

QUEEN'S MATHEMATICAL COMMUNICATOR

June
1984



QUEEN'S PUTNAM TEAM

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Queen's Mathematical Communicator

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Cover Picture. The Queen's Putnam Team for 1983 who placed eighth in the 44th Competition. From left to right, Michael Swain, Neale Ginsburg, and Teddy Hsu. Story on page 8. Photo by Ken Robinson of Flair Photographic, Kingston.

Research Award to Paulo Ribenboim

Professor Paulo Ribenboim of the Department of Mathematics and Statistics at Queen's University was one of two recipients of a 1983 Queen's University Prize for Excellence in Research. The award is valued at \$1,000 and was presented by Chancellor Agnes Benidickson at the Fall Convocation on October 29, 1983. The following citation was read.



"An outstanding researcher, a clear expositor and an influential teacher, Paulo Ribenboim personifies the best qualities of a mathematician. His early extensive contributions to the theory of valuations established him as an authority in this area. Now his impressive list of publications covers almost every branch of algebra. His mathematical taste, both in his choice and solutions to problems, is a hallmark of his work. His many books, including Ring and Modules and L'Arithmetique des Corps do not adorn shelves; they are well worn from use by students and experts alike. He is now involved in the monumental project of giving an encyclopaedic account of past and current work on Fermat's Last Theorem, a tantalizing problem dating from the 17th century. Attempts to solve this problem have created whole new mathematical subjects. Ribenboim's first volume on this topic has recently been acclaimed as a work of art - a masterpiece of historical erudition and lucid style. Without fanfare, he has directed the activities of the Algebra Group at Queen's and supported his colleagues by warm interest in their research. His organization of five major conferences, his invited lectures at over ninety universities, his election as a Fellow of the Royal Society of Canada and his honorary degree from the Université de Caen are testimony to the respect and admiration which his colleagues, students and many friends feel for this extraordinary person."

One of the duties of each prize winner is to give a public lecture. Professor Ribenboim's talk is presented on the following pages.

OAME Conference Held at Queen's

The 11th Annual Conference of the Ontario Association for Mathematics Education was held at Queen's from May 10 - 12, 1984. Some 800 math teachers from all over Ontario came to participate in a full and exciting program. Topics presented included the role of computers, curriculum change and problem solving. The Queen's people giving talks included Norm Rice, Doug Dillon, Dale Burnett, Bruce Kirby, Leo Jonker, David Gregory, Hugh Allen, Jim Whitley, and Peter Taylor. **John Coleman** gave the Keynote address, in Grant Hall entitled Bicycles, Mathematics and Computers. At the banquet on Friday night, recognition for distinguished service was accorded John Coleman, Norman Miller, Emeritus Professor in the Department (now 94), and Jim Boyd, a long-time teacher at LCVI and close friend of Queen's.

Fermat's Last Theorem: still unproven after 347 years.

Lecture of Paulo Ribenboim at Queen's University
on January 25, 1984

It is with pleasure that I address my colleagues and the general public on the subject of my own mathematics research. I must begin with a warning - mathematics is often quite difficult to explain to non-specialists, even to mathematicians in other disciplines. So, instead of trying to do what would be appropriate only for a series of specialized seminars, I will give here the motivating background.

Science develops when man is confronted with difficult problems. In mathematics, such problems often have the following character:

- the statement is easy to understand, and
- the methods invented to attack the problem turn out to have wide and surprising applications to diverse domains.

Such is the so-called Fermat's Problem, or Fermat's Last Theorem.

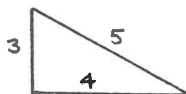
I begin with a plan of the lecture.

- (1) Statement of the problem
- (2) Sizing up the difficulty involved
- (3) Early attempts
- (4) The fundamental work of Kummer
- (5) Modern developments and the computer.

(1) Statement of the Problem

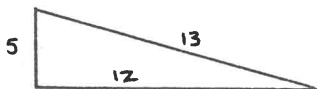
It was known since ancient times (perhaps before the Greeks) that there exist right-angled triangles with sizes measured by integers:

$$3^2 + 4^2 = 5^2$$



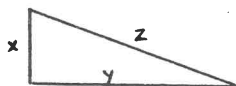
also

$$5^2 + 12^2 = 13^2$$



By the theorem of Pythagoras, the sizes x , y , z must satisfy the relation

$$x^2 + y^2 = z^2$$



So the problem here is to find all solutions in integers of the above equation. Such triples (x, y, z) are called Pythagorean Triples.

