QUEEN'S MATHEMATICAL COMMUNICATOR

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$$Q(n) = Q(n-1) + 3^{n-1}$$

DATABASES, RELATIONS AND FUNCTIONAL DEPENDENCES

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An aperiodical issued at Kingston, Ontario by the
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DATABASES, RELATIONS AND FUNCTIONAL DEPENDENCES

by

Tim Merrett

(Tim Merritt obtained his B.Sc. in mathematics at Queen's in 1964 and did his doctorate in theoretical physics at Oxford. He then worked as an application programmer for IBM in Scotland and is now an associate professor in the School of Computer Sciences, McGill University.)

Given n sets, $D_1, D_2, \ldots, D_n$, (not necessarily distinct) called domains, an n-ary relation, R , is defined as a subset of the (extended) Cartesian product,

$$R \subseteq D_1 \times D_2 \times \ldots D_n \quad .$$

Relations of this sort are of great interest to computer scientists specializing in databases. Databases typically store formatted data such as

STUDENT RECORDS (STUDENT COURSE GRADE)

STUDENT	COURSE	GRADE
HUNG	APL	B
HUNG	PASCAL	A
RAMAN	ALDAT	A
SMITH	ALDAT	B
SMITH	APL	A

STUDENT RECORDS is a relation which is a subset of STUDENT × COURSE × GRADE . The domains are the sets STUDENT = {HUNG, RAMAN, SMITH} , COURSE = {ALDAT,APL,PASCAL}, GRADE = {A,B,C} .

Mathematicians have been familiar with binary relations (n=2) for a long time and the properties of important classes of binary relations (notably with both domains the same set) are well known: equivalence relations, partial orderings, etc. Not much seems to be known about relations of order greater than two, in spite of their growing practical importance.

One important concept of binary relations that has been extended to n-ary relations is that of functionality. A function is a special binary relation in which no more than one element of the second domain (traditionally called the range) may be paired with any element of the first domain (traditionally called the domain: we will not use traditional terminology in this discussion). We introduce the functional dependence $A \rightarrow B$ for domains A and B to assert that, in the context of a particular relation, there is a many-one relationship between the elements of A and those of B. A functional dependence, $X \rightarrow Y$, may also hold between sets, X and Y, of domains. In STUDENT RECORDS, for instance, there is the functional dependence

$$\{\text{STUDENT, COURSE}\} \rightarrow \text{GRADE}$$

This may be interpreted as saying that if we know the student and the course we may find in the relation a unique grade (eg. HUNG, PASCAL gives A). From what is commonly known about grades, this functional dependency makes sense: a student can only receive one grade as a final assessment of his performance on a course. On the other hand, the dependence

$$\text{STUDENT} \rightarrow \text{GRADE}$$

does not hold for STUDENT RECORDS. Nor does it make sense: grades for a given student differ depending on the course he is in.

If we know that some functional dependences hold for a relation, can we derive additional dependences?

The following rules may be shown, from the definition, to apply to functional dependences

REFLEXIVITY	$X \supseteq Y$ implies $X \rightarrow Y$
UNION	$X \rightarrow Y$ and $X \rightarrow Z$ implies $X \rightarrow Y \cup Z$
DECOMPOSITION	$X \rightarrow Y \cup Z$ implies $X \rightarrow Y$ and $X \rightarrow Z$
AUGMENTATION	$X \rightarrow Y$ implies $W \cup X \rightarrow Y$
TRANSITIVITY	$X \rightarrow Y$ and $Y \rightarrow Z$ implies $X \rightarrow Z$

where W, X, Y and Z are sets of domains. Either reflexivity union and transitivity or reflexivity, augmentation and transitivity are axioms for functional dependences.

Now let us refine our definition of a relation to account for the fact that, in a database, it represents data physically stored on a medium such as a magnetic disc. It can be changed from time to time. A relation must now be defined as a set of subsets of the Cartesian product of its domains, together with some semantic invariants or properties that must always be satisfied by the n-tuples of the relation. A functional dependence is such a semantic invariant. Thus, for instance, it is reasonable that {STUDENT, COURSE} → GRADE should always hold. This functional dependence should be included in the definition of STUDENT RECORDS.

