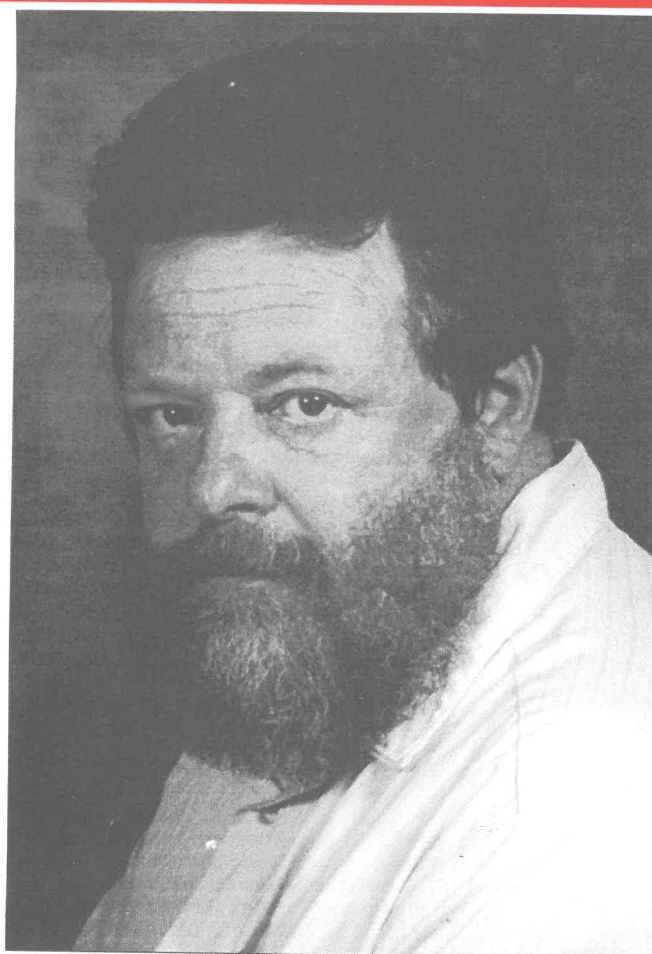


QUEEN'S MATHEMATICAL COMMUNICATOR



SUMMER 1999



NORMAN J. PULLMAN
1931-1999

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A JUNGLE OF GEOMETRY Adam Brown	1
JOBS NETWORK	7
MATHEMATICS AND ENGINEERING SEMINAR Dan Norman	8
HEAD'S REPORT Eddy Campbell	8
NORMAN J. PULLMAN 1931-1999	10
POOR LITTLE i George C. Bush	12
NEW PROBLEM Peter Taylor	12
PROBLEM FROM LAST ISSUE Peter Taylor	13
SOLUTION	13

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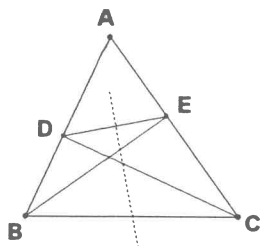
A Jungle of Geometry

Adam Brown

Adam Brown recently completed his Master's degree from Queen's with a project entitled "An Introduction to Geometric Transformations", and gave a Coleman-Ellis lecture entitled "Don't Underestimate Euclid" in February 1999.

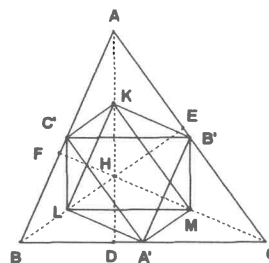
What can be meant by a 'jungle of geometry'? Well, remember that fascinating subject we met for the first time in high school with its congruent and similar triangles, surprising theorems and elegant proofs? Surely Euclidean geometry springs readily to mind. In this brief article, I intend to demonstrate some fancy problems, proofs and ideas. In short, the reader will be lead through a collection of geometric gems and jewels. They vary in size and colour, for the sake of variety, but all have a similar flavour. There are several excellent books available which contain interesting geometry problems and I will mention them at the end of this article. Now for a little warm-up problem. You may not use pencil and paper to solve the problem- just keep staring until the inevitable 'aha!' occurs.

Exercise 1: Let ABC be any triangle and drop perpendiculars from B and C meeting the sides at D and E as shown. Join DE . Now draw the right bisector of DE . Prove that it also passes through the midpoint of the base BC .



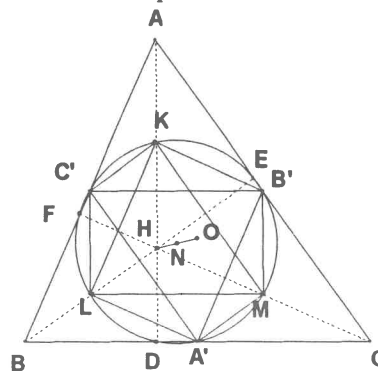
At the end of this article you can find the solution for this problem, but don't look until you know the meaning of suffering.

Our first topic of discussion concerns a fairly recent development in Euclidean geometry – the nine point circle. Euclidean geometry experienced a remarkable rebirth in the 19th and 20th century. A plethora of new theorems about old shapes were furnished; new points, lines and circles associated with the triangle, new features of quadrilaterals, new families of circles. One of the earlier discoveries was that of the nine point circle, and we will now devote our attention to it. Let us inspect the following figure carefully:



Here we have a triangle ABC , with altitudes AD , BE and CF meeting at H , the orthocentre of the triangle. We also have the midpoints of the sides denoted by A' , B' and C' and the midpoints of AH , BH and CH denoted by K , L and M respectively. Notice that $C'B'$ joins the midpoints of AB and AC in triangle ABC so it is parallel and equal to half of BC . For the same reason, in triangle HBC , LM is parallel and equal to half of BC . Also, as B' and M are midpoints in triangle AHC , we see that $B'M$ is parallel to AD ; similarly $C'L$ is parallel to AD . Since AD and BC are perpendicular, we have established that the opposite sides of quadrilateral $C'B'ML$ are parallel and adjacent sides are perpendicular. So it is a rectangle! Now rectangles are concyclic and their diagonals meet at the centre of the circumscribing circle. Thus $A'K$, $B'L$ and $C'M$ meet at a point, called N , which is the centre of the circle about all three rectangles. In fact we can do even better. As $\angle LEB' = 90^\circ$, the circle with $B'L$ as diameter also passes through E . In exactly the same manner, D and F are snared by this circle. Hence we have a circle passing through all 9 of these points. The 9 point circle is here with a vengeance. A few more comments before we move on. Triangle $A'B'C'$ is called the medial triangle of triangle ABC . Notice that the altitudes of this triangle are also the right bisectors of the original triangle ABC . Thus, the orthocentre of the medial triangle is the circumcentre of the original triangle. This point is always denoted by O . See the diagram below.

