Queen's University

Math & Stats Newsletter

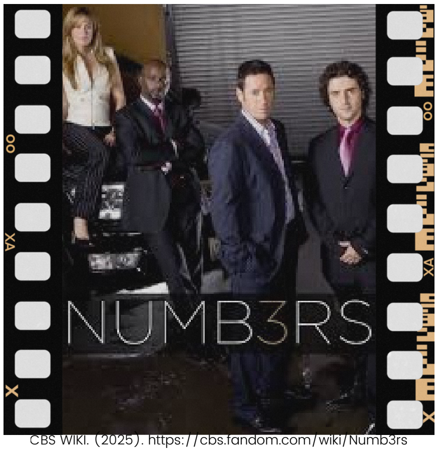
October

Quote of the Month

"The mathematician is fascinated with the marvelous beauty of the forms he constructs, and in their beauty he finds everlasting truth." - George Bernard Shaw

Math Meme of the Month:

Math Show Reccomendation



CBS Wiki. (2025). https://cbs.fandom.com/wiki/Numb3rs

NUMB3RS

This fictional procedural crime drama follows Don Epps, an FBI agent and Charlie Epps, his brother and a math professor at CalSci, who uses his knowledge of math and physics and their real world applications to help Don and FBI solve their cases, while also trying to avoid family drama.

Concept of the Month

Within this section of our newsletter, we want to write about aspects of math we find interesting. Wether we are writing of specific theorems, proofs, applications, or even the history of math, we wish to bring our love of the subject to a (relatively) broad audience.

Over the October and November newseltter's I am going to discuss two fixed point theorems, specifically, Banach's Fixed Point Theorem and Brouwer's fixed-point theorem. Although these theorem's are from two different field's of math (functional analysis and topology respectively), they follow a similar structure and gave me a similar sense of amazement when I learned them for the first time. This month I will focus on Banach's, and next month on Brouwer's (should people find this article semi-interesting).

Formally, **Banach's Fixed Point Theorem (1922)**:

Suppose that M is a closed nonempty subset of a Banach space $(X, \|\cdot\|)$. Let $F : M \to M$ be a contraction with contraction constant $K \in [0, 1)$. Then:

(a) There exists a unique fixed point for F, i.e. there is a unique $u \in M$ such that $F(u) = u$.

(b) For every $u_0 \in M$, the sequence $(u_n)_{n\geq 0} \subset M$ defined by

$$u_{n+1} = F(u_n), \quad n \geq 0,$$

converges to u (the unique fixed point from part (a)) with respect to the norm $\|\cdot\|$.

Where a function $F : X \to X$ is called a **contraction** if and only if there exists a constant $K \in [0, 1)$ such that

$$\|F(u) - F(v)\| \leq K\|u - v\|, \quad \text{for every } u, v \in X.$$

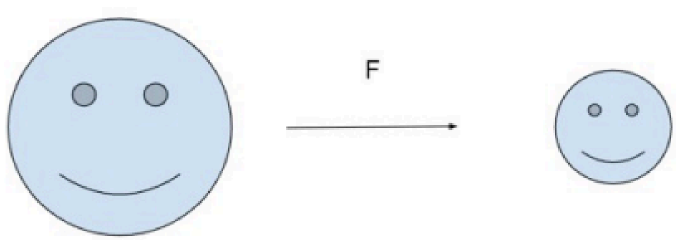
Additionally, we can use elements of $(u_n)_{n\geq 0}$ and our contraction constant K to get an arbitrarily good approximation of our fixed point u. Specifically: Specifically, we can use

$$\|u_{n+1} - u\| \leq \frac{K}{1-K}\|u_{n+1} - u_n\|$$

. Let's try to build some intuition for what this theorem is telling us. A Banach Space is a normed vector space (a norm gives a notion of size to a vector) in which Cauchy convergence of a sequence implies convergence. More simply, if elements of a sequence in a Banach space get arbitrarily close together, this sequence must converge to a limit. We can think of a Banach Space as a "nice" normed vector space like $(\mathbb{R}^n, \|\cdot\|_n)$. Where $\|\cdot\|_n$ is the euclidean norm:

$$x = (x_1, x_2, x_3, ...) \implies \|x\|_n = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Next, we can think of F as a function that pulls space together, sending input points closer together in the output.



