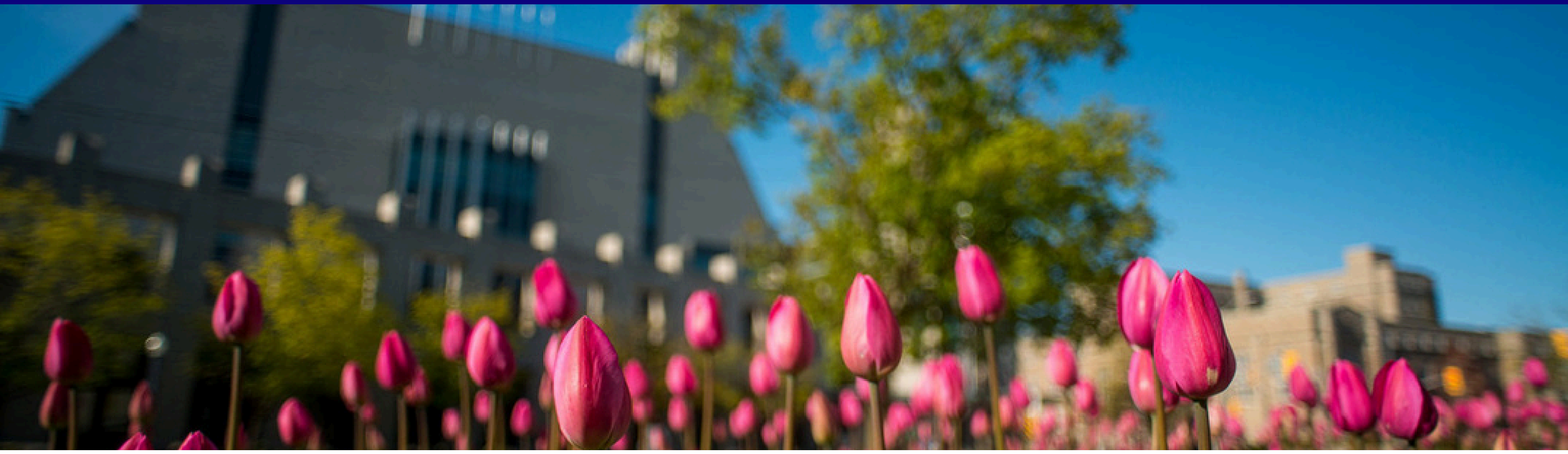
Queen's University

Math & Stats Newsletter

March



Visit Kingston. (2026). BEST IN BLOOM: 7 GREAT PLACES TO EXPERIENCE KINGSTON'S SPRING FLOWERS https://www.visitkingston.ca/stories/best-in-bloom-7-great-places-to-experience-kingstons-spring-flowers/

Quote of the Month

"Abstractness, sometimes hurled as a reproach at mathematics, is its chief glory and its surest title to practical usefulness. It is also the source of such beauty as may spring from mathematics" - Eric Temple Bell

Math Meme of the Month:

Math Show Reccomendation

Wikipedia. (2026). Worth (2020). https://en.wikipedia.org/wiki/Worth_(film)

Worth (2020)

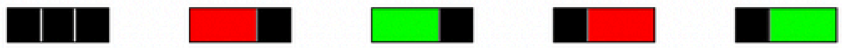
Worth is a BioPic about Kenneth Fienburg, who was the one repsonsible for figuring out exactly how much compensation the victims and families involved in 9/11 deserved. This will be an interesting watch for anyone thinking of going into actuarial sciences or insurance, as it explores the idea of how much exactly is a human life worth and what factors do we use to decide that?

No Upcoming Events

Good luck on exams everyone!!!

Concept of the Month

Within this month's newsletter, I will examine a combinatorial problem. I have always thought that solutions to combinatorial problems are interesting and somewhat unique. Today we aim to count the number of ways we can tile a block on length n (n can be any positive integer) with 3 separate tile options: a 1 unit black tile, a 2 unit red tile and a 2 unit green tile. We will denote our answer for any given n as a_n. To make this problem explicit, let's go over the answers to $n = 1, n = 2, n = 3$. When $n = 1$, we can only colour a block of length 1 with 1 black tile (since they are one unit long), thus $a_1 = 1$. When $n = 2$, we can colour our block with 1 red piece, 1 green piece, or 2 black pieces. Thus $a_2 = 3$. Let's visually enumerate our answer for $n = 3$.