It is not hard to show that in any solution, either x or y must be even; say y is even. We may also take x , y and z to be relatively prime (take out all common factors). Then it can be shown that all solutions of the equation are obtained as follows:

Take integers a , b , with no common factor and $a > b > 0$.

Then
$$\begin{aligned} x &= a^2 - b^2 \\ y &= 2ab \end{aligned}$$

$$z = a^2 + b^2$$

is a solution and all solutions are of this form.

Diophantus of Alexandria (250 AD) considered similar problems of finding solutions in integers to certain equations, now called diophantine equations. He wrote six books in arithmetics. Bachet, in 1621, translated Diophantus into French.

Now I come to Pierre de Fermat (1601-1665), Conseiller à la Cour de Justice de Toulouse (magistrate). Fermat studied Bachet's translation. In those days, exchange of scientific information was by letters between scientists. There were no scientific journals. Fermat corresponded with Pascal, Descartes, Roberval, Mersenne, Wallis, Brouncker, Carcavi, etc. He was, indeed, a true genius. He was one of the creators of differential and integral calculus (with Newton and Leibniz), and he advanced probability theory, geometric optics, and above all, number theory. Pascal wrote to Fermat: "Je vous tiens pour le plus grand Géomètre de toute l'Europe".

Two of Fermat's discoveries in number theory are

- (i) The equation $X^2 - DY^2 = 1$ ($D > 0$) has infinitely many solutions in integers; and
- (ii) If p is an odd prime then $p \equiv 1 \pmod{4}$, if and only if p is the sum of two squares. Thus

$$13 = 4 + 9$$

$$29 = 4 + 25$$

but the prime $p = 7$ is $\equiv 3 \pmod{4}$ and is not the sum of two squares.

An interesting problem posed by Fermat concerns the Fermat numbers:

$F_n = 2^{2^n} + 1$. Thus $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$, etc.

F_5 has about 10 digits, and F_6 has about 20 digits, etc.

Fermat asserted all such numbers are prime! This is true for F_0, \dots, F_4 , but the case of F_5 needed a table of primes up to $\sim 100,000$ which did not exist at the time. However, 100 years later Euler (1738) showed F_5 is divisible by 641. Since then many other F_n have been found to be composite, and no others have been found to be prime. The numbers F_n get large very quickly. For example F_{17} has about 300,000 digits.

Finally, I turn to Fermat's Last Theorem.

Around 1636 (?) while studying the Diophantus translation, Fermat wrote:

"A sum of two cubes cannot be a cube,
A sum of two biquadrates (4th powers) cannot be
a biquadrate, and more generally
A sum of two n th powers cannot be a n th power.
I have a marvelous proof of this fact, but
the margin is too small to contain it."

Later Fermat proposed as a problem his assertion about cubes and biquadrates. This problem (for all $n > 4$) is still open after 347 years. Fermat wrote a proof for biquadrates, but did not write a proof for cubes and never again mentioned the statement for $n > 4$. This assertion of his has been called "Fermat's Last Theorem" since all the assertions of Fermat (except the one on Fermat numbers) have been settled. This is the last one.

(2) Sizing up the difficulty involved

Suppose we are given the exponent $n > 2$. First, I point out that we cannot write up all the n th powers of integers and add them up two by two and see whether we get an n th power. That is a never-ending procedure, which also has to be repeated for all other n . So we must proceed by contradiction. Assume there are hypothetical numbers $x, y, z > 0$ for which $x^n + y^n = z^n$. From this hypothesis, derive by purely logical considerations some consequences which may be shown to be absurd - by contradicting some known fact. Sometimes we can derive consequences which are absurd only for certain types of numbers.

Let me remark at this point that we can restrict attention to the exponents $n = 4$ and n prime. Indeed if $n = ml$ ($1 < m, l < n$), and if there are $x, y, z \neq 0$ such that $x^n + y^n = z^n$ then $(x^l)^m + (y^l)^m = (z^l)^m$ which gives us an example for the factor m . Repeating this process, we arrive at exponents 4 or prime.

