DEPARTMENT OF MATHEMATICS AND STATISTICS
Summer 2023 Research Projects (in no particular order)

Last updated: 19 January 2023 – be sure to check back for updates

Project Title: Combinatorial aspects of inversion sets of roots

Supervisors: Ivan Dimitrov, Charles Paquette and David Wehlau

Project Description: Weyl groups form a class of groups that generalizes the permutation groups of n elements and arises in connection with various branches of mathematics, e.g. Lie theory and geometry. Many problems in these areas reduce to combinatorial problems related to suitable Weyl groups. Recent developments in geometric invariant theory reduced the problem of describing the Littlewood-Richadson (LR) cone associated with an algebraic group G to a purely combinatorial problem about sets of elements of the Weyl group W of G. Hence understanding the solutions of the latter is crucial for obtaining more explicit description of the LR cone and computing various numerical invariants related to it. In a previous very successful USRA project we resolved this problems in the case when W is the permutation group of n elements and made partial progress in some additional cases. However, a uniform treatment of all cases eluded us. A major step forward came out of a recent USRA project. Using the notion of Kostant roots, we were able to form exact conjectures about the expected results and to prove the first main result.

Student's role: Generate experimental data, form conjectures, verify such conjectures with more experiments, prove them or find counter-examples.

Prerequisites (req'd background/level of study): MATH 110, MATH 210; familiarity with the symmetric group and combinatorics related to it is a strong asset. Familiarity with a computer algebra system or the desire to learn such a system is expected.

Project Title: Algebraic and Geometric Study of Gentle Algebras

Supervisor: Charles Paquette and Kaveh Mousavand

Project Description: Gentle algebras and their applications have vastly appeared in various areas of mathematics and certain domains of theoretical physics. Here, an algebra is just a ring with a compatible structure of a vector space. These algebras are known for their rich combinatorial nature, which allows one to employ different tools and techniques while working with them. This immanent feature of gentle algebras has stimulated a lot of research. In particular, over any gentle algebra A, unlike arbitrary algebras, one can fully describe the *indecomposable A-modules* and present them in terms of combinatorial objects called strings and bands. That is to say, indecomposable modules over gentle algebras can be explicitly described through combinatorics of some words in certain alphabets which satisfy some algebraic conditions. In this project, we will use gentle algebras and their tractable module theory to study different triangulations of some orientable surfaces. For example, we treat triangulations on spheres with some punctures, or on surfaces with "handles" (such as the torus) and punctures. We then use this geometric model to answer some questions on the algebraic aspects of gentle algebras.

Student's Role: Learn the basic concepts on gentle algebras and their modules, explore through examples, formulate conjectures, prove some statements.

Prerequisites (req'd background/level of study): The student must have passed MATH 112, MATH 210, and MATH 310, or the equivalents

Project Title: Treatment effect estimation under covariate adaptive designs

Supervisor: Yanglei Song

Project description: In clinical trials evaluating a new treatment, randomization is required to balance both observed and unobserved confounding variables. Simple randomization, though popular, may lead to an imbalance between the number of subjects in treatment groups by chance. Adaptive randomization which allocates subjects sequentially based on the previous assignments is proposed to reduce the imbalance. Further, to achieve balance among important prognostic factors, covariates are used in making assignments, which is known as covariate adaptive randomization. This project concerns the estimation of treatment effects where data are collected under various covariate adaptive designs.

Student's Role: The student will first conduct a literature review, and discuss materials with the supervisor through regular meetings. Then the student will implement and compare several existing methods, and write a final report.

Prerequisites (req'd background/level of study): A strong background in probability/statistics (at the level of STAT 351/353); familiarity with at least one programming language.

Project Title: On the existence of periodic solutions to differential equations

Supervisors: Giusy Mazzone

Project Description: The aim of this project is to investigate the existence of periodic solutions to (systems of) nonlinear differential equations. It is well-known that there is no “universal” method to solve all possible differential equations. As a matter of fact, there is a relatively small number of differential equations for which a solution method has been developed. Nevertheless, differential equations are ubiquitous as they are mathematical models governing numerous physical and natural systems, and they are used for many engineering applications. Thus, whether it is for mathematical modelling or just numerical validation, it becomes necessary to obtain qualitative information about the solutions to a differential equation without being able to solve it explicitly.

While the existence and uniqueness of solutions to initial value problems is well-understood, less work has been done toward a comprehensive theory for the corresponding problem of finding nontrivial (i.e., non-constant) periodic solutions to nonlinear differential equations. The known results either assume conditions which are difficult to be checked (like finding uniform bounds on the solutions) or they are application specific (that is, they depend on the specific type of differential equation under study). In contrast, non-existence of periodic solutions may have intriguing consequences in the theory as well as in the applications. Take the “phenomenon of resonance”, for example. Consider the second-order linear differential equation modelling the undamped motion of a mass-spring system. It is well-known that if the system is subject to an external force which is periodic with the same frequency as the “natural frequency” of the system (i.e., the frequency of oscillations of the undisturbed motion), then the generic motion of the mass is characterized by oscillations with unbounded amplitude. In this case, it is said that the system is working at resonance, and then there is no periodic motion associated to that resonant force. Resonance is at the basis of many energy harvesting techniques utilized nowadays. From the mathematical viewpoint, this phenomenon could be easily described if one was able to write an explicit solution of the corresponding differential equation (like in the case of the mass-spring system, with the method of variation of parameters applied to the corresponding linear differential equation). However, many physical and natural phenomena are described by nonlinear differential equations for which an explicit solution is not available, examples include the occurrence of flutter in aeroelasticity, planetary motions, the modelling of the heartbeat or that of suspensions in wheeled vehicles as well as problems involving a delay. Aim of this project is to find the most general set of conditions (possibly easy to be checked) to determine whether a given differential equation admits a periodic solution. Applications of these results to real-world problems will be given.