Such intrinsic functional dependences must be distinguished from extrinsic dependences that may just happen to hold for a given set of n-tuples. For instance, the functional dependences

{STUDENT, GRADE} → COURSE

and

{GRADE, COURSE} → STUDENT

happen to hold for the above version of STUDENT RECORDS. They can be eliminated if we add to the relation the information that HUNG got an A in his ALDAT course. This illustrates the difference between accidental and essential functional dependences. It also shows how database workers consider relations to be time-varying in that they can be updated by addition of, deletion of or even changes to n-tuples.

An intrinsic functional dependence places a constraint on the number of n-tuples that may belong to the relation. If STUDENT RECORDS were unconstrained, it could have up to 27 n-tuples. The constraint $\{STUDENT, COURSE\} \rightarrow GRADE$ restricts it to at most 9 n-tuples, since there cannot now be more than one n-tuple for each value of the pair {STUDENT, COURSE}.

Conversely, if we know how many n-tuples there are in a particular instance of a relation, what can we say about the extrinsic functional dependences which can or cannot hold for that instance? STUDENT RECORDS can, if unconstrained, have up to 27 n-tuples. If it has more than 9 n-tuples, no possible functional dependence can hold apart from the trivial ones given by the reflexivity rule. If STUDENT RECORDS has fewer than 4 n-tuples, at least one functional dependence will certainly hold. From 4 to 9 n-tuples, whether or not there is an extrinsic functional dependence depends on which of the 27 possible n-tuples occur.

This can be seen by a geometrical interpretation of the relation and of functional dependences. Given the sets STUDENT, COURSE and GRADE and that STUDENT RECORDS is a subset of their cartesian product, we can represent the relation as a configuration of points in a three-dimensional space with three values on each axis. In such a configuration, if a line drawn parallel to the axis representing domain D intersects more than one occupied point (representing an n-tuple) then D cannot be functionally dependent on any other domain.

Now if more than 9 of the 3 by 3 cube of points are occupied we cannot find an axis such that all lines parallel to it intersect at most one point. Also if less than 4 of the points are occupied we can always find an axis such that all lines parallel to it intersect at at most one point.

These arguments generalize immediately to the n-ary relation $R(D_1,D_2,\ldots,D_n)$. The quantities corresponding to 9 and 4 are, respectively, $\prod_i |D_i|/\min_i(|D_i|)$ and $n+1$.

For numbers of n-tuples between these extremes we are faced with a large combinatorial problem. Even for STUDENT RECORDS there are 2^{27} possible instances to be examined in all or $\binom{27}{4} + \binom{27}{5} + \binom{27}{6} + \binom{27}{7} + \binom{27}{8} + \binom{27}{9}$ possible instances with from 4 to 9 n-tuples.

As an example of a complete enumeration we show the nontrivial functional dependences of instances of $R(A,B)$ where $|A| = 4$ and $|B| = 2$.

NO. N-TUPLES	0	1	2			3			4			5,6,7,8
FUNC.DEP.	all	all	$\phi\to A$	$\phi\to B$	$A\leftarrow\to B$	$\phi\to B$	$A\to B$	none	$\phi\to B$	$A\to B$	none	none
NO. INSTANCES	1	8	4	12	12	8	24	24	2	14	54	93

(A functional dependence of type $\phi \to A$ means that A is constant: it has the same value for all n-tuples). We see that the probability of finding a functional dependence is one for instances of 0, 1, or 2 n-tuples and zero for instances of 5, 6, 7, or 8 n-tuples. In between, it is 4/7 for 3 n-tuples and 8/35 for 4 n-tuples.

Problem. What is the probability of finding a functional dependence in an instance of an arbitrary relation $R(D_1,D_2,\ldots,D_n)$ of a given number, $|R|$, of n-tuples.

