Pulse shape in the SPC prototype

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Abstract

In this note we recall how the pulse shape is obtained from scratch equations as the pulse is given by the charge induced by the ions movement. The simulation of the electron drift shows that the measure of the time spread allows to obtain a quite good resolution on the radial position of its generation into the gas.

1 The electric field

A big sphere of a radius r_1 is set at 0 volt. Inside, a little sphere of a radius r_2 is set at V_0 volts. We want to calculate the electric field at a given r. We apply Gauss's Law :

$$\int \overrightarrow{E}\, d\overrightarrow{S} = \frac{Q}{\epsilon}$$

where ϵ is the gas permittivity (of the order of 0.1 pF/cm). The integration on a sphere of radius r gives :

$$E(r) = \frac{Q}{4\pi\epsilon} \frac{1}{r^2}$$

Using $E = -\nabla V$ we have

$$V(r) = -\int E(r)\, dr = -\frac{Q}{4\pi\epsilon} \int \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon} \frac{1}{r} + C^t$$

Now applying the boundary conditions $V(r_2) = V_0$; $V(r_1) = 0$, we obtain:

$$V = V_0\, \rho(1/r - 1/r_1) \tag{1}$$

with

$$1/\rho = 1/r_2 - 1/r_1$$

and

$$V_0 = \frac{Q}{4\pi\epsilon\rho}$$

From this equation we can deduce the capacitance of the detector

$$C = 4\pi\epsilon\rho$$

of the order of 1 pF, and E can be written as :

$$E(r) = \frac{V_0}{r^2}\, \rho \tag{2}$$

2 The detector response

Now we want to find the signal induced by the ions displacement created by an avalanche at time $t = 0$. The ion mobility in the gas, μ, is taken as independant of the electric field but inverserly proportionnal to the pressure P :

$$\mu = \frac{\mu_0}{P}$$

where μ_0 is the mobility at 1 atm of the order of $2\,10^{-6} cm^2 V^{-1} \mu s^{-1}$. By definition :

$$\mu = \frac{1}{E(r)} \frac{dr}{dt}$$

Replacing $E(r)$ by (2) :

$$r^2\, dr = \alpha\, dt$$

with

$$\alpha = \mu_0 \frac{V_0}{P} \rho$$

Note the parameter α is a function of V_0/P. The integration on the drift range gives :

$$\int_{r_2}^{r} u^2\, du = \int_0^t \alpha\, du$$

So the ions arrive at the time t at the distance r given by :

$$r = (r_2^3 + 3\alpha t)^{\frac{1}{3}} \qquad (3)$$

The duration of the ions drift corresponds to $r = r_1$:

$$t_{max} = \frac{r_1^3 - r_2^3}{3\alpha} \qquad (4)$$

For the tested SPC at Saclay with Argon, $t_{max} \approx 10\, s$ (Fig.1) !

Now we apply the Shockley-Ramo theorem to compute the charge on the small sphere induced by the movement of the ions:

$$dQ_{ind} = -q_{ions} dr \frac{E(r)}{V_0}$$

$$dQ_{ind} = -q_{ions} v_{ions} dt \frac{E(r)}{V_0}$$

$$dQ_{ind} = -q_{ions} \mu E dt \frac{\rho}{r^2}$$

$$dQ_{ind} = -q_{ions} \alpha \rho \frac{dt}{r^4}$$

If we substitute the expression (3) of r(t) in this equation, we get :

$$dQ_{ind} = -q_{ions}\alpha\rho(r_2^3 + 3\alpha t)^{-\frac{4}{3}}dt \tag{5}$$

By integration on time, we get the charge pulse given by the detector for an avalanche created at time $t = 0$:

$$Q_{ind} = -q_{ions}\rho[\frac{1}{r_2} - \frac{1}{(r_2^3 + 3\alpha t)^{\frac{1}{3}}}] \tag{6}$$

Note, at the maximum ions drift time t_{max} given by relation (4), which is the duration of the detector pulse, the induced charge is equal to the avalanche ones :

$$Q_{ind} = -q_{ions}$$

Finally, the detector response depends on the parameter α which is a function of V_0/P as shown on Fig.2.