Now we have drawn the nine point circle and marked O and N . With one tiny transformation, simply called a half turn, we can establish a remarkable fact. Notice that a half turn about N , the centre of the nine point circle, will map triangle KLM into triangle $A'B'C'$. Thus it will map the orthocentre of triangle KLM into the orthocentre of triangle $A'B'C'$; that is, it will map H into O . Thus N must



divide HO in half, and we have the nice theorem that the nine point centre is the midpoint of the orthocentre and the circumcentre of any triangle. While this little excursion was surely entertaining, let's mention a few afterthoughts. That the nine points all lie on a circle was a matter of nothing more than elementary geometry. It was applied with some deftness, but it is refreshing to reap so much with so little. And what of the half turn that was thrown in near the end? It also provides a very short proof to a nice theorem. This suggests the power of using geometric transformations to aid our reasoning when approached by theorems of geometry. Let's spend a bit more time investigating another transformation – the *rotation*.

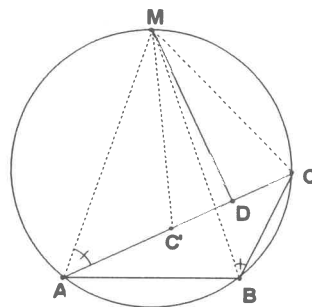
Let O be a fixed point of the plane, and α some angle, which we agree is directed counter clockwise. Then if we rotate A about O by an angle of α , we obtain the point A' so that $OA = OA'$ and $\angle AOA' = \alpha$. Notice that a line l will be taken to a new line l' so that the angle between the lines is exactly α . We can see this as follows. To find the image of l we drop a perpendicular from O to l at P . We rotate to find P' and then l' will be the line through P' and perpendicular to OP' . Suppose the lines meet at X . The pair of right angles at P and P' make quadrilateral $OPXP'$ cyclic, so $\angle POP' = \alpha =$ exterior angle at X , which is the angle between l and l' . It is now clear that if F and F' are figures related by a rotation about a point then if AB and $A'B'$ are any two corresponding line segments, then triangles OAB and $OA'B'$ are congruent. Also the angle between AB and $A'B'$ is α . We also have a converse theorem at work here; so if F and F' are two figures so that corresponding segments are equal and make an angle of α with each other, then F and F' are related by a rotation about some centre. To see this let segment AB in F be carried onto the corresponding segment $A'B'$ in F' after a rotation about O by α . Let X be an arbitrary point of F . Now by assumption X' will be a point of F' so that $X'A' = XA$ and $X'B' = XB$. Thus all the corresponding sides of triangles ABX and $A'B'X'$ are equal so they are congruent. This means that the rotation has also carried X into X' . Thus F is carried into F' .

Let us consider the following theorem by way of introduction to the application of these powerful ideas.

Theorem: (Archimedes' theorem of the broken chord). Let A and B be two points on the circumference of a circle and M be the midpoint of arc AB . Let C be another point on the arc AMB and D be the foot of the perpendicular from M to AC . Then $AD = DC + CB$.

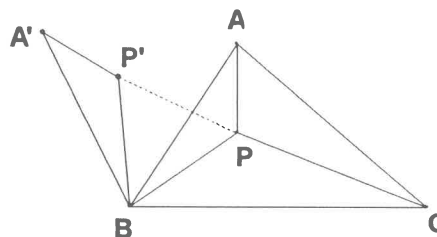
In other words, D is the midpoint of the polygonal path from A to C to B .

The dotted lines show construction lines. Notice the marked angles are equal since they are subtended



by the same arc. Now, rotate triangle MBC about M until B falls on A . This can be done since M is equidistant from A and B . It looks as though C is taken to C' which is a point on AC ; yet this must be the case by virtue of the equal marked angles. Now, since MD is perpendicular to AC and $MC = MC'$ by rotation, we see that triangles MCD and $MC'D$ are congruent. Thus $C'D = DC$. And now, $AD = AC' + C'D$. Yet $AC' = BC$ by rotation and $C'D = DC$. The result follows in a hail of arrows! Our next application is positively magnificent. Many years ago, Fermat posed the following

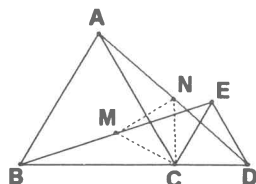
Problem: Find the point P inside acute triangle ABC which minimizes the sum $AP + BP + CP$.



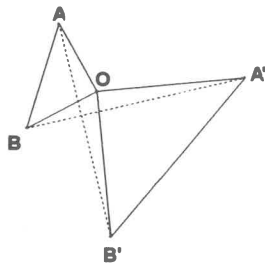
We begin by rotating triangle APB 60 degrees counter clockwise about B . Let this carry P to P' and A to A' as shown in the figure; clearly $AP = A'P'$ and $BP = BP'$. Also as $\angle P'BP$ is 60 degrees, this makes triangle PBP' equilateral so that $PP' = BP$. Thus we are now able to interpret the sum $AP + BP + CP$ as the length of the polygonal path from C to A' (notice that A' is fixed for all positions of P). This path has smallest length when it is a straight line, that is when C, P, P' and A' are collinear. Since triangle PBP' is equilateral, this will be the case when $\angle BPC = 120$ and when $\angle A'P'B$ (which is $\angle APB$) is also 120 degrees. Naturally, this means that $\angle APC$ is also 120. Thus the point that minimizes the desired sum is the point which

creates angles of 120 degrees with every pair of vertices. This point is called the Fermat point of the triangle.

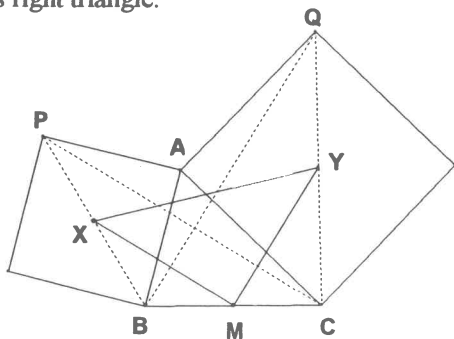
Exercise 2: Before continuing with the next theorem, here's something to whet the appetite. Let triangle ABC be an equilateral triangle, joined to equilateral triangle CDE as shown. Let M and N be the midpoints of BE and AD respectively. Prove that triangle MNC is also equilateral!



We will now indulge in some further rotation investigations, culminating in the remarkable theorem of Aubel. Suppose that two isosceles right triangles AOB and A'OB' are pinned together at the right angle O.



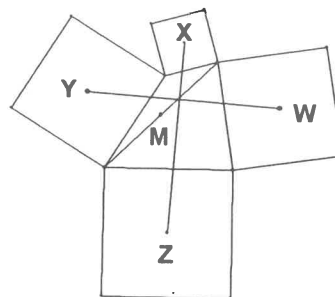
Then a rotation of 90 degrees about O will carry the segment AB' into BA'. Thus AB' is equal and perpendicular to BA'. This little lemma can establish the following result for us quite easily: let squares be erected externally on sides AB and AC of triangle ABC. Let M be the midpoint of BC. Then the centres of the squares along with M are the vertices of an isosceles right triangle.