Note. For the case $|R| = 9$ of the 3 by 3 cube representing, for instance, STUDENT RECORDS, or $R(A,B,C)$, the number of instances with the functional dependences $AB \to C$, $AC \to B$ and $BC \to A$ is just the number of latin squares of order 3 .

The purpose of this article has been to stimulate mathematical interest in database theory by describing one aspect and propounding a nontrivial combinatorial problem. The theory of n-ary relations deserves to be explored much further.

The Department is presenting a

SYMPOSIUM ON STATISTICS

to honour Professor Emeritus George L. Edgett

on Monday, November 5, 1979

Professor Edgett, who joined the staff of Queen's in 1930, was, as far as we can determine, the first mathematician in Canada to offer formal lectures on Statistics. Over the years he introduced a long succession of students to Statistics and engendered in them an enthusiasm for his favourite subject. His paper on missing observations proved seminal in stimulating much further research. We hope that on the evenings of November 4 and 5 he will reminisce about his 50 year involvement with statistics.

The Symposium will begin with a Reception, for out-of-town visitors, on Sunday evening at 8:00 p.m. in the home of Professor John Coleman at 108 Albert Street (immediately west of Victoria Hall).

The formal scientific programme will be from 9:30 to 5:30 p.m., Monday, November 5, 1979 with parallel sessions if necessary. There will be social events in the evenings of November 4 and 5 which will include a celebration of the decision of the Board of Trustees of Queen's University to rename the Queen's STATLAB as the George L. Edgett Statistical Laboratory.

Monday 9:30 - 12:00 (Miller Hall 201)	Principal Watts and Professor Jack Kiefer; Cofee; Professor Donald Watts; Contributed Papers.
2:00 - 5:00	will be devoted to contributed papers, coffee and an address by Professor Ralph Bradley.
7:00 - 10:00	Reception and Banquet, Queen'st Faculty Club

PEOPLE IN THE NEWS

Selwyn Caradus has been named Anglican chaplain at the university, effective this September.

He is an active member of St. George's Anglican Church, a choir member, vestry council member, and chairman of the church's education committee.

In August, he completed a master of theological studies from Queen's. After ordination, he plans to continue as a part-time teacher and part-time Anglican chaplain.

A specialist in a branch of mathematics called functional analysis, Caradus earned his first degree from the University of Auckland in his native New Zealand. His master's degree and doctorate are from the Universities of Southern California and California.

John Coleman narrowly lost the election to be Liberal candidate for Kingston and the Islands in the recent Federal elections; the party selected Peter Beeman, reeve of Kingston township, who was defeated by Flora Macdonald for the Parliamentary seat. John is so disappointed and morose that the is going to resign from the Departmental Headship after 20 years of outstanding service.

Ole Nielsen attended a conference on Harmonic Analysis and the Representation Theory of Topological Groups at Oberwolfach (W. Germany) from July 22-28/79; he gave a lecture there on the Topological Frobenius Property for Nilpotent Lie Groups.

He arrived back at Queen's on August 1, having spent a year's sabbatical leave at the Mathematics Institute of the University of Copenhagen.

Donald Watts - and a former Queen's student Douglas Bates, have been accorded the high honour of being invited to read a paper before the Royal Statistical Society. The paper, entitled "Relative Curvature Measures of Nonlinearity" is based on part of Bates' Ph.D. thesis, and will be read before the Research Section in London on October 17. Doug is now a member of the Department of Mathematics at the University of Alberta.

John Ursell - attended the New York meeting of the American Mathematical Society in April and the International Conference on Quantitative Genetics at Iowa State University in August. At the latter he presented a paper entitled "A new interpretation of recombination frequencies in genetics".

John C. Cartledge has just completed his Ph.D. thesis, entitled "Measures of information for continuous time estimation problems", under the supervision of Lorne Campbell. This research grew out of his Master's thesis which dealt with the performance of an optical fiber communication link. The thesis makes use of the recently developed theory of stochastic integrals for martingales. This is a theory which generalizes ordinary integrals to random functions. In the thesis, Dr. Cartledge developed a general theory of filtering for processes of the type which might result from the use of a photodetector in an optical system. He is now at Bell Northern Research in Ottawa, where he is working in the fiber distribution section.

