellow at Carleton University. In fact he is at the University of Ottawa.

MATH RIOTS PROVE FUN INCALCULABLE

Eric Zorn

(An excerpt from the Chicago Tribune, June 29, 1993)

News Item (June 23)—Mathematicians worldwide were excited and pleased today by the announcement that Princeton University Professor Andrew Wiles had finally proved Fermat's Last Theorem, a 365-year-old problem said to be the most famous in the field.

Yes, admittedly, there was rioting and vandalism last week during the celebration. A few bookstores had windows smashed and shelves stripped, and vacant lots glowed with burning piles of old dissertations. But overall we can feel relief that it was nothing — nothing — compared to the outbreak of exuberant thuggery that occurred in 1984 after Louis DeBranges finally proved the Bieberbach Conjecture.

"Math hooligans are the worst", said a Chicago Police Department spokesman. "But the city learned from the Bieberbach riots. We were ready for them this time".

When the word hit Wednesday that Fermat's Last Theorem had fallen, a massive show of force from law enforcement at universities all around the country headed off a repeat of the festive looting sprees that have become the traditional accompaniment to triumphant breakthroughs in higher mathematics. Mounted police throughout Hyde Park kept crowds of delirious wizards at the University of Chicago from tipping over cars on the midway as they first did in 1976 when Wolfgang Haken and Kenneth Appel cracked the long-vexing Four-Color Problem. Incidents of textbook-throwing and citizens being pulled from their cars and humiliated with difficult story problems last week were described by the University's Math Department Chairman Bob Zimmer as "isolated".

Zimmer said, "Most of the celebrations were orderly and peaceful. But there will always be a few — usually graduate students — who use any excuse to cause trouble and steal. These are not true fans of Andrew Wiles". Wiles himself pleaded for calm even as he offered up the proof that there is no solution in integers to the equation $x^n + y^n = z^n$ when n is a whole number greater than two, as Pierre de Fermat first proposed in the 17th Century. "Party hard but party safe", he said, echoing the phrase he had repeated often in interviews with scholarly journals as he came closer and closer to completing his proof.

IF UNDELIVERED RETURN TO:
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