Behold this fancy figure. We wish to show that XM is equal and perpendicular to YM. However, PC is twice as long as XM and parallel to it since XM joins the midpoints of two sides of a triangle. Also BQ is

parallel to and twice as long as YM. So it suffices to show that PC and QB are equal and perpendicular. But a rotation of 90 degrees about A carries triangle PAC into triangle BAQ. The result now follows. We now use this theorem to establish

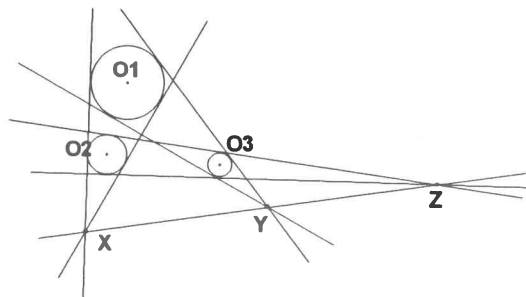
Aubel's theorem: Let squares be drawn externally on the four sides of a quadrilateral. Then the segments joining the centres of opposite squares are equal and perpendicular!



Consider the diagram shown above. Let the centres of the squares be X, Y, Z and W and let M be the midpoint of a diagonal. To avoid over-cluttering the diagram the lines about to be mentioned have not been drawn, but the visualization powers called for are not uncommon. Imagine drawing XM, YM and XY. From the previous theorem, we see that triangle XYM is an isosceles right triangle; the same is true for triangle ZWM. Now we have two isosceles right triangles pinned together at their common right angle. And by the innocent little lemma we developed at the outset, XZ is equal and perpendicular to YW. Clean. Real clean.

It's time now to wander somewhere else, encouraged by our triumphant success so far. Here's a fancy

Problem: Show that the three points where the common external tangents of three circles meet are collinear!



Now would be a nice time to indulge in another flight of fancy. Imagine the three circles as spheres whose centres lie in a plane. Clearly X, Y and Z lie in the plane of the centres of circles C_1 , C_2 and C_3 . Now, high above the spheres, drop a piece of paper and watch it float down until it just rests upon the spheres. This plane will lie on the cone generated by C_1 and C_2 , and so will pass through the vertex of the cone or X. Thus X lies on the piece of paper. Similarly Y and Z also lie on the piece of paper. We now have that these three points lie on two different planes which meet on aline!

A few more thoughts. Given the three circles, let X and Z be the centres of similitude of the circles as shown. Draw the line, say l , on XZ. The distances from C_1 and C_2 to l are in the ratio r_1 to r_2 ; and a

magnification of $\frac{r_2}{r_1}$ sends C_1 to C_2 . Then,

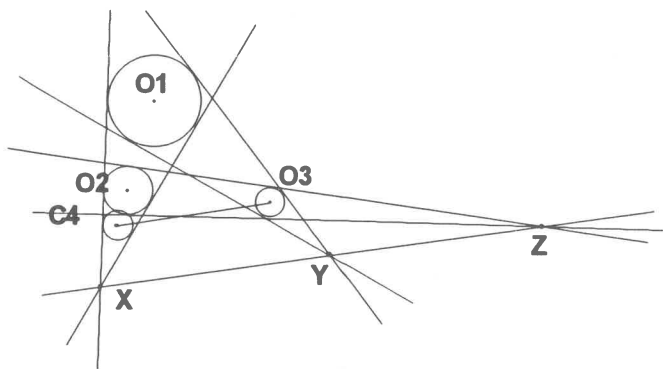
magnification about Z by a factor of $\frac{r_3}{r_2}$ will send C_2

to C_3 . Also the distances from C_2 and C_3 to l are in this same ratio. In all C_1 has been sent to C_3 by a factor of

the product, or $\frac{r_3}{r_1}$. Thus the ratio of the radii and

distances from C_1 and C_3 to l are the same: $\frac{r_3}{r_1}$. This

means that Y lies on l . Here's another neat idea. We draw an auxiliary circle as shown below tangent to the common tangents of C_1 and C_2 , yet having



the same size as C_3 . We call this newcomer C_4 . Imagine X, Y and Z as distinct points that we soon will show are collinear. Notice that

$$\frac{O_1X}{O_4X} = \frac{r_1}{r_4} = \frac{r_1}{r_3} = \frac{O_1Y}{O_3Y}, \text{ and so } O_4O_3 \text{ is parallel to } XY.$$

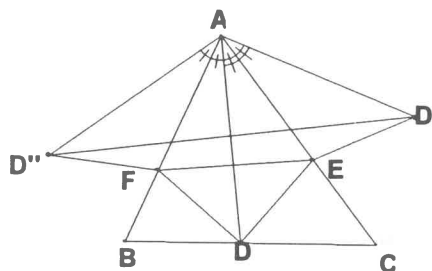
Now applying the very same argument with C_1

replaced by C_2 , we see that O_4O_3 is parallel to XZ. This establishes the collinearity of X, Y and Z with great charm. In all, three proofs of this engaging theorem!

We have already examined the problem of Fermat - to find a point inside an acute triangle which minimizes the sum of the distances to the vertices - and solved it using a rotation. Here's another interesting problem of a similar flavour. This time, we approach the problem with another bewildering transformation in mind, the *reflection*.

Problem: Let ABC be an acute triangle. Find the inscribed triangle of smallest perimeter.

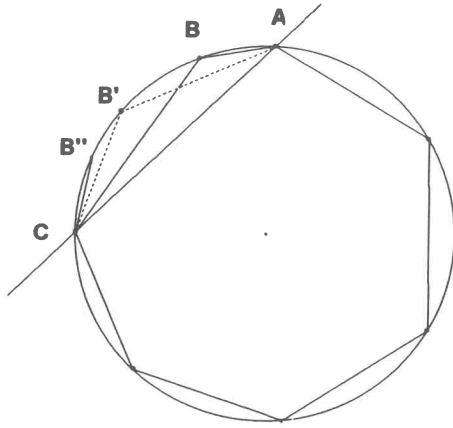
First select any point D on BC; we will find the inscribed triangle of smallest perimeter using D as one



of its vertices, and E and F arbitrary points on their respective sides. Let D' be the reflection of D in AC, and D'' the reflection of D in AB. This makes $DE = D'E$ and $DF = D''F$. Now for all choices of E and F, the perimeter of DEF has been expressed as the polygonal path from D'' to D' . This path is least when it is straight, or when $D''F + FE + ED' = DD'$ and $F = AB \cap DD'$. Now grant D the freedom to roam on BC. Notice that $AD'' = AD = AD'$, and that $\angle D''AD'$ is twice $\angle BAC$, by reflection. So triangle $AD''D'$ is always isosceles with a fixed vertex angle. To minimize its base then, we equivalently wish to minimize the length of the equal arms. This is accomplished by making AD as small as possible, so by choosing D to be the foot of the perpendicular from A. As nothing was special about choosing the base to be BC, the inscribed triangle of smallest perimeter will have E and F as altitude feet as well.

Now, a prelude for the famous isoperimetric theorem. Notice the cunning use of reflection!

Problem: Show that of all polygons inscribed in a circle, the regular one has largest area.