Thus $a_3 = 5$. Now that the problem is clear, let us think about how to count a_n in general. Our goal is to find a function $f(n)$ such that $f(n) = a_n$ for every positive integer n. Such a formula is called a *closed-form solution.* Before jumping into the solution, I must introduce some necessary machinery. To study this problem, we introduce the idea of a *formal power series.* A formal power series is an expression of the form

$$\sum_{n=0}^{\infty} b_n x^n = b_0x^0 + b_1x^1 + b_2x^2 + ...,$$

which we think of as encoding the sequence $(b_n) = b_0, b_1, b_2, ...$

In our case, we define the generating function

$$A(x) = \sum_{n=0}^{\infty} a_n x^n,$$

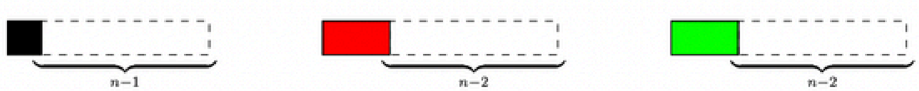
where the coefficient a_n represents the number of ways to tile a block of length n. Formal power series are somewhat mysterious at face value. We are simply viewing our series $(a_n) = a_1, a_2, a_3, ...$ in a slightly different light, yet we can now perform algebraic manipulations to help solve our problem. A few important notes: (1) when dealing with formal power series, we do not consider convergence in an analytical sense, (2) $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$. (2) is the only identity we use for our problem, however, when problems get more complicated, many other algebraic identities must be used, some derived from differentiating formal power series. Let's quickly prove (2):

$$\sum_{n=0}^{\infty} x^n = F(x) = x^0 + x^1 + x^2 + ... = 1 + x^1 + x^2 + x^3 + ... = 1 + x(x^0 + x^1 + x^2 + ...) = 1 + x\sum_{n=0}^{\infty} x^n = 1 + xF(x)$$

thus,

$$F(x) = 1 + xF(x) \implies F(x) - xF(x) = 1 \implies F(x)(1-x) = 1 \implies F(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Now, back to our problem. We will use a very powerful tool to begin, recursion. If we have some block of length n, then to begin counting all combinations of tilings, we can start by identifying our starting options. We can start with a black, red, or green tile.



Now, if we start with a black tile, we have $n - 1$ empty spots left to tile in a_{n-1} possible ways. Similarly, when we start with a red or green tile, we have a_{n-2} ways to tile the remaining $n - 2$ squares. If we sum over all three possible starting pieces, we get

$$a_n = a_{n-1} + a_{n-2} + a_{n-2} = a_{n-1} + 2a_{n-2}$$

Equivalently, we can shift indices to get

$$a_{n+2} = a_{n+1} + 2a_n$$

What we have derived here is a *recursive solution* to our problem. Theoretically, since we have by hand calculated the base cases when $n = 1$ and $n = 2$ we could now use the recursive formula to find the solution for any n. The problem is that for any n we must start by using a_1, a_2 to find a_3 then we can use a_2, a_3 to find a_4 and so on. In general, we will need to implement the equation n times to find a_n. Whether you are going by hand or implementing this on a computer, you are wasting time and energy as n gets large. This is why we are instead in search of a simple closed form solution that is a function of n. The recursive solution is however a mean to our final solution. We will now enlist the help of power series, using $A(x) = \sum_{n=0}^{\infty} a_n x^n = a_0x^0 + a_1x^1 + a_2x^2 + ...$ Since $a_{n+2} = a_{n+1} + 2a_n$, we must also have

$$\sum_{n=0}^{\infty} a_{n+2}x^{n+2} = \sum_{n=0}^{\infty}(a_{n+1} + 2a_n)x^{n+2} = \sum_{n=0}^{\infty} a_{n+1}x^{n+2} + \sum_{n=0}^{\infty} 2a_nx^{n+2}$$