(3) Early attempts

Exponent 4.

Fermat gave the proof for $n = 4$ with a truly marvelous method, "the method of infinite descent". The idea is this. Fermat produced an algorithm which converts any positive integral solution (x, y, z) into a new positive integral solution (x', y', z') with $z' < z$. The process is repeated to get a sequence of solutions with $z > z' > z'' > \dots > 0$. Since all the z values are integers > 0 , we get a contradiction.

Exponent 3

Euler (1770) provided a solution for $n = 3$ with the method of infinite descent in his famous book "Algebra". There was an interesting controversy around a gap in his reasoning which took many years to clarify.

Exponent 5

This case was settled by Legendre and Dirichlet (1825/8) in competition. At that time there was great activity in Paris over this problem. The question was put to a prize (Gold Medal) by the French Academy of Sciences.

Exponent 7

This was settled by Lamé (1839). Proofs were becoming increasingly more complex and long, and it did not seem to be getting any clearer how to attack the case of an arbitrary prime exponent p .

Then there arrived on the scene the first woman mathematician of high

calibre, Sophie Germain (early 1800's), alias Monsieur le Blanc. She maintained correspondence with Gauss and Legendre, but, because she was a woman, could not publish her results at the Academy. Legendre wrote: "Sophie Germain, with a very ingenious new method, proved in a 'trait de plume', the 1st case of Fermat's Last Theorem for all exponents $p < 100$. By way of explanation, Fermat's Last Theorem had come to be divided into two cases, case 1 in which p does not divide into any of x , y and z , and case 2 in which p divides one of these numbers.

Sophie Germain's method used previous results of Abel, a Norwegian genius who died at age 27. Abel showed that if we have case 1 and $x^p + y^p = z^p$ then $x+y$, $z-x$ and $z-y$ must all be p th powers. It follows that $(x^p + y^p)/(x+y)$ is a p th power, and similarly for $(z^p - x^p)/(z-x)$ and $(z^p - y^p)/(z-y)$. These are quite stringent conditions and allowed Sophie Germain (with an extension by Legendre) to show Fermat's Last Theorem holds in case 1 for p prime provided any of the numbers $z^{kp} + 1$ for $k = 1, 2, 4, 5, 7, 8$ is prime. It's not hard to verify that this condition holds for any prime $p < 100$, and the result follows.

(4) The fundamental work of Kummer

In 1847 Lamé published a proof of Fermat's Last Theorem for all exponents n , thus solving the problem. Alas! his arguments were fallacious. Liouville, Dirichlet and Kummer criticized his proof. Poor Lamé - after many attempts to repair his arguments he had to give up. His mistake was profound, and revealed a fundamental difficulty and the need for deeper understanding of numbers. Fermat's problem was becoming even more interesting.

The work of Kummer was monumental. His starting idea is simple. If there are integers x, y, z for which $x^p + y^p = z^p$, then

$$x^p = z^p - y^p.$$

How are we to compare these? On the left we have a product (a power) and on the right we have a difference!

Kummer's idea is to change the difference into a product, which can be done by using the complex numbers ζ , the p th roots of 1, used by Gauss in the division of the circle:

$$\zeta = \cos \frac{2\pi}{p} + i \sin \frac{2\pi}{p}.$$

Then $x^p = z^p - y^p = \prod_{j=0}^{p-1} (z - \zeta^j y)$. The numbers $(z - \zeta^j y)$ appearing in the

product on the right are not integers in the usual sense but let's treat them as if they were. If they have no common factor, then each one should be a p th power in some sense, and this gives us a strong condition to work with.

To justify such arguments, one needs a theory of number derived from the division of the circle. This can be done and leads to the definition of cyclotomic numbers and cyclotomic integers. Of course, we also need

the cyclotomic integers to be, in unique way, products of prime cyclotomic integers. However, this is sometimes, but not always true. For example, it fails for $p = 23$.