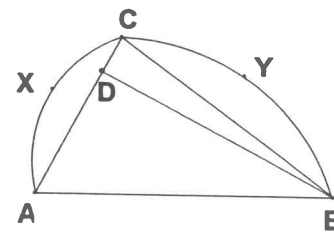
Here is an n -gon inscribed in a circle. First note that the sides of the polygon can be arranged in any order, yielding another n -gon with the same perimeter and area. This can be seen by joining all vertices to the centre of the circle, and then arranging pieces of the pie in any order. If the n -gon isn't regular, there must be at least one arc smaller than an n th of the circumference and also one larger. Cut them up and place them together; say the shorter one is AB and the longer BC . Locate the point B' so that arc AB' is exactly an n th of the circumference. Now reflect triangle ABC in the right bisector of AC producing B'' , the image of B . Then B'' actually lies beyond B' on the directed arc AC . This is the case since arc AB'' equals arc CB , by reflection, which was chosen to be larger than an n th of the circumference. That is, B' lies inside the arc BB'' . Since B and B'' are images after reflection in the right bisector of AC , B' must be further from AC than each of B and B'' (which are equidistant from AC). This in turn means that triangle $AB'C$ has larger area than triangle ABC , and so the new polygon with B' in place of B and all other vertices unchanged has larger area as well an arc AB' an n th of the circumference. Repeating this argument at most $n - 1$ times, a regular n -gon is reached having a larger area than the original n -gon.

As promised,

The Isoperimetric Theorem: Of all closed curves with constant perimeter, the circle has the largest area. We will examine one of Steiner's proofs of this theorem, and show that if there is a figure which has an area greater than all other figures, it must be the circle. We have already supposed here that such an extremal figure exists, as did Steiner. Later, the ominous Karl Weierstrass objected to Steiner's proofs because of their assumption of the existence of a maximal figure. He later supplied a non-constructive proof that the assumption can be made rigorous, but as yet no one has found an intuitive geometric way to see this. So, on the faith of geometric intuition, suppose that there is

a figure having largest possible area with a fixed perimeter. In the footsteps of Steiner, then, let's first show (1) that the closed curve of largest area, C , must be convex. For if C is not convex we can find a pair of points, say (A, B) on the curve so that the whole curve lies on one side of the line AB . Reflecting all points of the curve in the line AB will yield another curve with the same perimeter of C , yet a larger area. Thus C must have been convex.

Another property (2) of C must be true; namely that any line which splits the perimeter of C in half must also split its area in half. To see this, choose any line l passing through points A and B of C which bisect its perimeter. Since C is convex, each half of the boundary of the curve lies completely on one side of l . If either of these halves has larger area, then using the reflection of this larger side in l along with the original larger half, we again have a curve with the same perimeter as C but a larger area. The maximality of C ensures that this cannot happen. From this we can infer something a bit stronger: any 'arc' AB of C with half the perimeter of C , must provide us with a figure which encloses the largest possible area of a curve half the length of C and line AB . Otherwise we can increase the area of C by reflection in AB . With this in mind, let AB be any such perimeter-splitting line and let C be any point on this 'arc'.

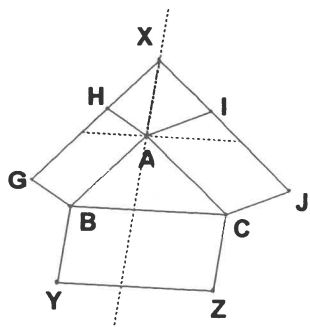


We have established that this figure must have its largest possible area. By convexity, CA and CB lie within the figure, forming lunes AXC and BYC . The angle at C in this figure could potentially be acute, right or obtuse. Let D be the foot of the perpendicular from B to AC . The area of the figure arises from the lunes, which are immutable as the angle at C varies, and triangle CAB . Thus the total area is only dependent on the area of triangle CAB . If angle C is acute, as shown above, then the area of triangle CAB , half the product of CA and BD ; can be increased by increasing the height BD . As the lunes are swiveled about C , the furthest B can be from AC occurs when angle ACB is right. In that case the altitude BD will coincide with side BC . A similar situation occurs when the angle at C is obtuse; and consequently the area of the shape can be increased lest the angle at C is right. So the shape of largest area has the property that

every C on arc AB makes angle ACB right. The shape is a semicircle with diameter AB! The full circle springs forth when the other half of the figure is retrieved by reflection in AB.

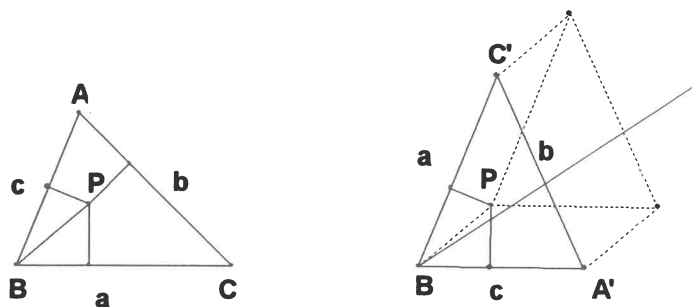
Our final coup de grace emerges from the Erdos-Mordell Inequality: If P is any point in triangle ABC and if p_a , p_b and p_c are the distances from P to the sides of triangle ABC, then $PA + PB + PC \geq 2(p_a + p_b + p_c)$. This problem was posed in 1935, and solved by Mordell in 1937, although the proof was not elementary. Later in 1945 an elementary proof was discovered by N.D. Kazarinoff, based on the idea of reflections.

First we need to derive Pappus' generalization of the Pythagorean theorem. Let ABC be a triangle, and place parallelograms on the sides AB and AC. Let the sides produced meet at X as shown. Now append vectors each equal to AX to vertices B and C forming parallelogram BCYZ. Then the sum of the areas of the first two parallelograms equals the area of the third. Extend BY and CZ to product points M and N as shown. Then XACN and XABM are parallelograms, since opposite sides are parallel. Now we see that $(IACJ) = (XACN)$ and (where the brackets denote area)



$(HABG) = (XABM)$, since these pairs are obtained from each other by shearing. Since XA is parallel and equal to MB and NC, we simply pull down the segment XA until A rests on side BC. Then parallelograms XABM and XACN merge into MNCB. This latter parallelogram is congruent to BCZY, and so the result follows. Crafty. We can now assail the Erdos-Mordell inequality.

Our first step is something of a shocker – reflect the triangle in the angle bisector of $\angle ABC$. We leave the point P fixed, but interchange A and C accordingly. This transformation does not change p_a or p_c (the distances from P to BC and AB).



This figure shows the before and after effects of our reflection. Also BC' has length a , and $BA' = c$. We now construct parallelograms on sides BC' and BA' sharing the common edge BP . Notice the length of AC , which has mapped to $A'C'$, remains that of b . Now apply Pappus' theorem to the trio of parallelograms and $ap_c + cp_a = \text{area of parallelogram on } A'C' \leq b \cdot BP$, since the height of this parallelogram cannot exceed BP . Similarly, $bp_c + cp_b \leq a \cdot AP$ and $ap_b + bp_a \leq c \cdot CP$. Dividing and adding.

$$AP + BP + CP \geq \left(\frac{a}{b} + \frac{b}{a}\right) \cdot p_c + \left(\frac{c}{b} + \frac{b}{c}\right) \cdot p_a + \left(\frac{c}{a} + \frac{a}{c}\right) \cdot p_b.$$

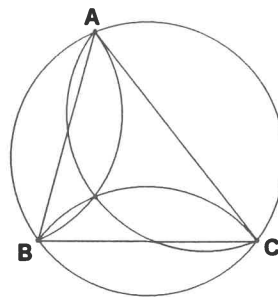
Noting that each bracketed expression is at least 2, being the sum of reciprocals of two real numbers, we conclude that

$$AP + BP + CP \geq 2(p_a + p_b + p_c).$$

A wonderful proof.