Examining our left hand side, we have

$$\sum_{n=0}^{\infty} a_{n+2}x^{n+2} = a_2x^2 + a_3x^3 + a_4x^4 + ... = A(x) - a_0x^0 - a_1x^1 = A(x) - 1 - x$$

Notice that while a_0 is a weird idea (how many tilings of a block of zero length?) you can either find it using our recursive formula and our earlier derivation of a_1, a_2 or one can say there is one way to place zero tiles. Examining both terms on our right hand side, we get

$$\sum_{n=0}^{\infty} a_{n+1}x^{n+2} = a_1x^2 + a_2x^3 + a_3x^4 + ... = x(a_1x^1 + a_2x^2 + a_3x^3) = x(A(x) - a_0x^0) = x(A(x) - 1)$$

and

$$\sum_{n=0}^{\infty} 2a_nx^{n+2} = 2a_0x^2 + 2a_1x^3 + 2a_2x^4 + ... = 2x^2(a_0x^0 + a_1x^1 + a_2x^2 + ...) = 2x^2A(x)$$

Putting everything together, we have

$$A(x) - 1 - x = x(A(x) - 1) + 2x^2A(x) = xA(x) - x + 2x^2A(x)$$

$$\implies A(x)(1 - x - 2x^2) = 1 \implies A(x) = \frac{1}{1 - x - 2x^2} = \frac{1}{(1-2x)(1+x)}$$

Now, we can leverage partial fractions. We know that $\frac{1}{(1-2x)(1+x)} = \frac{A}{1-2x} + \frac{B}{1+x}$, for some constants A and B. I will save you from looking at any more algebra, we can (quite easily) derive that $A = \frac{2}{3}, B = \frac{1}{3}$. Thus, after all these algebraic manipulations, we have found that

$$A(x) = \sum_{n=0}^{\infty} a_nx^n = \frac{2}{3}\left(\frac{1}{1-2x}\right) + \frac{1}{3}\left(\frac{1}{1+x}\right)$$

Now, we will implement our trick from (2):

$$\frac{1}{1-(2x)} = \sum_{n=0}^{\infty}(2x)^n = \sum_{n=0}^{\infty} 2^nx^n \text{ and } \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^nx^n$$

Putting everything together, we get that

$$A(x) = \sum_{n=0}^{\infty} a_nx^n = \frac{2}{3}\left(\sum_{n=0}^{\infty} 2^nx^n\right) + \frac{1}{3}\left(\sum_{n=0}^{\infty}(-1)^nx^n\right) = \sum_{n=0}^{\infty}\left(\frac{2^{n+1} + (-1)^n}{3}\right)x^n$$

And thus we have found that $a_n = \frac{2^{n+1}+(-1)^n}{3}$. Thus we can quickly calculate the number of ways to tile any block of length n.

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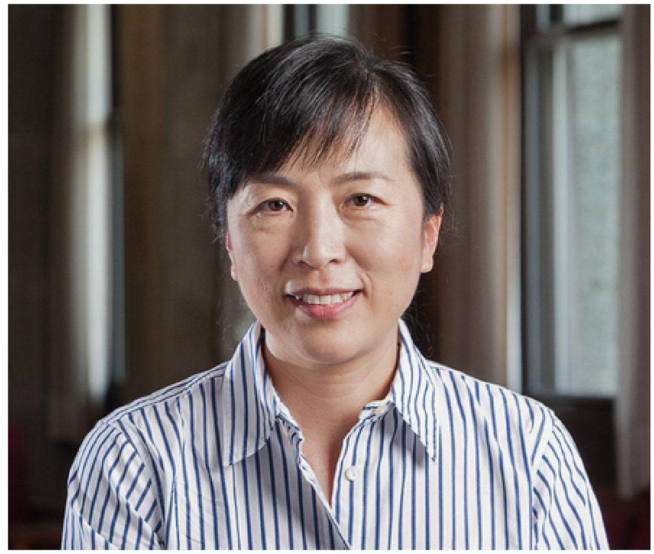
Professor Spotlight: Ping Li



With her clear care for students and thoughtful advice, Professor Li was a wonderful interviewee to conclude this year's professor spotlights.