Norm Pullman participated in the Franco-Canadian Conference on Combinatorics (Universite de Montreal, June 1979); the Workshop on Algebraic Aspects of Combinatorics (Simon Fraser University, July 1979) and some meetings of the Math Club (Thousand Islands Secondary School, Brockville, Ontario).

Jim Verner has presented papers recently at the Conference on Numerical Ordinary Differential Equations at Urbana, Illinois, in April, at the Department of Computer Science, University of Toronto in May, and at the Conference on Numerical Mathematics at Winnipeg in Steptember.

Tony Geramita gave a series of lectures at the Universita di Roma and the Universita di Catania in Italy this spring, as well as lecturing at Oberwohlfach in Germany. His book written with Jennifer Seberry, University of Sydney, "Orthogonal Designs: Quadratic forms and Hadamard Matrices" has just been published by Marcel Dekker.

Thomas Stroud attended the First International Conference in Bayesian Statistics in Valencia, Spain (in May) and the annual Combined Statistical Meeting in Washington, D.C. in August.

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MASTER'S DEGREES AWARDED RECENTLY BY THE DEPARTMENT OF MATH AND STATISTICS

NAME	SUPERVISOR	TITLE
DECAEN, Dominique	N. Pullman	Prime Boolean Matrices
DICKINSON, Robert	J. Davis	Iterative Methods For Matrix Spectral Factorization
DONALD, Alan W.	N.J. Pullman	Edge and Arc Partitions of Arbitrary Graphs and Digraphs
GUINAND, Paul Scott	D. Norman	Morse Theory on Banach Manifolds
HARVEY, Jack	P. Taylor	Applications of Sperner's Lemma
MACLEOD, W. Bentley	J. Davis	On Observability For Non-Linear Systems
POUND, Anne	T. Smith	A Study Of Statistical Methods For Ordered Categorial Data
SWIFT, Robert W.	M. Orzech	Smooth Algebras
TAYLOR, Christopher	L. Jonker	Anosov Diffeomorphisms

NEWS FROM GRADUATES

1968

Rick Bunt (Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598) writes:

After graduating from Queen's I entered the graduate program in Computer Science at the University of Toronto, from which I received my M.Sc. and Ph.D. degrees. In 1972, I joined the faculty of the Department of Computational Science at the University of Saskatchewan, Saskatoon, where I presently hold the rank of Associate Professor. This past year I have been on sabbatical leave with IBM Research in Yorktown Heights. Among my colleagues here at Yorktown, although I do not work in his group, is Ken Iverson, whose series of lectures while I was at Queen's was, in fact, one of my early introductions to the computing field. I have fond memories of hours spent during my final year on the department APL terminal which was located, I believe, in the humanities building at that time.

I have recently co-authored a textbook entitled "An Introduction to Computer Science: An Algorithmic Approach" which has been published by McGraw-Hill and is intended for freshman majors in computer science. I am also at present an Associate Editor of the INFOR journal, Canada's major research outlet in the field of computing.

1941

Harry Occomore (15 Chelford Crescent, Belleville, Ontario, K8N 4J8) writes:

After two years Faculty of Applied Science and Engineering University of Toronto, I spent 30 years in Outside Plant Design Engineering with Bell Canada. The last 12 years I was Supervisor Engineering Design at the Bell Canada Technical Training Centre, 11 Bay Bridge Road, Belleville, Ontario. Also during my "declining years" I had the pleasure of teaching calculus, statistics and probability and Business statistics at Loyalist College here in Belleville.

1973

Robert Preston (2514-2 Forest Laneway, Willowdale, Ont., M2N 5X7)

After graduating with an Honours B.Sc. degree in Mathematics, has been employed by IBM Canada Ltd. in Toronto both as a Systems Analyst and Systems Engineer.