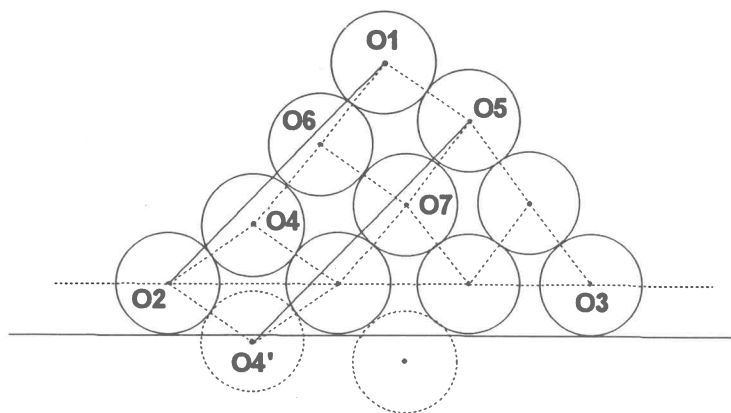
As an amusing follow-up, Kazarinoff stated in his paper that the same proof carried over when P is outside the triangle. It has been proven that the result holds when P is indeed external to triangle ABC, but as yet, no one has been able to push this proof through to that case. The dreaded wonders of margin notes!

Exercise 3: Draw the circumcircle of any triangle ABC, this clearly splits the circle into three arcs determined by the three sides of the triangle. Now reflect each of these arcs in their respective sides of the triangle. Prove that the three reflected arcs all pass through a common point!



Our final little theorem is one of my very own. Is that arrogant? Self-indulgent? Pretentious? Of course!

A circle stacking theorem: Start with a row of equal circles on a line, and place a row of one fewer circles tangent to circles of the bottom row two at a time. Carry on in this way until 1 circle at the summit is reached. Then provided the circles on the bottom are positioned so that none of the succeeding circles slip through any gaps, the centre of the top circle lies half-way between the centres of the extreme bottom two.



Notice that it suffices to show that O_1O_2 is equal in length to O_1O_3 . By joining the centres of tangent circles we obtain chains of rhombi. Thus O_1O_5 is equal and parallel to O_6O_7 , and so on. To transfer this segment all the way down to O_2 , we reflect O_4 in O_2O_3 yielding O_4' . Thus O_2O_4' is parallel and equal to O_1O_5 which implies that O_1O_2 is parallel and equal to O_5O_4' . So, this sequence of translations has enabled us to transfer O_1O_2 onto a new equal and parallel segment. By carrying on this reasoning, we will transfer O_1O_2 onto O_3O_1' , where O_1' is the reflection of O_1 in O_2O_3 . Then, by reflection in O_2O_3 , O_3O_1' is carried onto O_3O_1 . So we have carried O_1O_2 onto O_1O_3 by a sequence of translations, followed by a reflection, so $O_1O_2 = O_1O_3$!

Hints:

Exercise 1: Notice that quadrilateral DECB is cyclic.

Exercise 2: Perform a rotation of 60 degrees clockwise about C.

Exercise 3: Is this a familiar point of the triangle?

If your curiosity about plane geometry has been piqued, I am grateful. So, below is a treasury of riches. Don't let them lie dormant! They are waiting for you.

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Mathematics and Engineering Seminar

Dan Norman

Math & Engineering graduates continue to provide most of the content for our Fall Term seminar for graduating students in the Math & Engineering program. The Department of Mathematics and Statistics is very grateful to the following graduates, who spoke in our seminar last fall:

- Dr. Ron Kerr (Sc '87, now at the Communications Research Centre, Ottawa) on *Error Correction Coding*
- Dr. Hugh Cameron (Sc '73, now with Nortel in Montreal) on *Speech Recognition Technology and Applications*
- Mr. David Singer (Sc '92, now with Technology Solutions Canada, in Toronto) on *Call Centre Consulting for Fun and Profit*
- Mr. Mark Baker (Sc '91, now with Sun Microsystems) on *The Past, Present and Future of Wireless Computing and Communications*
- Dr. Kevin Deluzio (Sc '88, post-doctoral fellow in Clinical Mechanics, Queen's) on *Statistics and Gait Analysis in Clinical Mechanics*
- Mr. Chris Beveridge (Sc '95, now with Celestica in Toronto) on *Printed Circuit Board Design for Manufacturing*
- Dr. Jennifer Moore (Sc'89, post-doctoral fellow, Mechanical Engineering, Toronto) on *Blood Flow Modelling with Realistic Arterial Geometry*
- Ms. Paula Preston (Sc '79, now with Nortel in Ottawa) on *Managing the Product Cycle for Public Data Networks*

The fourth year students really appreciate these talks --- they find them interesting, and many find their horizons expanded about the kinds of careers they can aspire to. (The students also get to practice reporting skills by submitting reports on most seminars, and shorter summaries on the rest.)

We are looking for speakers for this coming Fall. It is valuable to have a mix of speakers, some recent graduates, some with more mature careers. Talks may focus on a design problem, or research, or development, or consulting, or managing in an engineering enterprise, or manufacturing, or

entrepreneurship, or a mixture of the above. We have had speakers come to talk two or three times.

To volunteer to speak, get in touch with Dan Norman:
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Head's Report

Eddy Campbell

These two years have been marked by the truly exceptional accomplishments of our faculty and students.

- Ram Murty won a Killam Fellowship last year, one of five such awards nation-wide across all academic disciplines. Ram's award is for two years and allows him to pursue his research in number theory full-time. Concurrently, Ram won a distinguished lectureship at Brown University.
- Two members of our faculty won an Ontario Council of University Faculty Association Teaching Award this year: Morris Orzech who is a long standing member of the Department of Mathematics and Statistics at Queen's and Jim McLellan who is a recent Mathematics and Engineering graduate from Queen's and currently holds a cross-appointment with the Department of Mathematics and Statistics and the Department of Chemical Engineering.
- Norm Beaulieu won a Steacie Fellowship from the Natural Sciences and Engineering Research Council of Canada, one of four such awards nationally. Norm is cross-appointed into our department from the Department of Electrical and Computer Engineering.
- Troy Day, a student of Peter Taylor, won a Natural Sciences and Engineering Research Council of Canada Doctoral Prize, one of four such awards nationally, Troy also won Killam Postdoctoral Fellowship, one of eight such awards nationally. Finally, Troy was awarded the Canadian Applied and Industrial Mathematics Society Doctoral Prize for 1999 for his thesis, written under Peter's direction. Troy took his Killam to UBC and is now in a tenure-track position in the department of Zoology at the University of Toronto.

