Queen's University

Math & Stats Newsletter

November



Queen's University. (2023). Wellness For The Winter and The Year Ahead. https://www.queensu.ca/artsci/news/wellness-for-the-winter-and-the-year-ahea

Quote of the Month

"Winter is the time for study, you know, and the colder it is, the more studious we are"
- Henry David Thoreau

Math Meme of the Month:

How I feel going to profs office hours



Math Show Reccomendation



Prime Target

This story is centered around Edward Brooks, a graduate math student at Cambridge, whose research into prime numbers could soon give him access to every computer in the world. Of course, this earns him unwanted attention from the NSA, in particular, a woman named Taylah Sanders, whose meant to monitor mathematicians worldwide. If you love action and pondering the possibilities of math, you'll love this show.

Upcoming Events

Video Game Night - Nov 27th - 6-8 pm

Check our instagram @queensmastdsc for specific dates and continuous updates

Concept of the Month

Last month we reviewed Banach's Fixed Point Theorem. Banach's theorem is a celebrated result in Functional Analysis that studies fixed points of contractions over Banach spaces. This month we will review Brouwer's Fixed Point Theorem, a topological result regarding continuous maps. Mirroring last month, we will break down the meaning of the theorem and share some interesting results. Unlike last month, the proof of this theorem is extremely interesting. In a completely non-rigorous manner, we would like to share a rough sketch of the proof that gives the reader some intuition as to why this theorem is true. Formally, Let

$$D_n = \{x \in \mathbb{R}^n : ||x|| \le 1\}$$

be the closed unit ball in \mathbb{R}^n . If

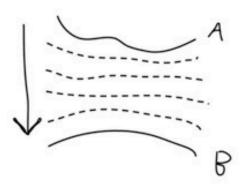
$$f: D_n \to D_n$$

is continuous, then there exists a point $x^* \in D_n$ such that

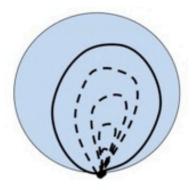
$$f(x^*) = x^*$$
.

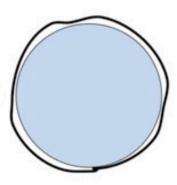
Although the formal statement uses the notion of unit balls for clarity, the theorem easily extends to all compact convex subsets of \mathbb{R}^n . Hopefully a reader has some idea of convexity (no inward dents—standard polygons are convex; a star is not). Compactness simply requires the subset to contain its boundary and to be bounded (i.e., contained in some closed ball). Now, let's think of our continuous function as a map that can bend, squish, spin, but never break our subset. If two points x_1, x_2 are close together in the domain of f then $f(x_1), f(x_2)$ will be close together in its range. Now, for an interesting application, take a cup of coffee (convex and compact) as our subset of \mathbb{R}^3 . If we take a spoon and stir our coffee (a continuous transformation), then, by our theorem, once the coffee has settled again, one "particle" of coffee will end up in the exact same spot as it began. This result is certainly surprising. Brouwer's theorem is a foundational topological result; it helps prove deep results regarding differential equations and even has important applications in game theory and economics.

I hope you are convinced of the beauty and importance of Brouwer's theorem. I hope to now share some of the beauty of one of its proofs. For our purposes, I will sketch a proof of the 2-dimensional case; continuous functions on the closed disc in \mathbb{R}^2 have at least one fixed point. Stay with me while I first try to give you some necessary basic knowledge of algebraic topology. Consider paths in \mathbb{R}^n . We say that two paths, A and B, are **homotopic** if path A can be continuously (no breaks!) deformed into path B. See the diagram depicting a simple homotopy between two line segments A and B for some clarity:



Now, let's apply this idea to loops in \mathbb{R}^2 (that begin and end at (0,-1)) on surfaces like a disc (full interior), and a circle (empty interior). Let's call the two dimensional disc D^2 and the two dimensional circle S^1 . Notice that on the disc, every loop is homotopic to the "loop" that never leaves (0,-1) (just a single point at (0,-1)). However, on a circle, if a loop goes around the whole circle, one cannot deform it to a single point, as the empty interior would force said loop to break. Hopefully the diagrams help explain this idea better:

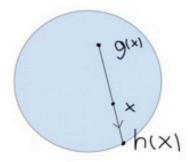




Since loops (that begin and end at (0,-1)) on the disc are all homotopic to a single point, topologists say the disc has fundamental group 0. On the circle, loops are homotopically distinct if they go around the circle a different number of times (either in a negative or positive direction). Thus the circle has fundamental group \mathbb{Z} . Concretely, an integer a corresponds to a loop that goes around the circle a times (clockwise or counterclockwise depending on whether a is positive or negative). To better understand this idea, think of why a torus corresponds to $\mathbb{Z} \times \mathbb{Z}$.