Professor Li's journey in mathematics began early. Her mother, a chartered accountant, taught her to count when she was a toddler, and math quickly became her favourite subject. She also credits her mother's strong work ethic for inspiring her own dedication. This combination of passion and discipline carried her through her undergraduate studies in China, where she completed degrees in both Mathematics and Computing. Afterward, she spent a few years working in industry and raising a family before returning to academia to pursue a master's degree in topology, followed by a PhD at Queen's.

Her PhD research was highly specialized, focusing on Cohomological properties of various affine semigroup rings and how these properties relate to topological, geometric and combinatorial properties of the semigroups in the cone that they span. Although proud of her research and grateful for the experience, Professor Li admitted that her research often occupied her thoughts constantly, even in her sleep, so after finishing her degree she decided to shift her focus more toward teaching.

Professor Li has taught many courses over the years, particularly in calculus, including nearly ten years of teaching APSC 171/172 and, most recently, coordinating and teaching MATH 121. She has also taught numerous upper-year courses in both Arts and Science and Applied Science.

Her favourite course to teach has been MATH 224, Applied Mathematics for Civil Engineers, because it offered "a bit of everything": MATLAB programming, differential equations, and probability and Statistics, and practical applications in industry. It's clear, however, that what she values most is connecting with students. She says they help her feel young, inspire her, and teach her new things every year.

Beyond the classroom, Professor Li also supports students as a Co-Chair of Undergraduate Studies in the Math and Stats department. In this role, she helps students navigate degree requirements, provides academic advising, assists with course selection, guides graduation planning, and addresses academic concerns as they arise.

When asked what advice she would offer students, she emphasized two key messages. First: "Don't be afraid to reach out." Second: take advantage of the many resources available today. She noted that AI can be a helpful tool when used wisely—much like a calculator—so long as students understand why an answer is correct rather than relying on it blindly.

Thank you again to Professor Li for the lovely interview. Wishing her and all her students a wonderful end to the semester.

Industry Spotlight: Operations Research and Logistics Optimization

For most of us when we order something online, board a flight, or even wait for our food order to arrive, we rarely think about the mathematical decisions happening behind the scenes. Despite this, modern logistics depends on constant choices: which warehouse should a product be stored in, which route should a truck or delivery driver take, how many workers should be scheduled, the list goes on. In essence, how can a company move goods quickly without wasting money, fuel and time. The field that tackles these problems is operations research, often called OR, and is one of the clearest examples of mathematics shaping our world and day to day lives.

Operations research (OR) is the science of attempting to make the best decision within a complex system. I n practice, that means taking a problem with multiple constraints and converting it onto a mathematical model that can be analyzed and optimized. In logistics, these constraints can be anything but some classic examples include delivery deadlines, vehicle capacity, fuel costs, traffic, warehouse space, and labour availability. The goal is usually to minimize cost, time or waste while still maintain a relative balance, ensuring efficiency. Organizations use OR alongside analytics to turn data into decisions, drawing especially on optimization, simulation, probability, and statistics.

At the mathematical level, logistics optimization brings together several areas of math and stats. Linear programming and integer programming are used to find the best solution among many possibilities when variable and constraints can be expressed mathematically. Network models help describe supply chains as connected systems of routes and hubs. Probability and statistics are used when demand is uncertain, or travel times vary. Simulation is also important because in large systems companies often need to test how a plan performs under realistic conditions before actually adopting it. In other words, OR is not just solving the equations, but also about designing systems that still work when things outside the chalk board get complicated.

One well known example is UPS's Project ORION (On-Road Integrated Optimization and Navigation), a large-scale route-optimization project used to improve package delivery. Rather than relying only in driver habit or simple mapping software, ORION uses advanced optimization to determine more efficient delivery routes across a huge transportation network. INFORMS reports that at full deployment, ORION is saving UPS $300 to $400 million USD annually, while simultaneously reducing fuel use by about 10 million gallons ad cutting carbon emissions by roughly 100,000 metric tons per year. It is a beautiful and satisfying example of how abstract optimization problems can translate into massive practical gains when applied at an industrial scale by large companies.