- Jian Shen, a student of David Gregory, won the Governor General's Gold Medal for 1999 for the best PhD thesis at Queen's in any academic discipline. Jian was awarded a NSERC Post-Doctoral Fellowship which he has taken to the University of Wisconsin at Madison. Jian was also awarded the Doctoral Prize of the Canadian Mathematical Society for the best PhD thesis in Canadian Mathematics in 1999.
- Michael Levi graduated in the Mathematics Physics program this year. He was awarded the Governor General's Silver Medal as the outstanding student in any undergraduate academic discipline, the Prince of Wales prize as the top student in the Faculty of Arts and Science, and a special medal in Mathematics and Physics.
- Daniel Veiner graduated as the top student in the Faculty of Applied Science this year. Daniel was a student in the control and communications option of our Mathematics and Engineering degree.
- Andrew Granville has won the first Ribenboim prize of the Canadian Number Theory Association. Paulo Ribenboim, now Professor Emeritus, is more productive than ever, and his books "The Book of Prime Number Records" and "The Little Book of Big Primes" are mathematical bestsellers. His collected works, comprising seven volumes, has appeared in our series "Queen's Papers in Pure and Applied Mathematics". Andrew was one of Paulo's students, graduating from Queen's in 1987. The prize is to be awarded once every four years.
- Agnes M. Herzberg has received the 1999 Service Award from the Statistical Society of Canada. The award recognizes Dr. Herzberg's dedicated service to the society.

Twenty-eight students graduated from the Mathematics and Engineering Program. Seven of these students plan further graduate work, four of them at Queen's. Two students have firm plans to become teachers, and seven students have taken jobs. One student is involved in the Queen's International Development Program.

Forty-one students graduated from Arts and Science. We have less definite data on the employment prospects of these students.

We made two appointments this year, and we are very proud of them. They are Tamas Linder, who studies communications and information theory. We also appointed Andrew Lewis, who studies control

theory. Both Tamas and Andrew are expected to qualify as professional engineers in due course, and their chief teaching duties are expected to be in the Mathematics and Engineering program.

You will read elsewhere on our Jobs Network initiative. In brief, we are asking our alumni and others to provide summer, work experience, and permanent employment for students at all levels in our degree programs. Our degree programs are among the most challenging at Queen's, and consequently our students are among the best at Queen's, which is justly famed for the quality of her students. Math and Stat students make fabulous employees! The initiative has led us to explore ways in which we might strengthen our actuarial degree and to investigate the possibility of a joint degree with the School of Business.

This year, the department will participate in a pilot project with the Development Office at Queen's. We would like to stay in closer contact with our alumni, in part to generate employment opportunities for our students, and in part to realize our goals in the capital campaign.

We're also very excited by the new prospects for statistics at Queen's. Regular readers of this column will know that this beautiful and important discipline has suffered because of government cutbacks. We now have just four statisticians on faculty. However, we have been able to obtain funding for a senior position in statistics, and we expect to rebuild around this new appointment. We are especially grateful for extremely generous bequest from Geraldine Mitchell, Arts '29 part of which will be used to help fund this position.

The provincial government's Access to Opportunities Program (ATOP) is intended to double the number of graduates in high technology programs. We developed a new option in our Mathematics and Engineering Program called "computers and communications" as part of the university's response. The first group of thirteen students will begin second year in September. We also do a lot of service teaching in the Faculty of Applied Science (Engineering) and we expect that we will be given new faculty and staff positions new in support of increased enrolments due to ATOP. We have already received a number of computers for the communications laboratory as well as the robotics and control laboratory. I have to say that it's just great to belong to a Department of Mathematics and Statistics that owns a computer-driven milling machine.

The Campaign for Queen's.

Our first and most costly goal is very ambitious indeed. We would like to honour A. J. Coleman who was head of this department for 20 years. If we are successful in raising \$1,000,000 then we will be able to provide an endowment fund in support of a named professorship. If we manage to raise \$2,000,000 we will be able to support a named chair. A chair is a highly prestigious faculty position over and above our usual complement. Finally, each \$1,000,000 beyond \$2,000,000 to a maximum of \$4,000,000 will support a named Post-Doctoral Fellow in support of the chair. The Department has put aside \$100,000 from the Mitchell bequest in order to get this fund-raising effort started.

In addition to this, there are a number of more modest objectives. For several years, we have employed Gill tutors, senior undergraduates chosen for their ability and personality to help foster a sense of community among our junior undergraduates. We would like to have support in order to hire outstanding students for work within the department, either in administrative, research or curriculum development positions. Each \$100,000 in endowment will generate roughly \$5,000 in salary support. I note that the initial bequest from the Gill family was \$25,000.

A very generous donation by Graham and Stevie Keyser has provided vitally needed funding for our reading room over the past two years. The room is now being rewired to support a network of computers for our faculty, graduate students and visitors. The Keyser's support is intended to help provide the kind of environment in which research flourishes, where chance conversations can lead to profound discoveries.

We were able to launch the Jobs Network initiative because of a very generous donation from Mark Baker. Mark graduated from the Math and Engineering program in 1989. The initiative will need on-going funding from our alumni if it is to flourish. And, of course, in these days when tuition fees are increasing so rapidly, we want to ensure that our students have every opportunity to succeed.

Bob Erdahl has stepped down from his position as graduate coordinator after 7 very successful years. We're all grateful to Bob for his hard work. In particular, I'd like to point out that Bob is deserving of a lot of credit for the list with which I began this report. Thanks, Bob. Bob is succeeded by Leo Jonker, who was head of the department from 1990-1995.

Bob and others, notably Grace and Morris Orzech, have been contemplating how we might best incorporate mathematics and statistics problems from industry and elsewhere into our undergraduate and graduate curriculums. There have been a number of successful experiments in the US that we intend to emulate. Most involve teams of students working together on highly selected problems suitable to the length of term and the sophistication of the students. It's interesting to contemplate the ingredients required for the success of this initiative at Queen's. Chief among these will be the extent to which we can involve the faculty, and in particular, the extent to which faculty are able to engage their research interests in the initiative.

Finally, I want to mention a very interesting initiative of Professor Agnes Herzberg. Agnes is in the process of creating an interdisciplinary research group based out of our department. It is widely thought that the really big problems in science and society today live at the interface of a wide variety of disciplines, including both mathematics and statistics. Agnes' group would meet regularly to discuss such problems and to hear from internationally famous researchers from around the world. Professor Herzberg hosts each year a conference on Science, Statistics and Public Policy. The list of attendees is truly astonishing. It includes Nobel prize winners, renowned scientists, politicians, and public policy experts from many countries and cultures, to present papers and discuss each year a specific area of concern. Agnes originally planned just one conference, but the first proved so successful that she has been prevailed upon to continue on an annual basis. Funding is needed to help continue this important work.

Norman J. Pullman 1931 - 1999

A letter from Sylvia Monson (Ph.D. 1995) and Rolf Rees (Ph.D. 1986).

Professor Norman Pullman, born in New York in 1931, obtained his Ph.D. at Syracuse University in 1962. He taught for three years at McGill University in Montreal before taking up a postdoctoral fellowship at the University of Alberta in 1965. He then took a faculty position at Queen's University at Kingston, Ontario, where he was promoted to Professor in 1971. He retired from Queen's in 1994 with the status of Professor Emeritus.