To rectify the situation, Kummer had the striking idea of introducing "ideal numbers". There is an interesting comparison here with chemistry, in which cyclotomic numbers correspond, for example, to the "fluoride radical", and ideal numbers correspond to the element "fluoride" - at that time not yet isolated. Using ideal numbers, it is now true that every ideal number is, in a unique way, the product of prime ideal numbers.

This new arithmetic is a bit similar to ordinary arithmetic. There are two important deviations.

(i) There may be ideal numbers which are not identifiable with actual cyclotomic numbers. But essentially there are only a finite number h_p of classes of ideal numbers. This number h_p is called the class number of p .

(ii) There are cyclotomic integers $\neq 1$ which divide 1 like ζ, ζ^2, \dots

and also

$$\sqrt{\frac{1-\zeta^k}{1-\zeta} \times \frac{1-\zeta^{-k}}{1-\zeta^{-1}}}, \text{ etc.}$$

Kummer proved his main theorem in 1847. A regular prime is a prime p which does not divide its class number h_p . Kummer showed that Fermat's Last Theorem holds for all regular p .

Are all primes regular? No, for example, 37 and 59 are irregular. It is known, in fact, that there are infinitely many irregular primes. But it is not known whether there are infinitely many regular primes. During his whole life Kummer worked hard on Fermat's Last Theorem for irregular exponents. He made extensive and amazing computations, and created the theory of cyclotomic numbers.

(5) Modern developments

One thing leads to another; one unsolved problem yields ten! This is how mathematics progresses. To better understand cyclotomic numbers, we were led to the theory of algebraic numbers. To better understand these, we were led to class field theory.

I will mention four important modern developments.

(a) At the beginning of this century, Wieferich showed (using Kummer's theories) that if the 1st case fails for a prime p , then $2^{p-1} \equiv 1 \pmod{p^2}$. One year later Mirimanoff showed that p must also satisfy: $3^{p-1} \equiv 1 \pmod{p^2}$. Subsequent computations showed that no prime $p < 4800$ million has these two properties. We conclude that the 1st case of Fermat's Last Theorem holds for all $p < 4800$ million. The type of computations required for these methods are exceedingly long.

(b) With other criteria using Bernoulli numbers (from probability theory), Vandiver found a criterion which could be tested by computer. After a year of calculation on a monster IBM, Wagstaff obtained the result that Fermat's Last Theorem holds for all $p < 125000$.

(c) On the other hand, a careful theoretical analysis, shows that if Fermat's Last Theorem is false, i.e. if $x^p + y^p = z^p$ for some p , with $x < y$, then x must have at least 1,900,000 digits! Compare this with the number of nuclei in the universe, which has only 123 digits.

(d) The "theorem of the century" by Faltings, again a genius of 29 years, is that for every exponent n there can exist only finitely many x, y, z (without common factor) for which $x^n + y^n = z^n$. How many may in fact exist is not specified, nor is there any estimate given. These results are quite deep and I would need one year to explain them to the best equipped of people present here!

Let me close with a word about my own work. The problem of solving equations in integers leads to a study of algebraic numbers and prime ideal numbers. Tools of analysis are required: the so-called p -adic analysis and valuation theory. This is where my work began.

Queen's Eighth in Putnam Exam

The prestigious William Lowell Putnam Mathematical Competition is held every year in December. Candidates sit in two 3 hour sessions and attempt to solve 12 problems. The 44th competition, in 1983, had teams from 256 colleges and universities in North America. Queen's placed eighth. Members of the team were **Neale Ginsburg**, a first year math student, **Michael Swain**, in fourth year math and physics, and **Teddy Hsu**, in fourth year physics. Neale was one of the five-man Canadian Team at the International Math Olympiad in Paris last year. Michael leaves Queen's to pursue a Ph.D. in Computer Science at the University of Rochester, and Teddy plans to begin a Ph.D. in Physics at Princeton.

The first five teams in the competition were (in rank order): Cal. Tech, Washington Univ. (St. Louis), Waterloo, Princeton, and Chicago. The next five (in alphabetical order) were Alberta, Harvard, Memorial (Nfld.), Queen's, and Yale. It is noteworthy that 4 of the top 10 are Canadian universities.

Previous to 1983, Queen's record in the Putnam Competition is one first (1952), one third, one fourth, and one honorable mention (6th-10th).