Lastly, our subset M must be closed, i.e M contains it's limit points. We can draw some intuition by thinking of basic closed subsets in $\mathbb{R}$: $[a, b], a, b \in \mathbb{R}$ Our Theorem now tells us that F has some fixed point $u, F(u) = u$ and allows us to find a very accurate approximate of this u. Hopefully by now you have some preliminary understanding of the theorem, let's now look at some fun examples:

Within these photos, we can see we are working in a Banach Space, $(\mathbb{R}^2, \|\cdot\|_2)$, and we can think of M as some closed ball that entirely contains the largest drawing. Now our function F take our image and shrinks it onto itself (thus the smaller image is also in M). Therefore we see that F is a contraction from M to M. From here, our theorem tells us that when we apply F and shrink our image, some point remains in the exact same position after the transformation. If we continue to apply F (represented by the nested images) we get a nice visual representation of where our fixed point lies. I have highlighted it with a red dot. Another interesting thought experiment and application of our theorem is to imagine a "doll-house" model of your home sitting on the kitchen table in your home. Our theorem tells us that some point sits in the exact same spot of the doll-house as it sits in our real house. Think about this idea, what is our Banach Space? What is our function?

Interesting examples/visualizations aside, Banach's Fixed Point Theorem is a fundamental result of Functional Analysis. It is a powerful result that can be used to show existence and uniqueness of solutions, and to estimate solutions to certain differential, integral, and non-linear equations. The Theorem has application from Quantum Mechanics to Dynamic Systems and Control Theory.

Although I could not cover it here, the proof for Banach's Theorem is not all that complicated or long. If you have taken an advanced calculus course you should be able to grasp the key ideas behind the proof. I would encourage you to look up the proof. Tune in next month for when I discuss Brouwer's Fixed Point Theorem.

This article relies heavily on concepts taught to me in Francesco Cellarosi's Functional Analysis class here at Queen's University.

Professor Spotlight: Dr. M. Ram Murty

Professor Murty was always interested in math, as well as many other subjects such as physics, computer science and philosophy. He says part of what encouraged his love for math was when a retiring teacher at his high school gave him a collection of old American Mathematical Monthly magazines that he was able to pour over in his free time. From there he completed his undergraduate studies in math at Carleton, where a teacher again had an impact on him. His professor reportedly badgered him until he applied to MIT, where he ended up getting in and completing his Phd.

More recently his other high school loves, computer science and physics, have started coming back into his life and research, as well as philosophy, especially in the case of AI. He has a vested interest in the definition of consciousness, and how we can prove something is conscious through math and philosophy. His life has been, what he liked to call "serendipitous", and this reflects in his advice to students: whatever you're meant for will come to you by “an inscrutable law” .

When asked why he wanted to become a Professor, Murty made it seem like it was obvious; he was being paid to learn, which he was going to do anyway. He even took it as far as to say learning and teaching are the meaning of life. Professor Murty’s love and prioritization of learning is clear in both his current fall courses MATH 381: Math with a historical perspective and MATH 418: Number theory and cryptography, where he de-emphasizes the importance of grades and instead wants students focused on the process of learning. He often says “math must be done with pen and paper in hand” to encourage students to work through their problems. Thank you Professor Murty for diving into the “seductive” and “limitless” qualities of math with us.

Industry Spotlight: Credit-Risk Modelling

Borrowing money is an essential part of financial life - it allows people to buy homes, purchase cars, and, for many of us here at Queens, pay for our education with student loans. For most of us as borrowers, the process appears simple: apply, wait for approval, pretend to read and understand the agreement, then sign on the dotted line. But behind the scenes, for every “yes” or “no” there is a sophisticated system - built on the back of mathematics and statistics – that determines how risky each borrower may be. This is the field of credit-risk modelling.

Every time a bank, credit card company, or online lender decides whether to extend credit, it’s engaging with the credit-risk industry - a global network of financial institutions, regulators, and analytics firms that quantify the chance a borrower might default on their payments. At its centre, credit-risk modelling aims to translate uncertainty into numbers: the probability of default, the potential loss if default occurs, and the overall exposure of a lender’s portfolio. These calculations influence everything from individual loan approvals to international banking regulations. Credit-risk modelling has evolved the industry from rule-of-thumb assessments to data-driven systems that are based in probability theory, optimization, and machine-learning models - ideally designed to make lending safer, fairer, and more efficient.

To dive into credit-risk models, we first need to know how they quantify risk. The most common approach uses logistic regression, a statistical method that predicts the probability of default based on borrower characteristics such as income, credit history, and existing debt.More advanced techniques extend this with survival analysis to estimate when a default might occur, or with machine-learning algorithms that can detect subtle, non-linear patterns across large datasets.