The same ideas appear across the broader logistics industry. Amazon's Supply Chain Optimization Technologies team, for example, describes their work as using OR and machine learning to decide what products to buy, how much inventory to hold, and where items should be stored across its network of warehouses. At this scale logistics becomes a giant mathematical coordination problem involving forecasting, placement, routing and timing.

For students in math and stats OR is a reminder that some of the most useful mathematics is not always the most visible. Optimization, probability, graph theory, stats, and computation all come together to in a field that helps humanity make faster, cheaper and smarter decisions. Career paths in the area include things like OR analyst, supply chain analyst, logistics planner, and optimization scientist. As math or stats majors at Queen's the core skills are exactly the kind we build here: problem formulation, quantitative reasoning, and analysis. Operations research demonstrates math goes beyond a way of understanding our world but also serves as a method of organizing it more effectively.

Sources:

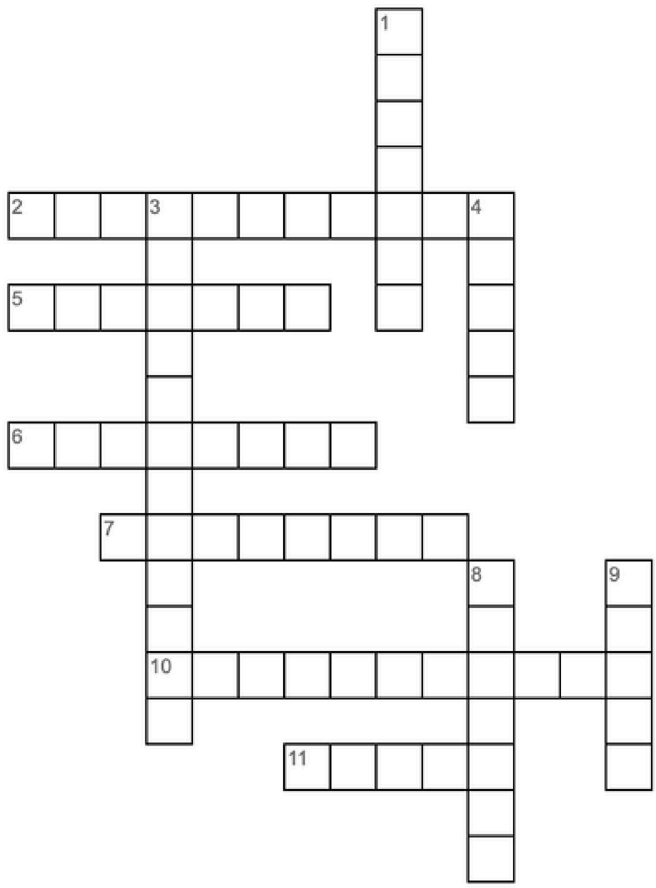
INFORMS. "Operations Research & Analytics."

Safely. "Project ORION by UPS: A Game-Changer in Delivery Efficiency and Route Safety." April 21, 2025

INFORMS. "UPS On-Road Integrated Optimization and Navigation (ORION) Project." INFORMS.

Amazon Science. "reMARS revisited: Amazon's supply chain optimization." October 18, 2022.

Puzzle of the Month



Across

2 a special ratio = 1.61803398875

5 how I hope everyone does on their math and stats exams; also a type of number

6 a quantile with 4 groups

7 the types of numbers Z represents

10 The man who proved Fermat's Last Theorem

11 horizontal line on a 2D graph

Down

1 a polynomial of the fourth degree

3 what the D in ODE stands for

4 the last letter of the greek alphabet

8 statistical distribution; reminiscent of a fish

9 The calculator you are allowed for exams at Queen's

Printable Crossword Maker. (2026). https://www.printablecrosswordmaker.com/crossword/march_math_and_stats_crossword/1414391/crossword.jsp.

March Recap!!

Escape Jeffery

Pi Sale