In individual results, **Teddy Hsu** ranked 20 1/2 out of 2055 contestants. One Canadian, David Ash, from Waterloo University, stood in the top 5.

The coaches of the Queen's team are **Leo Jonker** and **Peter Taylor**.

Robert Richard Dingle Kemp (1932 - 1984)



Professor Robert Kemp of the Department of Mathematics and Statistics at Queen's University died in January after a brief illness.

Doctor Kemp was awarded the Bachelor of Arts degree in mathematics by McMaster University in 1953 and the Doctor of Philosophy degree by the Massachusetts Institute of Technology in 1956. After further work at M.I.T. and at the Institute for Advanced Study in Princeton, he joined Queen's in 1958. Except for a sabbatical leave at Imperial College in London, he has been at Queen's since then.

While at Queen's, he continued the research into differential equations and differential operators which he had begun in his Ph.D. studies and he lectured regularly to students in the Faculties of Arts and Science and Applied Science. He also contributed very substantially to the operation of the university. He was for many years Chairman for Graduate Studies in the Department. He was a member of Senate from 1972 to 1975; he served a term as Chairman of Division IV of the School of Graduate Studies and Research; he was President of the Faculty Club; and he was a member of several Faculty and University committees. On the national scene, he was at one time a member of the council of the Canadian Mathematical Congress.

His students will remember him as a teacher who set high standards for them. His colleagues will remember him as a researcher who set high standards for himself and also as a source of much information about analysis.

Outside the university, Bob Kemp was probably best known as a first class bridge player. He was among the best in Kingston and did very well in national competitions.

We have lost a valuable colleague whose early and unexpected death has shocked the Department. Our sympathy goes out to his wife and daughters.

Adelaide hall - Symbols in Stone

How many have cast their eyes up when entering or passing by Adelaide Hall and seen the inscription

$$e^{i\pi} = -1$$

carved above the entrance? Is this the math building? No, it's a women's residence. How curious.

At the time Adelaide Hall was begun (1951) the Dean of Women at Queen's was **Dr. A. Vibert Douglas**, now Emeritus Professor in the Department of Mathematics and Statistics. Let me quote from an article written by Dr. Douglas in 1961, with the title I have reproduced above.

'When plans for the extension to Ban Righ Hall were nearing completion it became apparent from the architect's drawings that five large rectangular and five smaller square stones were to be carved and placed above the first and second bay windows immediately over the arch of the entrance at the corner of University and Stuart Streets. Instead of conventional geometric and leaf designs, it seemed appropriate that these ten stones should convey in symbolic form some of the ideas and the ideals most closely associated with a university whose traditions are those of learning and of religion, a university whose traditions are those of Sapientia et doctrina stabilitas, words based upon a verse in Isaiah 33: "Knowledge and wisdom shall be the stability of thy times." The verse which precedes this reads: "The Lord is exalted, He dwelleth on high, He hath filled Zion with judgment and righteousness."

Dr. Douglas then goes on to describe each of the stones. In particular:

'The historian Esdras relates a contest which took place in the court of Darius, King of Persia, when the young men in attendance upon the king competed as to which of them could say the wisest thing. One said, "Great is truth and mightily above all things....it endureth and is always strong, it liveth and conquereth forever more", and King Darius gave judgment, "Thou are found wisest". How best can truth be symbolized? Not relative truth but absolute truth, independent of epoch and the relativity of human affairs? The marvel and the mystery of numbers, the beauty of the mathematical relationships between numbers, between even irrational and imaginary numbers, can be illustrated by the following equation $e^{i\pi} = -1$. The letter e represents an infinite series whose sum is an irrational number, the base of Naperian logarithms; i is the imaginary number whose square is equal to -1 ; and π is the well known ratio of the circumference of a circle to its diameter in Euclidean geometry. This equation is carved on the central stone on the upper row. There is sublimity in this amazing relationship which the mind of man has discovered but did not invent - it partakes of the nature of absolute truth and the more one thinks about it the more one feels something of that awe which Moses felt when he seemed to hear a voice saying, Take off thy shoes from off thy feet for the ground on which thou standest is holy ground.'