Each model focuses on three key metrics: probability of default (PD), loss given default (LGD), and exposure at default (EAD). Together, these form a lender’s expected loss, the foundation of modern risk management. Even small improvements in these estimates can translate into large savings or avoided losses, which is why analysts continually refine their models.translate into large savings or avoided losses, which is why analysts continually refine their models.

In September 2025, researchers Agus Sudjianto and Denis Burakov introduced a new approach to credit-risk modelling that addresses one of the field’s most pressing challenges: combining predictive power with interpretability and fairness.Their framework reinterprets common industry metrics - such as Weight of Evidence (WoE), Information Value (IV), and Population Stability Index (PSI) - through the lens of information theory, a branch of mathematics that attempts to quantify uncertainty.

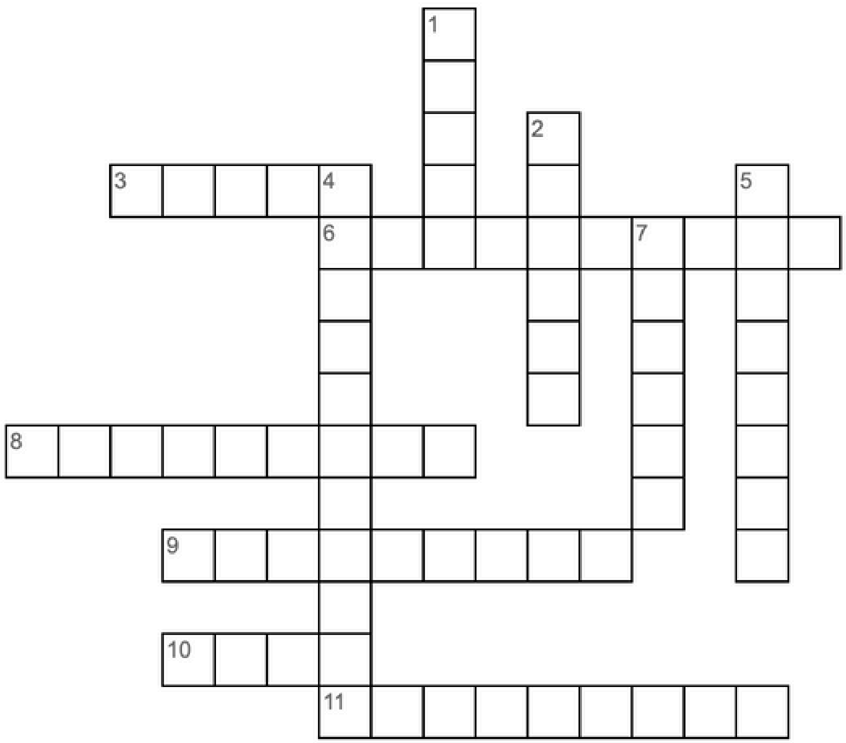
This perspective allows analysts to treat these traditional metrics as formal statistical divergences, complete with standard errors and hypothesis-testing capabilities. In practical terms, it means credit-risk models can now be rigorously evaluated, compared, and validated while maintaining regulatory and ethical standards. Additionally, the framework provides tools to explicitly measure and mitigate bias, ensuring that models do not unfairly disadvantage groups of borrowers.

For the industry, this contribution is significant. By providing a mathematically sound way to unify predictive accuracy, interpretability, and fairness, it strengthens trust in lending decisions and improves
financial institutions’ risk management.

For students in math and stats it is a clear example of how advanced mathematical concepts - in this case, from information theory and statistics - can have real, and important applications in the world. Roles such as Credit Risk Analyst, Quantitative Analyst, or Risk Modeller involve designing and validating models, analyzing borrower data, and ensuring regulatory compliance. Core skills include probability and statistics, regression analysis, optimization, coding in Python or R, and an understanding of finance or economics.

Credit-risk modelling shows how math and stats extends far beyond our lectures in Jeffrey Hall (or Stirling...). Probability, regression, optimization, and statistical inference are not just abstract ideas; they are tools that can shape industries, inform policy, and promote ethical decision-making.

Puzzle of the Month



Across

3 number of sides on a heptagon

6 The type of number pi is

8 Opposite of prime

9 if $a \equiv b \pmod{n}$, a and b are this

10 A type of Hypothesis; meaning zero

11 One of the DSC copresidents; older sister of the Olsen twins

Down

1 The type of coordinates that would include a point in the form (r, θ)

2 University level math competition

4 Famous statistician and nurse

5 1 in standard normal distribution

7 One of the DSC copresidents; Title of a One Direction song

(2025). https://www.printablecrosswordmaker.com/crossword/october_math_dsc_crossword/1336039/crossword.jsp.