VISITORS

Among visitors to the Department who have arrived recently are the following:

1. Michael Bulmer - a theoretical biologist from Oxford working with Peter Taylor and planning to be here until December 31, 1979.

2. Barry Dayton - an algebraic geometer from Northwestern working with Leslie Roberts and planning to be with us until June 1980.

3. Bernice Auslander - from the University of Massachusetts, Boston, was back with us for a few days working with Morris Orzech.

4. Sergei Sergevich RYCHKOV - a world famous geometer from the Steklov Institute in Moscow and

   Arcadij Anatolievich MALTSEV - a topologist and editor of Uspehi Matematiceskih.

   Sergei and Arcadij are the first visitors we have had the privilege of receiving on the Exchange arrangements into which the Department has entered with the Steklov Institute. They are living with Bob Erdahl in Portsmouth, and expect to be in Canada to mid-November. They will be based at Queen's, but will occasionally make trips to other mathematics departments.

5. Ferruccio Orecchia , Genova, and Alain Bouvier, University of Lyon, whose fields are commutative algebra, will be with the Department during the first term.

6. John Randall - University of Warwick, will be with us for the year working on singularity theory with Leo Jonker.

7. Gwyn Evans - from Aberystwyth is the Visiting Senior Statistician with Statlab this term.

8. Vikki French , the University of North Carolina,is a postdoctoral visitor in statistics who will be with us for the year.

9. Takashi Agoh - is from the Science University of Tokyo, an institution about the size of Queen's devoted to science and technology. He studied with Taro Morishima and is a specialist in diophantine equations, class numbers; he has also worked on Fermat's last theorem and Bernoulli numbers. While at Queen's he is working with Paulo Ribenboim.

10. Wilhelm von Waldenfels of the Institute for Applied Mathematics at the University of Heidelberg, Germany, is visiting Queen's for the academic year 1979-80. He is investigating some connections between probability theory and quantum theory.

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The 28th Summer Research Institute of the American Mathematical Society will be held at Queen's from July 14 - August 1, 1980. It will be devoted to operator algebras and their applications to such subjects as group representations, physics, eigodic theory, and foliated manifolds. The chairman of the organizing committee is Professor R.V. Kadison of the University of Pennsylvania. Professor E.J. Woods of the Queen' Mathematics and Statistics Department will provide the local liaison. An attendance of 250-500 mathematicians from perhaps 15 countries is anticipated.

LETTERS

We were pleased to receive so many letters in response to the first issue of the "Communicator". They contained lots of news from old friends and graduates, and many suggestions for future issues.

Here are a few excerpts:

I have recently been preparing a senior academic high school course which focuses on current applications and trends in mathematics, as compared to the largely "pure" mathematics courses which have been traditionally offered to students. My work has made painfully obvious to me how out of touch many of us in the high schools can become with "real" and "new" mathematics. The "Queen's Mathematical Communicator" has great potential to help remedy this situation.

Yours sincerely,

Peter Wright
Pure and Applied Sciences-
Mathematics
Queen Elizabeth Park School
2301 Yolanda Drive
Oakville, Ontario
L6L 2H9

I am currently the Head of the Mathematics Department at Fort William Collegiate in Thunder Bay. We are a small school (700 students) but have produced some very talented math graduates.

I enjoyed Ron Horn's article and would appreciate more articles in this vane. Math applications are very important but information about them is not really available to me at the high school level. I would like to see a future issue devote some time to mathematics and the micro processor.

Yours sincerely,

Russ Garrett
1431 Cuthbertson Place
Thunder Bay, Ontario
P7E 5L3

While pursuing my academic career I found that my 'Course J' background was very relevant and gave me an excellent background. The conceptual understanding of control systems, the pragmatic influence of the electrical engineering courses and labs and the analytical skills developed during the math courses were all useful and have continued to be useful into my present job.