Two New Positions for the Department

Since 1972 Queen's University has been pursuing a conservative policy with respect to hiring new faculty. This is part of the university's larger decision to maintain enrolment at a fixed level of about 11,000 students. The Department of Mathematics and Statistics has experienced some decline in numbers under this regime, but the untimely death of Professor Kemp this January and the early retirement of Professor Caradus (to take up full time work with the Anglican Church) at the end of the current academic year have prompted the Faculty of Arts and Science to grant us two new positions. Though we are not quite back in an era of expansion, we do have the opportunity to try out some new procedures for making appointments.

Naturally these new appointments are seen by members of the department as having tremendous significance. They open a multitude of possibilities reflecting the variety of opinions you'd expect in a department that is diverse and large. For example, we could make the appointment in a field in which we are deficient, or in an area in which the department already has strength. We could hire a promising young person or a person with a well-established reputation. Some are keen to hire a woman, since at present there are only two in a department of size 45. On all these questions, the department does not speak with one voice.

One point of agreement is our decision to designate one position for mathematics and one for statistics. The new procedures alluded to above, call for the final decision to be made by an Appointments Committee consisting of the Head, various members of staff and three members of our student body, one in graduate studies, one in Engineering Math, and one in Arts.

The Committee's tasks are to seek and evaluate applicants, and make a recommendation to the Dean of Arts and Science. The Committee could recommend that no appointment be made. In this case, the positions will be used for nonrenewable appointments and the procedure set in motion again next year or even later.

Normally, advertising for the following year would begin in September or October. In spite of the fact that our ads only appeared in late February, we have received about fifty applications. Throughout the process the Committee has had several open meetings with the department to seek advice.

On May 8, the Committee met to conclude its deliberations. For the position in mathematics it recommended the appointment of **Dr. H.E.A. (Eddy) Campbell**. Dr. Campbell, who received his Ph.D. from the University of Toronto in 1981, has been an Adjunct Assistant Professor at Queen's since September 1983. His teaching has been praised by students and his expertise in Algebraic Topology is welcomed by his colleagues. For the position in statistics the recommendation was for no appointment at this time.

Although the work of this Appointments Committee is finished, the need to consider new applications has caused the Department to focus more sharply on its future. Discussion of a number of issues has taken on a more urgent character. Such discussion will continue in a lively manner during the rest of the 1980's.

Grace Orzech

News

Professor Tony Geramita was invited to spend a week at Bucknell University, Lewisburg, Pennsylvania, as a Distinguished Visiting Professor. He was there from April 8 - 14, gave 2 lectures on the area of his research, and consulted, on an informal basis, with members of staff.

The Curves Seminar which Tony has organized now for four years had a very active fall time. There were visitors to it from around the world including: Prof. S. Greco, Politecnico di Torino, who was here as an NSERC exchange scholar for seven months, Prof. N. Chiarli, Politecnico di Torino, Prof. E. Davis, S.U.N.Y. at Albany, Prof. B. Singh, Tata Institute for Fundamental Research, Bombay, Prof. M. Boratynski, Academy of Sciences, Warsaw, Poland, Prof. A. Bouvier, Université de Lyon, and Prof. G. Paxia, Catania. The research of these people and of L. Roberts and A. Geramita, at Queen's, have common threads, and by being together and talking about such work, we understand better where our research can go. It was an exciting time for these people and for the two graduate students, Anna Lorenzini and Amar Sodhi, who are also working in the field.

The announcement of the early retirement of Sel Caradus does not mean (happily) that we'll lose him immediately. He plans to spend the next four years as Adjunct Professor at Queen's and continue, as at present, to teach part time in the Department and to serve as Queen's Anglican Chaplain. He's not committing himself past that, (at least not to us) but his long-term plans certainly include a growing involvement with the Church and a desire for new challenges.

We received a letter from Brian Manning (B.Sc. 1975) who has become Head of Mathematics at Adam Scott CVI in Peterborough. He poses the following question.

"There are numerous examples where the graphs of $y = \sin x$ (and $y = \cos x$) occur as natural phenomena. Is there any phenomenon in which the graph of $y = \tan x$ occurs? (or $y = \csc x$, x , $y = \sec x$)"

"My first reaction to the question was that such occurrence was unlikely due to the discontinuous nature of the tangent function, but there are discontinuities in nature. Of course, we are aware of numerous applications of the tangent function in mathematics, but we are looking specifically for the graph of $y = \tan x$."