Formally, category theory captures this idea. The functor π_1 sends the spaces S^1 and D^2 to the groups \mathbb{Z} and 0, and it sends continuous maps to group homomorphisms. In this sense a functor acts as a bridge between fields: for us, π_1 translates topology into algebra. Due to the short nature of this article, we cannot go deeper into category theory.

Now we can begin the sketch of our proof. The proof is **non-constructive**, utilizing a proof by contradiction. This means that unlike in Banach's fixed point theorem, where we could find an arbitrarily close approximation of our fixed point, Brouwer's theorem simply tells us that a fixed point must exist (we have no algorithm to find it). For a contradiction, let's assume that there exists a function $g: D^2 \to D^2$ such that g has no fixed points. Therefore, for every $x \in D^2$, $g(x) \neq x$. Therefore, we can define a function $h: D^2 \to D^2$ that uses g to send points on the disc to the boundary of the disc. Notice the boundary of the disc is S^1 (the circle). See diagram:



Notice that we can only define such an h because g has no fixed points. Furthermore, we can show h is continuous (although I will omit this from this proof). Now let's define the inclusion map $i: S^1 \to D^2$ such that for every $x \in S^1$, i(x) = x. Now, take $h \circ i: S^1 \to S^1$, $x \mapsto h(i(x))$. Notice that since h does not move boundary points (points on S^1), for every $x \in S^1$, h(i(x)) = x. Therefore, $h \circ i = id_{S^1}$ (the identity map). Now, using our previous work relating the disc to 0 and the circle to the integers, since h is continuous, this construction induces a homomorphism

$$\pi_1(h \circ i) = \pi_1(h) \circ \pi_1(i) : 0 \to 0$$

where $\pi_1(i): 0 \to \mathbb{Z}$ and $\pi_1(h): \mathbb{Z} \to 0$.



It is not possible for $\pi_1(h \circ i) = id_{\mathbb{Z}}$ to send every integer to 0 and then somehow recover every integer again. Thus no such map exists, and we have a contradiction. This implies that g(x) must have a fixed point!

Professor Spotlight: Dr. Maria Teresa Chiri

It's safe to say that any math or stats major knows Maria Teresa Chiri, she's a memorable professor to anyone who has been in her courses. Thankfully, she is not memorable in the stressful way teachers can stay with us, but rather she sticks with you — from her Italian accent and "illegal mistakes" to her philosophical additions to math class that focus on teaching us about life beyond math and grades. Her goal as a professor being "at the end of the semester, I hope you and your friends are yelling at each other in front of a black board on the best way to solve a problem".

Chiri began her bachelor's in mathematics at the University of Salento. When asked why she chose math as her major, she was completely honest: she said she picked it because she was "a lazy person," explaining that, in high school, she never struggled and didn't feel that math was too difficult.

Following her graduation from Salento, she moved to the University of Pisa, where she began her master's. Chiri explained that it was here that math became a challenge, however, through determination she took it as a personal challenge to prove to



After 2024 120 Midterm, Student Photo

Following her graduation from Salento, she moved to the University of Pisa, where she began her master's. Chiri explained that it was here that math became a challenge, however, through determination she took it as a personal challenge to prove to herself that she could accomplish it and she did. Following this, she connected with her passion for math with the help of her PhD advisor who reminded her of why she should do math, for the sake of math itself, math being full of intellectual honesty and truth. And in 2019, she earned her PhD at the University of Padua.

Professor Maria Teresa Chiri describes her work as sitting "not too far from applied problems," while still abstract and theoretical. Her papers are pure mathematical analysis, but the questions she studies are often inspired by real phenomena such as traffic, supply chains, contamination, and even wildfires. What motivates her is not the real-world application itself, but the mathematical structure that emerges from those situations.

A central part of her research focuses on conservation laws, a class of first-order partial differential equations. These equations describe how a quantity—such as the density of vehicles on a highway—evolves over time and space.

She explains that traffic can be modeled in two ways. In a microscopic model, one tracks the motion of each individual vehicle. In a macroscopic model, traffic is treated as a continuous flow, like fluid moving through a pipeline. Chiri works with the macroscopic perspective, studying how the density of cars changes across time and space. The same mathematical tools apply not only to traffic, but to supply chains, blood flow, and other systems that can be viewed at both microscopic and macroscopic scales.