I currently have a 1977 Course J graduate, Bruce Giles, working in my section. He has adapted very well to the industrial environment and has demonstrated a good grasp of computer application and control concepts.

C. Whitfield
139 Laura Street
Sault Ste. Marie, Ontario

[Charles Whitfield graduated from the Mathematics and Engineering program in 1968. After completing his M.Sc. in mathematics at Queen's he did a Ph.D. in Systems Engineering at Case Western Reserve University. His thesis topic was "On the Role of Co-ordination in On-line Control: An Application to Furnace Temperature Control". He worked for Goodyear Tire and Rubber for a year and then moved to Algoma Steel in 1974. For the last two years he has been supervisor of a group charged with the on-going support and optimization of the process computers in the Rolling Mills Area at Algoma Steel].

CONFERENCE ON RECENT PROGRESS ON NUMBER THEORY

From July 2 to 20, 1979, the Queen's Mathematics and Statistics Department was host to an international conference on Recent Progress in Number Theory. About 125 mathematicians were in attendance, including more than 50 from the U.S., 30 from France, 25 from Canada, and about 20 from other countries including Brazil, the Netherlands, Germany, Australia and Yugoslavia. The focus in the first two weeks of the conference was provided by three series of ten lectures each: John Coates on p-adic L-functions, Hugh Montgomery on the Riemann Hypothesis, and Michel Waldschmidt on Transcendental Numbers. All three of these speakers are well-known as leading experts in their respective fields, and their lectures gave an up-to-date survey of current research and important open problems. The third week was highlighted by a series of five lectures on Sieve Methods by Enrico Bombieri, a recent winner of the Fields Medal, the equivalent in Mathematics of the Nobel prize in other sciences. There were also, in the third week, about 20 additional hour lectures presenting recent research in a wide variety of fields within Number Theory. There were, as well, numerous informal research contacts among the participants throughout the conference; such exchanges of ideas are the lifeblood of ongoing work in fields, like Number Theory, in which there is enormous current research activity.

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There is a new largest known prime, found last October by two California high-school students who announced their discovery at the West Coast Number Theory Conference held last Christmas.

It is

$$2^{21701} - 1$$

and consists of 6533 decimal digits.

ALAIN CONNES RECEIVES HONORARY DEGREE FROM QUEEN'S

At the afternoon convocation on June 2, 1979 the degree of Doctor of Laws (honoris course) was awarded to the French mathematician Alain Connes. The following citation was written by A. J. Coleman and E.J. Woods.

Alain Connes was born and raised in the south of France, received his university education in Paris, and now holds a position at the University of Paris. At the age of 28, he became world-famous by solving in a very beautiful and constructive manner a longstanding problem related to the central issue of the classification of operator algebras. In achieving this, he manifested great courage to persevere and brilliance as his mind soared among recondite abstractions whose complex intertwinings few of us can imagine. The significance of this work was quickly recognized by the International Mathematical Union and by the French Academy of Science. His great theorem was obtained while he visited Queen's in 1974-75 when, under the enlightened policies of France, he was permitted to discharge his military service by bring French culture to the barren shores of Lake Ontario. Together with contributions from other mathematicians, the work of Alain Connes has brought order and structure to an area where previously there had been chaos and confusion.

His more recent work relating operator algebras to the geometrical objects known as foliated manifolds indicates his ability to create new and unforeseen relationships.

Alain Connes, now in his 32nd year, is, according to our records, the youngest person on whom Queen's has bestowed an Honourary Degree. We honour him not only for his past achievements, we honour him for the great potential we see in him for his future impact on mathematics and science, but above all we honour Alain Connes as a man of modest and generous spirit who is a heart-warming exemplar of the high culture which France has contributed to Canada and the world.

PROBLEM SECTION

by

Peter Taylor

The following problem was posed in the last issue.

Problem No. 1

Suppose we have p standard coins, all identical except that one has a different weight from the others. We wish to use a simple balance to find the non-standard coin. Assume we have an inexhaustible supply of standard coins which we can use in the weighing.