We were not much help to Brian. In fact we couldn't think of a thing. Can you?

* * *

Figure it Out "I hear your class donated \$10.50 to the charity drive."

"Yes, some students gave 35¢, some gave 50¢, and some gave nothing."

"How many students are there in your class?"

"If I told you, you would be able to figure out how many gave each amount."

"I see. Well would you tell me whether there is an odd or even number of students?"

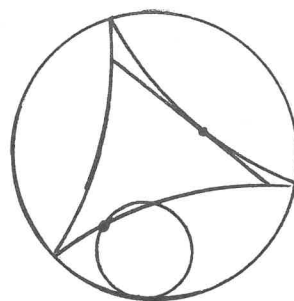
"No, if I told you, you could figure out how many gave each amount."

So how big is the class, and how many gave each amount?

Norm Rice

The Kakeya-Besicovitch Needle Problem.

The problem concerns subsets S of the plane in which a needle of unit length can be rotated through 180° . For example S could be taken to be a circle of radius $1/2$ or an equilateral triangle with altitude 1. The circle has area $\pi/4$ ($\approx .785$) and the triangle has area $1/\sqrt{3}$ ($\approx .58$). The problem is to find such a set S with minimum area.



A rather interesting example of an S is what is called the three-cusped hypocycloid (drawn above). Take a circle of radius r and run it around inside a circle of radius $3r$. A fixed point on the small circle generates the hypocycloid. If $r = 1/4$, then the needle can be rotated inside the hypocycloid with both ends constantly in touch with the boundary. It turns out that this S has area $\pi/8$ ($\approx .3925$). Kakeya conjectured in 1917 that no S could have area smaller than this.

He was wrong. In 1927 Besicovitch produced the remarkable result that sets S of arbitrarily small area could be found. Besicovitch's sets are like trees with long skinny branches and the needle in turning must frequently slide back and forth between trunk and branch.

But let me propose a variant. Suppose the set S is required to be convex. What then is the minimum area? Of the above examples, the circle and the triangle are convex, so $1/\sqrt{3}$ is the best candidate so far. What more can you say?

I am grateful to Tony Geramita for bringing the original needle problem to my attention.

Solution to Last Problem (October 1983)

The last problem was about the lovely game of Wythoff. To discover how to play the game, it turns out one must "get ahold" of the sequence (x_n, y_n) of losing positions, the first few terms of which follow.

n	(x_n, y_n)	n	(x_n, y_n)	n	(x_n, y_n)
0	(0,0)	6	(9,15)	12	(19,31)
1	(1,2)	7	(11,18)	13	(21,34)
2	(3,5)	8	(12,20)	14	(22,36)
3	(4,7)	9	(14,23)	15	(24,39)
4	(6,10)	10	(16,26)	16	(25,41)
5	(8,13)	11	(17,28)	17	(27,44)

That is, one must find a formula for x_n in terms of n (then $y_n = x_n + n$). A clue comes from noticing the close relationship of the above sequence to Fibonacci numbers. Indeed the answer is that

$$x_n = [\tau n] \quad \text{and} \quad y_n = [\tau^2 n]$$

where $\tau = 1.6180\dots$ is the positive root of $x^2 - x - 1 = 0$. A nice reference to these ideas (and many others) is the paperback, *Mathematical Recreations and Essay*, by W.W.R. Ball and H.S.M. Coxeter, 12th Edition, University of Toronto Press, 1974.

Microcomputers for Queen's Engineering

The computing facilities at Queen's have been evolving for over 20 years. In 1961, Queen's first "mainframe" computer -- an IBM 1620 -- was installed in Ellis Hall. This computer has since been replaced by computers of increasing capacity. Our present configuration of twin IBM 4341's acquired two and one-half years ago are to be replaced by an IBM model 3081 this summer. The accelerating demand for computing power has been partially a result of the increasing availability of easily usable software -- applications programs of many different types -- in many different areas. Not only is the scientific software more sophisticated; word processing, for example, now makes the writing of a report a far simpler process.