One problem she is particularly interested in involves conservation laws where the flux is discontinuous in the conserved quantity. She gives an intuitive analogy: imagine a conveyor belt in a factory carrying products. If the belt suddenly stops, everything behind it also stops immediately. This instantaneous transmission of information is captured by a conservation law with discontinuous flux. Similar discontinuities appear in computer network failures or cyberattacks, where a shutdown in one place triggers a cascade of shutdowns elsewhere.

Another major theme in Chiri's work involves control problems—situations where one must determine how best to influence or contain a system. She offers an example: imagine a contamination spreading over a region and suppose you can fight it with pesticide. Where along the boundary is the best place to apply the pesticide so the contaminated region shrinks as effectively as possible?

This leads to geometric optimization questions. If the contaminated region is convex, her results show that the optimal strategy is to place the pesticide along the part of the boundary with the greatest curvature, because this causes the region to contract faster. She notes that proving this requires several pages of mathematical argument. What interests her now is the harder question: what happens when the region is not convex? Can one still identify an optimal placement? And is maximum curvature still the correct principle?

She explains that these problems are highly sensitive. Small changes in where the control is placed along the boundary can significantly affect how quickly the contaminated region shrinks and how much area remains after a given time. She is also exploring related questions in wildfire confinement, an issue especially relevant in Canada.

In addition to theory, she also studies numerical methods for conservation laws—schemes that approximate solutions to complex PDEs. Her work examines how such schemes can be constructed and how to prove that the numerical approximations converge to the true solution. This fits into her broader interest in combining analysis with computational approaches.

Across all her work, Professor Chiri is drawn to problems in which applied phenomena give rise to mathematically subtle or irregular behavior. She collaborates with researchers from different areas, but what excites her most is the underlying mathematical structure—especially where it becomes discontinuous, geometric, or otherwise challenging. Her research sits at the boundary between applied motivation and theoretical, driven by curiosity and the desire to understand the mathematics in its own right.

You're sitting in class, listening to Chiri lecture in her normal tone and rhythm, soft but powerful, quick but not too quick. As you rapidly write the example she is explaining in your notes, she pauses for a second where she normally wouldn't, you know she is about to tell the class something that is a mild diversion from where the example was going. You glance up and she begins to say something about a mistake made frequently by her other students, or perhaps on the midterm. If you have had the pleasure of taking one of Chiri's classes you know exactly what she is going to say next, "This is an ILLEGAL MISTAKE". For a second you worry, 'did I make this mistake? Have I made this mistake?' But what follows is always an odd error in algebraic notation, something like $x^3 \cdot x^3 = x^9$. And maybe in the past you have made such a simple error when working quickly or maybe not, either way you know you are not making it again.

It turns out this unique and memorable phrase from Chiri comes from her past high school teacher. The teacher had the class write the illegal mistakes on boards and put them on the wall and would tell students when they made them "You need to go to math jail!". Thankfully, for us, Chiri has reduced it to just a fun way of pointing out mistakes that works effectively at helping move beyond them.

When Professor Chiri speaks about mathematics, she does not describe it as a toolbox of techniques but as something far more complex. For her, mathematics is a language—precise, expressive, and capable of conveying truths that ordinary speech cannot reach. She illustrated this with a beautiful comparison to poetry:

"Just think with the same language, you can write so many beautiful poetry with the same language in mathematics, you can write so many beautiful correct prose. So, to me a proof is the same as a poetry in the sense you're trying to convey a message and need to find the art, the rigorous and correct way of doing that and the path you choose has some sort of art, so I believe that it's a language with a creative component."

Further on the creative aspect of mathematics Chiri gave an excellent analogy:

"You see a mountain far there you see the top of the mountain and maybe the top of the mountain is real, so I need to understand how to cross the woods to walk up there. And this walk—this path that you are creating—is your proof or the top of the mountain is an illusion, doesn't exist and then you keep walking until you get to where you saw something and there is nothing. So, to me the creativity is in seeing something and trying to reach it somehow creating the path, a path that is not there. Or thinking that you see something trying to reach that something and realizing that something is not there."

This connects directly to what she calls the intellectual honesty of mathematics. A proof either works or it doesn't. The mountain is real or it isn't. But the journey—your attempt to reach it—produces understanding regardless of the outcome.