Professor Pullman's research spanned a wide range of topics in matrix theory, linear algebra and graph theory. He made significant contributions to the theory of tournaments, to the study of clique and biclique covering numbers of graphs and their relationship to the problem of determining the boolean and real ranks of binary matrices, and to the study of linear operators that preserve some prescribed property of a matrix. His outstanding achievements were celebrated in a *festschrift* entitled "Graphs, Matrices and Designs", published as Volume 139 of the Dekker series "Lecture Notes in Pure and Applied Mathematics". The list of contributors to this volume includes many well-known mathematicians: LeRoy Beasley, Charles Colbourn, Paul Erdos, Douglas Stinson and others.

Norman Pullman had an uncompromising work ethic which he passed on to us early in our relationships with him as his doctoral students. He said "You owe the mathematical community the very best job that you can do". It is a standard that we will always strive to achieve. We learned many things from him over the years: he taught us how to write a research paper and how to properly swallow a raw oyster with draught beer.

Professor Pullman passed away peacefully in the early morning hours of May 28, 1999. He will be missed terribly by all of us who knew him well. To have known Norman Pullman was truly to have loved him, for he was that kind of man. Much more than an outstanding mathematician, he was a man of great kindness and compassion. He also had a contagious sense of humour. Whenever he dined out at a Greek restaurant, he would ask the waiter if the *moussaka* had been made with fresh moose.

His sense of humour did not wane in spite of his recent illness from which he knew that he would not recover. We were both privileged to have visited with him during his last days in the hospital. Rolf recalls: I last saw Norman on May 18th, ten days before his passing. I had just returned from a conference in Utah. I was telling him about how our mutual friend and colleague, LeRoy Beasley, had shown us around the great Temple Square in Salt Lake City. Norman was unable to speak, but could not resist the urge to summon pad and pencil so that he could ask, jokingly, how many of us had been converted to Mormonism. Sylvia recalls: My second to last visit with Norman was on Sunday, May 23rd. He had rallied that weekend, was sitting comfortably in a chair and was in very good spirits. His throat was beginning to heal after the removal of the respirator and he was able to speak in a soft voice. Norman delighted us that

evening with a flawless recitation of one of his favourite poems, "Jabberwocky".

Such was the way in which Norman lived and died. Goodbye Norman, our teacher, our mentor, our colleague and our very dear friend.

A letter from a long time friend and collaborator, Professor LeRoy B. Beasley, Utah State University. Read at the Memorial Service on June 10, 1999.

Norm Pullman was my coauthor, collaborator and colleague, but more importantly, my friend and mentor.

I have been trying to identify those traits that made Norm Norm. Perhaps the most impressive quality he had was his devotion to the things he considered important: his family, friends and mathematics.

In his treatment of these, he was kind, considerate, generous and enthusiastic. He was proud of his family and their accomplishments, showing pictures and telling of their activities each time I visited. He was never critical of his friends, always accepting us and our shortcomings, complementing us for the good and ignoring the bad. The enthusiasm he showed for his family and friends was also reflected in the way he approached mathematics. His enjoyment of mathematics was obvious. The energy he put into a paper was awesome. He worked tirelessly to perfect a proof or clarify a paragraph. When he visited me in Logan, we would work all day until I could barely think, then the next morning he would present me with write-ups or rewrites that must have taken most of the night. He never lacked for ideas or for suggestions on how to proceed. Most of the time these suggestions proved fruitful.

I remember the many happy times I spent with Norm and Barbara: dining out, picnicking, eating oysters in Florida, visiting the brew pubs in Kingston, driving to the Rideau locks, walking on the beach, the many happy hours relaxing in their living room after a days work, trying to help solve a cryptic crossword puzzle. I will miss Norm's visits and I will miss visiting him. I will miss the inspiration he always left me with. I will miss being able to bounce ideas off him. No matter how far fetched they might have been he always had a helpful response.

I am fortunate to have known Norm and to have been a friend and colleague. I will miss him.

A memorial page for Professor Pullman may be found at www.mast.queensu.ca/~pullman

Poor Little i

A mathematical fantasy by George C. Bush

The following contribution was sent to us by George C. Bush. George received his Ph.D. from Queen's in 1961. He was a member of the Mathematics and Statistics Department from 1961-1966 and he was back in this capacity on a temporary basis from 1971-1973.

Let me tell you about my family. It is very large, more of a clan than a family. We are extremely hard-working. Most of us seldom get a rest. The various branches of the family work very well together, but there are serious class distinctions among us. The tension comes from the names given to us by our uncles. These people are not really our uncles. They are not part of the family, but they treat us as their personal servants and playthings.

It was Uncle Euler who gave me my name, *i*. When you refer to yourself you use a strong capital, *I*, but for me there is only a small letter – poor little *i*. However, *i* am getting ahead of my story. *i* am one of the youngest in the family. My oldest cousins are the integers. Now there is an attractive name, suggesting integrity and having it all together. Even among the integers, however, the uncles sowed seeds of discrimination by branding half of them as *negative*. Next came the *rational*s with a name of which they could be proud. In contrast, their cousins were named *irrational*. That name goes back all the way to the time of Uncle Euclid. The uncles insist that they do not use the name as an insult. They mutter something about not being ratios, but it is hard to escape the unattractive overtones of the name.

It was Uncle Cardano who discovered our part of the family. He played games with some of my brothers and sisters, but he did not want to admit us to the family. For the next three centuries the uncles continued to put us to work, but almost competed in vilifying us. Uncle Descartes called us “imaginary”; Uncle Newton found us “impossible”; Uncle DeMorgan said we were “self-contradictory and devoid of meaning, but useful”. They showed us very little respect, even though they were learning that they could not continue their work without us. Uncle Gauss was an exception. He changed our name from

imaginary to complex because he felt that names like negative and imaginary caused prejudice against us.

A large part of our problem arose from the family picture gallery. The members of all the other parts of the family had their portraits arranged in a long row. The uncles called this row the *number line*. The cousins who had a place on the line called themselves the *real* family. We were outcasts because no one knew how to draw our portrait. It was Uncle Wessel who first discovered how to enlarge the picture gallery to provide places for us. He was not very influential among the uncles, so his discovery did us very little good until Uncle Gauss added his prestige to the idea.

Once we had a place in the family picture gallery, most of the uncles were willing to accept us as full members. Uncle Hamilton, however, was not satisfied. He felt that there was still something unexplained about us. Instead of recognizing us by our portraits, he preferred to label each of us with two members from the real family. He felt that he had made our presence in the family respectable. *i* bolster my ego by pointing out that it takes *two* from the real family to equal *one* of us complex numbers.

New Problem

Peter Taylor

Tossing the water filled balloon.

This amusing problem was provided by Norm Rice. It seems he was at a church picnic and one of the games was tossing a water-filled balloon back and forth and of course because of the delicate nature of the missile and the fact that you are after all dressed up in church picnic clothes, you want to minimize the chance that it will break when you catch it. What that means is that you want the balloon to arrive with the minimum speed.