Student's role: The student is expected to review/learn classical results in the theory of ordinary differential equations, this includes existing theorems regarding initial value problems as well as techniques typically used to prove the existence of periodic solutions (like fixed point theorems). Under the direction of the supervisor, the student will try to find novel conditions that would ensure that a given differential equation admits a periodic solution. Results will be corroborated by real-world applications of the student’s choice.

There will be regular meetings with the supervisor to discuss the material and present the new results. A final written report is required.

Prerequisites (req’d background/level of study): MATH/MTHE 280, MTHE 237 or MATH 231. For students in their 3rd or 4th year of study.

Project Title: The North Pole Problem

Supervisor: Jamie Mingo

Project Description: If one considers S, the two sphere in three dimensional Euclidean space, one can randomly rotate it by a random orthogonal matrix, sampled according to Haar measure. This action will move the north pole to a random point on the sphere and the distribution will be normalized area measure on the sphere. It was noticed that if one squares the orthogonal matrix before applying it to the north pole we get a different distribution, where one is more likely to stay on the same hemisphere where one started. There has not been a systematic investigation of this phenomenon in higher dimensions or a modern approach to the three dimensional case. The project is to understand the existing results in The ‘‘north pole problem’’ and random orthogonal matrices, by Eaton and Muirhead, Statistics and Probability Letters, 2009, and to see to what extend the results can be simplified and extended.

Student's Role: At the outset this will require some reading of background material to get familiar with the problem; after that participation in the research.

Prerequisites (req'd background/level of study): A suitable background will be some linear algebra, some probability theory, and some programming experience.

Project Title: Interaction of Information, Control and Probability

Supervisor: Serdar Yuksel

Project Description: The interaction between control, information, and probability is a general phenomenon that arises in many problems involving decision making and control under probabilistic and informational uncertainty. The general goal of this project is to study the optimal design of information structures and decisions/controls in stochastic systems. Depending on student interest, we will focus on one of the following three main themes:

- (i) Optimal stochastic control with partial information: Non-linear filtering, approximations, robustness, and learning

- (ii) Decentralized stochastic control and stochastic games: Arriving at optimality/equilibrium under decentralized information and learning

- (iii) Optimal control under information constraints and optimal design of coding and control policies in networked systems

Student's Role: The expectation is that the student(s) would study a wide variety of resources that will be provided, take part in research discussions through regular and frequent meetings, and write a comprehensive technical document at the end of the summer to report their studies. Implementations of some of the findings through Matlab or another program is also likely.

Prerequisites (req'd background/level of study): A strong interest in probability and analysis. 3rd or 4th year students would likely be more suitable for the project, though this is not necessary.

Project Title: Input-to-State Stability in Terms of Two Measures

Supervisors: Kexue Zhang

Project Description: The notion of input-to-state stability (ISS) has been proven powerful in stability analysis and control synthesis for dynamical systems. This notion characterizes the effects of external inputs on the stability of control systems. On the other hand, the notion of stability in terms of two measures allows us to study Lyapunov stability, partial stability, orbit stability, and stability of the invariant set for nonlinear systems simultaneously. In this project, we will study the bridge notion connecting the above two, namely, ISS in terms of two measures, in both discrete-time and continuous-time system settings. We will explore the similarities and differences in the stability analysis between continuous-time and discrete-time systems, and then extend the notion of ISS in terms of two measures to systems described by differential equations on time scales.

Student's Role: Study various stability notions and time-scale calculus; explore ISS in terms of two measures for continuous-time and discrete-time systems; extend this unified notion to differential equations on time scales and hopefully construct some preliminary stability results. We seek two undergraduate students to join this team.

Prerequisites(req'd background/level of study): A background in differential equations. Knowledge of Lyapunov stability and experience in MATLAB will be a plus.

Project Title: Markov genealogy processes

Supervisor: Felicia Magpantay and Troy Day

Project Description: During an epidemic the emergence of "variants of concern" is shaped by evolutionary pressures such as vaccination and competition. The pandemic has led to an explosion in genomic surveillance, highlighting the need for efficient and mathematically rigorous analysis of epidemiological time series coupled with genetic data. In this project we will look into the recently developed theory of Markov genealogy processes. We will also examine the implementation of this theory for statistical inference in the R package phylopomp and apply it to datasets.

Main reference: https://doi.org/10.1016/j.tpb.2021.11.003

Student's Role: The student will participate in our study of Markov genealogy processes and conduct a review of different approaches in phylodynamic analysis. The student will also learn about phylopomp and high-performance computing, and apply these to a disease modeling project using real data.

Prerequisites (req'd background/level of study): Solid background in probability, basic mathematical or statistical programming

Be sure to check back as additional projects may be added.