Let $Q(n)$ be the maximum value of p for which the problem can be solved in n weighings.

To get a feeling for the problem discover that

$$Q(1) = 2$$
$$Q(2) = 5$$
$$Q(3) = 14 .$$

Find $Q(n)$ and prove that it holds.

Can you think of any other, more difficult versions of this problem?

Solution (by P.J. Cahen, Tunisia)

Consider first the simpler problem in which we know the odd coin is heavier (or in which we know it is lighter). Let $Q_1(n)$ be the maximum number for this problem. It is easy to see by induction (make three equal parts and compare two of them) that $Q_1(n) = 3^n$. Now consider the original problem. Take the $Q(n)$ coins. Set aside $Q(n-1)$ of them. (Clearly for the first weighing we cannot set asside any more than this.) The remaining coins can be weighed against the same number of standard coins. If we get a balance we are reduced to the original problem with $Q(n-1)$ coins. If we get an imbalance we are reduced to the simpler problem with $n-1$ weighings. In this case we can have no more than 3^{n-1} coins. Thus

$$Q(n) = Q(n-1) + 3^{n-1} .$$

The solution of this recurrance is $Q(n) = (3^n+1)/z$.

Comments.

Cahen's solution is neat and correct except that the underlined statement requires a bit of justification. The 3^{n-1} coins we use in our first weighing need not have been weighed against 3^{n-1} standard coins. They could have been divided up between the two pans. In this case the result, in case of an imbalance, would be to label some H and some L , so that an H coin is either normal or heavy, and an L coin either normal or light. Our simpler problem really ought to concern the case where there is one abnormal coin, but each coin is labelled either H or L . Call the function for this problem $Q_2(n)$. It is still true that $Q_2(n) = 3^n$, but the argument is a bit more complicated than for the Q_1 problem.

A correct solution was also received from Wendy Grava (nee Ramsay) of Southhampton, Ontario. Both Cahen and Grava pose the variant in which no standard coins are available and Cahen gives the formula $Q_3(n) = (3^n-1)/2$ for this problem. The problem is more difficult but can be solved in a similar way. The Q_2 function without standard coins is rather more difficult to find. Cahen also proposes that we are required to determine whether the odd coin is light or heavy (without standard coins), and provides the formula $Q_4(n) = [(3^n+1)/2] - n$ for this problem.

Partial solutions we submitted by Joseph Hagge of Milton, Ontario and G. Boyle of Ottawa. We are very pleased to get these solutions and hope to get even more to the following problem.

Problem No. 2

The Iterated Mother Problem

Let X be the set of all n-vectors whose entries are non-negative integers. Define the transformation T from X to itself as follows

$$T(x_1,x_2,\ldots,x_n) = (|x_1-x_2|,|x_2-x_3|,\ldots,|x_{n-1}-x_n|,|x_n-x_1|)$$

Here is an example for $n = 5$.

$$\begin{aligned} x &= 4 \quad 7 \quad 8 \quad 1 \quad 4 \\ Tx &= 3 \quad 1 \quad 7 \quad 3 \quad 0 \\ T^2x &= 2 \quad 6 \quad 4 \quad 3 \quad 3 \\ T^3x &= 4 \quad 2 \quad 1 \quad 0 \quad 1 \end{aligned}$$

What happens, in general, if T is iterated indefinitely? I am especially interested in receiving any concise proofs. I am indebted to Joel Hillel of Concordia University for this nice problem.

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The Mathematics and Engineering students are planning quite a field trip this year. Not content with the usual excursion to Toronto or Montreal, they hope to spend a week in Florida over the New Year. While there, they have been invited to Disney World and the Harris Corporation. Harris is a large corporation with interests in most fields of engineering. They fabricate semiconductors, design satellite systems, as well as engage in research in almost every field. Walt Disney world, as well as being the world's foremost amusement park, is the first city which was completely designed before it was built. It embodied state of the art technology, and is a fascinating engineering project. They have been invited for "behind the scenes" tours at both locations.