- (a) Let's suppose the balloon is released and caught at the same height. The problem is what angle should the balloon be projected at? If the angle is just above zero, you have to give it a lot of speed as it can't fall too much. If the angle is large, it will go quite high and then gravity will give it lots of speed on the way down. It seems there will be an intermediate angle which minimizes the speed of arrival. What would that be? Take the acceleration due to gravity to be $g = 10 \text{ m/s}^2$.

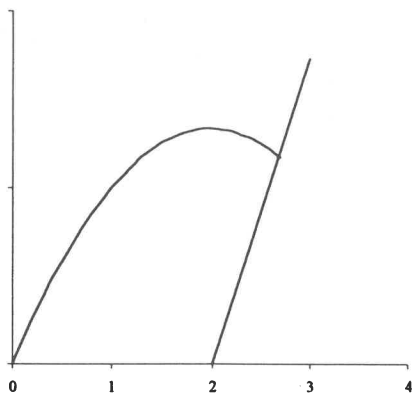
Okay. It turns out that this is really an old problem (but a good one) in disguise. In this case the final speed will be the same as the initial speed (by symmetry) and the problem of choosing the angle to minimize this (with a fixed horizontal distance) has the same answer as the problem of choosing the angle to maximize the horizontal distance traveled with a fixed initial speed. And that's 45 degrees. By all means work it out if you want. It will provide good technical preparation for what follows.

So how are we to make this an interesting problem? Well what if the ground isn't flat and the thrower and receiver are at different heights? Well, that makes things more interesting but it's still standard high school physics. But here's a thought. Suppose the ground isn't flat and I can position the receiver anywhere I want subject to some simple constraints (like he can't be too close to me). Where should he stand so he receives the balloon with the minimum speed. The idea here is that the more I get him above me, the more I am able to get the balloon to him at its maximum height which is when the speed of any trajectory is a minimum. So maybe the problem should be:

(b) Find an interesting problem along these lines.

Well, (b) is the problem I took for myself and here's what I came up with. So maybe your real job is to solve the following:

(c) I am standing on flat ground, but 2 meters away a hill starts to rise in a straight line at an angle of 60 degrees to the horizontal. I can put the receiver (who'd better be a competent climber) anywhere on the hill. Where should he be so that (if I project the balloon optimally) he will receive it with minimum speed?



The Plank Problem

(from Summer '98 issue)

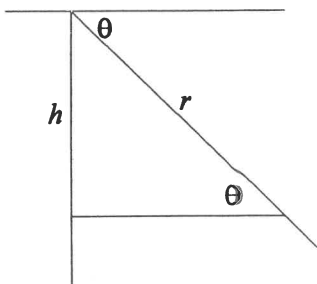
Peter Taylor

A whole family of planks radiate from a point at different angles down to the ground below. On each plank there is a ball sitting at that top point (so the different balls are all coincident at the beginning). At $t=0$, each ball begins to roll down its plank in a frictionless manner. [There is a vertical plank which doesn't even have to be there, and its ball simply falls to the ground.] The problem is to find the locus of the family of balls at any time. That is, at any moment the balls will all lie along a natural curve. What is that curve?

Solution

Well this problem turned out to be more interesting than I thought when I set it. What on earth does it mean to "roll in a frictionless manner"? If you want to roll, you need friction. So either we have no friction and a sliding ball (which in the true frictionless inclined plane genre should probably be a block) or we have friction and roll. We can't have both. Now in both cases, we'll still assume no energy lost to friction, but in the second case there will be energy spent in giving the ball rotational momentum. Anyway, we wind up with two cases, and the interesting (and unexpected) result is that in both cases the locus we are after is a circle. Complete and quite impressive solutions were received from Len Minty (B.Sc. B.Ed. '69), Ross Ethier (Apple Math '80), W.C. Bentley (B.Sc. '77, B.Ed. '86, now a physics teacher in Bancroft), G.J. Young (B.A. Harvard, Duncan B.C.) and Rick Pim (B.Sc. '81, mathematical physics, M.Sc. '84, physics, PhD '90, physics). Rick and Jim Whitley (retired math prof, now teaching "Jforce," the extended section of 1st year Applied Science) were the only ones to notice the rolling ball solution.

First let's assume no friction. Consider the plank at an angle of θ to the horizontal. At any time t let h be the distance fallen and r be the distance along the plank, as in the diagram.



Then if the velocity of the ball is v , conservation of energy gives us

$$\text{KE gained} = \text{PE lost}$$

$$mv^2/2 = mgh$$

and what we need from this is:

$$v^2 \text{ proportional to } h.$$

Now v is dr/dt and h is $r \sin \theta$, so

$$dr/dt \text{ prop to } (r \sin \theta)^{1/2}.$$

This is a differential equation which can be solved to give

$$r \text{ prop to } t^2 \sin \theta.$$

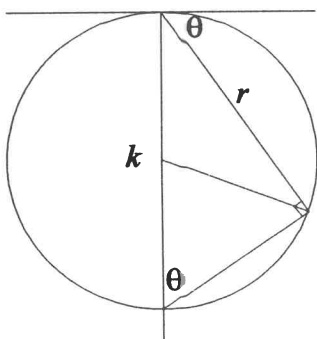
We conclude that at any fixed time,

$$r = k \sin \theta$$

where the "constant" k will increase with time.

Now this implies that the ball lies in a circle with centre $k/2$ units below the starting point and radius $k/2$. There are at least three ways to see this. One is to recognize the above equation as the equation of a circle in polar coordinates. A second is to convert to cartesian coordinates using the transformation $x=r \cos \theta$, $y=r \sin \theta$. We get the equation

$$x^2 + y^2 = ky$$



which is the above circle. A third way is to use a little Euclidean geometry. Start by drawing the circle, and then recall that the diameter always subtends a right angle at the circumference. Then the relationship $r = k \sin \theta$ emerges from the triangle in the diagram.

Now for the rolling ball. In this case, the PE lost will be gained in KE in two different ways, translational energy and rotational energy. The translational component $mv^2/2$ is familiar enough, but the rotational part is less well known. It turns out that rotational KE is $I\omega^2/2$ where I is the moment of inertia about the axis of rotation and ω is the angular velocity (in radians/sec). Now for a ball of radius R rotating about a central axis, $I = 2mR^2/5$, and if the ball is rolling down a plank with speed v then $v = \omega R$. Thus the rotational KE is $I\omega^2/2 = mv^2/5$, and the total KE is the sum:

$$\text{KE} = mv^2/2 + mv^2/5 = 7mv^2/10.$$

The point is that this is still proportional to v^2 and hence v^2 is still proportional to h and the argument for case 1 works here too. We conclude that in both cases we have a circle, though in the first case, the radius of the circle grows faster than in the second.

Rick Pim pointed out that in the second case of the rolling ball, the ball that falls straight down won't be on the same circle as the others because it won't pick up the rotation (in fact it will be on the case 1 circle). We could, I suppose, arrange for it to be on the second circle by winding a string around it and letting it fall yo-yo fashion.

Correction: In last year's Communicator we incorrectly stated that David Hamilton whose contribution was printed under the heading "Our Alumni" graduated from Queen's some 30 years ago. As a matter of fact he graduated in 1980. Our apologies to David!

IF UNDELIVERED RETURN TO:
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