3 The amplifier transfer function

As the transfer function of the amplifier, we can take an exponantial :

$$A_0 = e^{-X} \tag{7}$$

with

$$X = \frac{(t - t_0)}{\tau}$$

or a more general function which parametrizes the rising t_r and the falling t_f time :

$$A = e^{-X_1} - e^{-X_2} \tag{8}$$

with $X_1 = a(t - t_0)$ and $X_2 = b(t - t_0)$, $a = 1/t_f$ $b = 1/t_f + 1/t_r$.

For the Canberra amplifier used during the test at Modane, the rising time is very small respect to the falling time, so the transfer function is approximated by A_0 with $\tau = 121\mu s$ (thanks to Igor Irastorza) , see Fig.3b.

4 The avalanche signal

The electronic signal given by an avalanche created at $t = 0$ is then given by the convolution of the detector response by the amplifier transfer function :

$$S(t) = \int_0^t A(t-u)\,\frac{dQ_{ind}}{du}(u)\,du \tag{9}$$

With the Canberra amplification we need to compute :

$$S(t) = -q_{ind}\alpha\rho e^{-\frac{t}{\tau}} \int_0^t e^{\frac{u}{\tau}}(r_2^3 + 3\alpha u)^{-\frac{4}{3}}\,du \tag{10}$$

The numerical integration shows that the avalanche signal can be parametrized by an expression similar to (8) :

$$S(t) = -q_{ions}k(e^{-at} - e^{-bt}) \tag{11}$$

with the scaling constant k close to the deficit balistic. The parameters a, b and k are function of V_0 and τ. Two typical signals from an avalanche are shown on Fig.3 and Fig.4. The variation of the signal parameters versus V_0/P (detector) are shown on Fig.5,6 and versus τ (amplificator) on Fig.7,8. The main feature is that the deficit balistic is always greater than 50%.

5 The photon signal

In case of a photon converted into N electrons in SPC, the electrons drift towards the small sphere where they create avalanches at a mean time taken as time origin, $t = 0$, with a gaussian of σ time-spread due to longitudinal diffusion. So the overall signal S_{tot} is the sum on the N electrons of the convolution of the expression (11) with a normalized gaussian :

$$S_{tot} = \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{t} S_i(t-u)e^{-\frac{u^2}{2\sigma^2}}du \tag{12}$$

with

$$S_i(t) = -q_i k(e^{-at} - e^{-bt})$$

where q_i is the avalanche charge created by the i^{th} electron. If Q is the total charge of all the avalanches :

$$Q = \sum_{i=1}^{N} q_i$$

we obtain :

$$S_{tot} = -\frac{Qk}{2}\left(e^{(\frac{\sigma^2 a^2}{2}-at)}[erf(\frac{t-\sigma^2 a}{\sigma\sqrt{2}})+1] - e^{(\frac{\sigma^2 b^2}{2}-bt)}[erf(\frac{t-\sigma^2 b}{\sigma\sqrt{2}})+1]\right) \tag{13}$$

Note, for $\sigma = 0$, the signal is given by the relation (11) where q_{ions} is replaced by Q.

The Fig.9 shows the expected effect of a typical electron drift diffusion on the signal according to the previous formula.

Let's call t_i the time corresponding to the first inflexion point on the curve $S_{tot}(t)$. This time t_i is closed to the σ time-spread. As we do not know the start time, we better consider the time-difference $\Delta t = t_i - t_0$ between t_i and t_0 the time intercept of the tangent at t_i with the time base-line.

The Δt time intercept can be parametrized (Fig.10) :

$$\Delta t = \frac{p_1 \sigma}{\sigma + p_2}$$

The two parameters $p_1 \approx 55\mu s$ and $p_2 \approx 27\mu s$ are roughly independent of the detector response (V_0/P) (Fig.11) and also of the amplifier (τ) (Fig.12).

So the measurement of Δt will give the value of the time dispersion σ :

$$\sigma = \frac{p_2 \Delta t}{p_1 - \Delta t} \tag{14}$$

6 Measurement of the photon conversion position

Once the time-dispersion σ is measured, the simulation gives the conversion position r as a function of σ and V_0/P.

To simulate the drift of the electrons towards the small sphere, we need a good estimation of the drift velocity and diffusion into a large range of E/P between about 1 and 10000 Volt/cm/atm. Their values are given by Magboltz (thanks to David Attié) and parametrized as shown on Fig.13,14 and 15. The simulation consists to track each electron by small steps (1 mm) untill it arrives on the small ball. The simulation shows that the conversion position of a photon is strongly correlated to the time dispersion of the electrons (Fig.16,17,18 and 19).