There has been a parallel evolution of instruction in our academic programs. Within the Faculty of Engineering, computer programming was introduced as a formal requirement in 1968, and taught by the Department of Engineering Drawing. This responsibility was fully assumed by the Department of Computing and Information Science in 1979. Until the present, all formal instruction in computer programming has been supported by mainframe computing power. However, a number of Departments within the Faculty have found it convenient to acquire microcomputers for the development and use by students of discipline oriented applications programs. Moreover, an increasing number of households have been acquiring personal computers for recreation and domestic applications. As a result many students entering Queen's have already become familiar with the skills required for using a microcomputer.

To take advantage of the past experience of these students, and to make computing a more stimulating experience for students in engineering, the Faculty of Applied Science has been investigating the opportunities that increased use of microcomputers might provide in the undergraduate program. A group of faculty members concerned with the future role of computing in the faculty initiated this project. Some of them visited Clarkson College, a U.S. institution which now requires that each entering undergraduate acquire a particular personal computer for use in most (if not all) subjects studied. Following a series of discussions and study of the concept, this group submitted a report to the Faculty of Applied Science. This report has since been reviewed, and two Committees have studied proposals for possible implementation.

As a result of the discussions and reports which have occurred during the past academic session, most members of the Faculty believe that a student who has immediate access to a computer at all hours of the day would have a substantial advantage in his or her academic program. First, a student could allocate time required for solving computer problems without worrying about access to the facilities. Secondly, students could easily use software provided, to perform drill exercises in other academic subjects. Thirdly, software could be developed for direct interface to engineering equipment provided on campus. This would allow much more realistic laboratory experiments to become part of the academic program. Reports would be written on and printed by the microcomputer. This would encourage editing of both content and grammar in reports for improved communication skills. Finally, the students would be encouraged to

explore the capabilities of a microcomputer through less formal and perhaps even recreational computing, and through this exercise become more inclined to use a computer when it is appropriate to do so.

In April, the Faculty Board approved a proposal to introduce the use of personal computers into the courses of the program in Applied Science. Each student entering the Faculty of Applied Science in the Fall of 1985 would be "strongly encouraged" to purchase a recommended personal computer. Through bulk purchase agreements with the supplier, these computers will be available to Queen's students at a very attractive discount.

In addition, the Faculty will supply clusters of personal computers for free use by Applied Science students who do not wish to purchase their own computer. The particular computer to be recommended will be announced by the summer of 1985; it will have a 16-bit processor, more than 100K bytes of memory, and one or two disk drives. Software to run FORTRAN, BASIC, a word processor, and a spreadsheet facility will also be provided, and the operating system will be MS/DOS.

Essentially, this microcomputer will provide each student with sufficient computing power to execute most of the computing he will need in a four-year program. Supplementary computing power will be provided on the new IBM computer for running large scale packages. During the years at Queen's each student will have the opportunity to acquire applications software developed for this computer by faculty and fellow students. It is expected that the experience a student gains on the microcomputer and the software developed and acquired at Queen's will provide each of Queen's engineers with a valuable resource to offer to society.

Jim Verner

Important message on back.
Detach and mail to:

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IF YOU WANT THE COMMUNICATOR TO KEEP COMING ...

In response to what is known as "the squeeze" Queen's, along with all other Ontario Universities has been cutting back its budget in nearly every category. At the beginning, a few years ago, this was, on the whole, beneficial, but I think it's fair to say that current cuts are in some way lowering the quality of undergraduate instruction. Needless to say, in this atmosphere, all non-teaching expenditures are closely scrutinized and questioned. So it is with the **Communicator**. It represents at the moment the largest non-teaching item in our budget; its costs are equally divided between printing and mailing. Currently we mail out some 1700 copies of each issue, 1100 to alumni, and 600 to schools.

If this magazine is of interest to its readers, we would like to continue it. But we are considering, at the very least, restricting its distribution to those who really want to receive it. In fact, to those who are sufficiently enthusiastic that they are prepared to take a moment to fill out and mail the tear-off portion below! If the response is favorable we will most likely continue to produce it, sending it to all high schools in Ontario and to all alumni who return a form.

So all **Communicator** supporters please fill out and return:

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