Chiri's teaching reflects this philosophy. She wants students to experience mathematics as a language and a creative act, not merely a set of procedures. She encourages them to struggle productively, to explore, to communicate, to build paths even when the summit looks uncertain. Mastery, she reminds them, comes not from memorizing answers but from learning to think in the language of mathematics.

And ultimately, she hopes students will share this journey with each other. Once again, her goal is that by the end of the term, students are "yelling at each other in front of a blackboard", not out of frustration, but because they have discovered the joy of working through an idea together, creating a path toward a mountain that may or may not be there, but which is always worth approaching.

For Professor Chiri, mathematics is a language, an art form, a journey of creativity, and a climb toward something just beyond the horizon. And it is this philosophy that shapes both her research and her teaching.

Industry Spotlight: Biostatistician

Math is the backbone of everything, one of them. This spotlight may be check <u>The Canadian Society for</u> something that Queen's offers. especially interesting to student in <u>Epidemiology and Biostatistics</u> or However, the Biology Mathematics specilization, as this the following examples: month will highlight what being a • The history of Eugenics in in the field. Communication is biostatistician is like.

Biostatisticians have a important career. Think of medical invention you're grateful for - that was tried and tested by If you're thinking of pursuing a job in learner, because as biostatisticians to make sure it was the field of biostatistics, being in this technology and the world evolve safe and effective. There are, of department is a good start. There so must you. options course. biostatisticians, but many do work in math and stats department that developed, if you want to give the medical field the pharmaceuticals. So if you're looking biostatistics: BIOM 300, and STAT entry level positions such as to aid in improving healthcare, and 486. BIOM 300 is called Modelling research assistant or coordinator putting your knowledge of statistics Techniques in Biology, and it's in health related organizations. to use, this career could be right for exactly what it sounds like, you will According to the Government of

may be found doing research: STAT 486: Survival Analysis focuses across Canada. collecting, analyzing and reporting largely on learning how to analyze their findings on biological data. Or, data one might get from a clinical biostatisticians! they may go into academia and trial. It allows lots of application become

a professor. To get a more but there are some subjects that it specific idea of some projects field, many people get a masters pairs with more often, and biology is biostatisticians have worked on, or PHD in biostatistics, which is and news in the field of biostatistics, like important, you also need

- **Biostatistics**
- very Working on cures for Leukemia
- any Risk factors for mental health challenges

for are really interesting courses in the or may allow you taste a learn how to analyze and form Canada, the prospects in this In their day to day biostatisticians models for biological functions. field are moderate to based coding practice and new insight.

To continue in the biostatistics while is developing soft skills, to help you vital, as your work will span fields, and you need to be able to <u>childhood</u> explain your analysis to anyone. You also have to be a keen

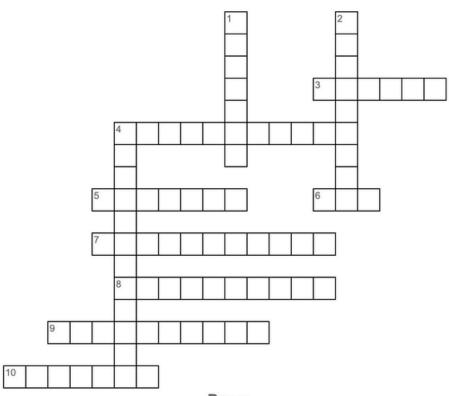
> After all these of field a try, you can aim for some

Good luck to all the aspiring

Indeed. (2025). What Does a Biostatistician Do? (Plus Requirements) https://www.indeed.com/career-advice/careers/what-does-abiostatistician-do

(2025). Government Canada. Biostatistician in Canada https://www.jobbank.gc.ca/marketreport/outlook-occupation/5442/ca;jsessionid=2DB2C53AF3ED52375BB5CBAA86139C10.jobsearc

Puzzle of the Month



Across

- 3 He's famous for his little theorem.
- 4 The October DSC event
- 5 The lesser known Subject of Isaac Newton's work
- 6 Abbreviation of the latin way to end a proof
- 7 Math 310
- 8 The word for E in MLE
- 9 A well known math toy with different coloured sides
- 10 The shape of the planet's orbit, as proved by Kepler

Down

- Abbreviation for the national statistics institution of Canada
- 2 Proper Vocab word meaning one-to-one
- 4 $f(a) + f'(a)/1 \cdot (x-a) + f''(a)/2! \cdot (x-a)...$

October Events Recap