For the two gas mixture Ar-Isobutane(98-2) and Ar-CH4(98-2), the conversion position of the photon is given, with a precision of the order of 5 cm, by (units= $cm, \mu s, V, atm$):

$$r = c\sigma^{\frac{1}{3}}(\frac{V_0}{P})^{0.4} \tag{15}$$

where the constant c is equal to 1 for the mixture with Isobutane, and close to 1.1 with Methane.

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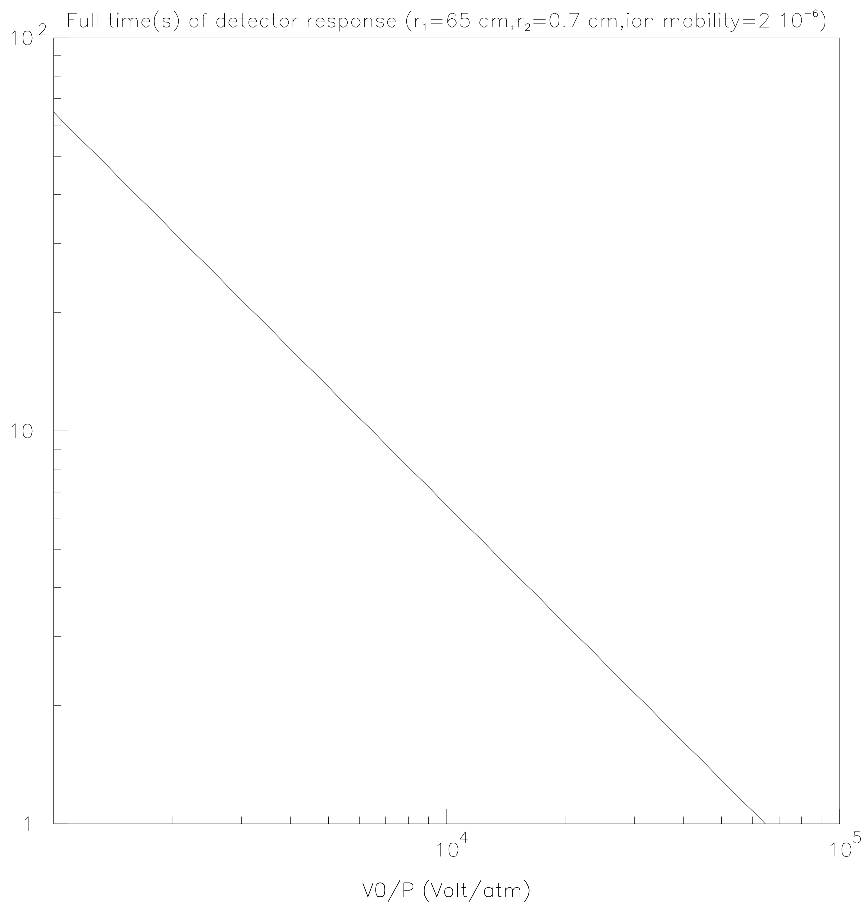


Figure 1: Simulation of full time of detector response.

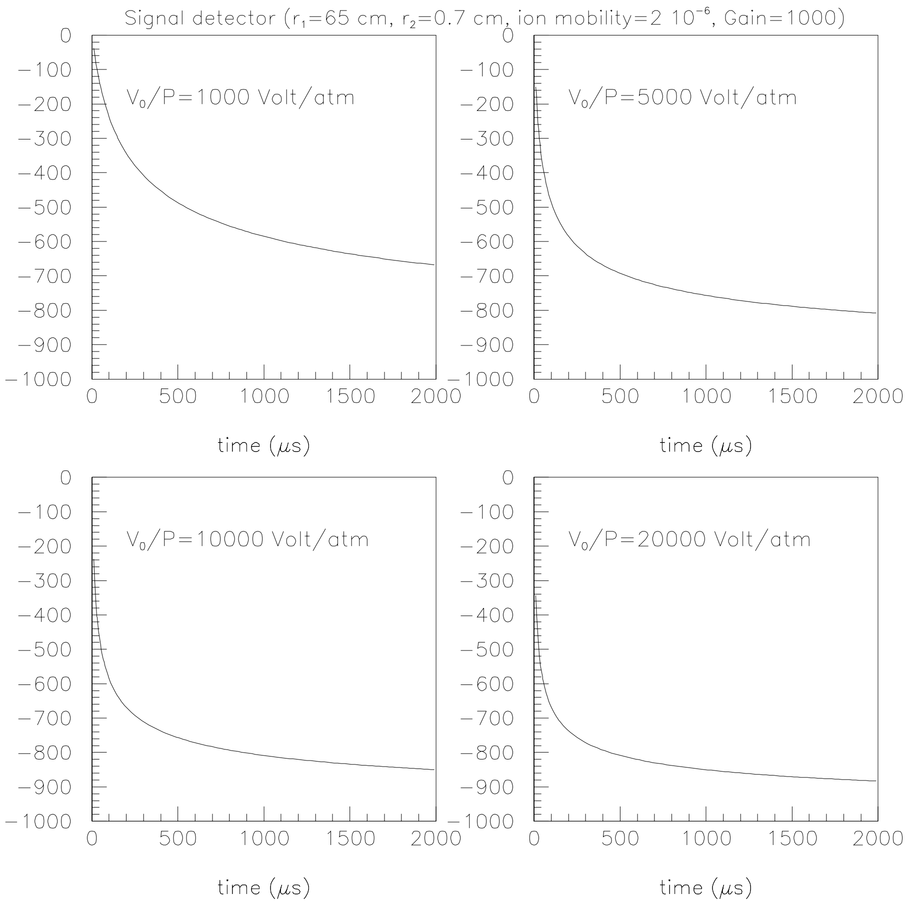


Figure 2: Simulation of the detector response.

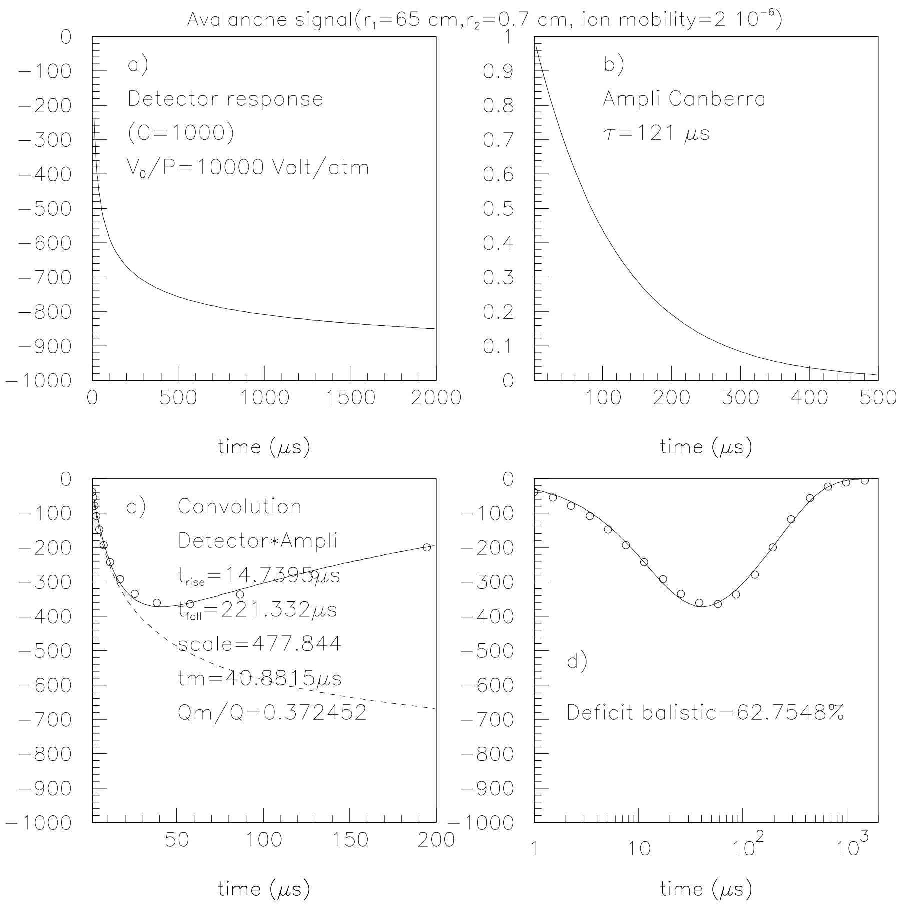


Figure 3: Simulation of the signal given by an avalanche(t=0), scale=q_{ions} k (see text relation (11))

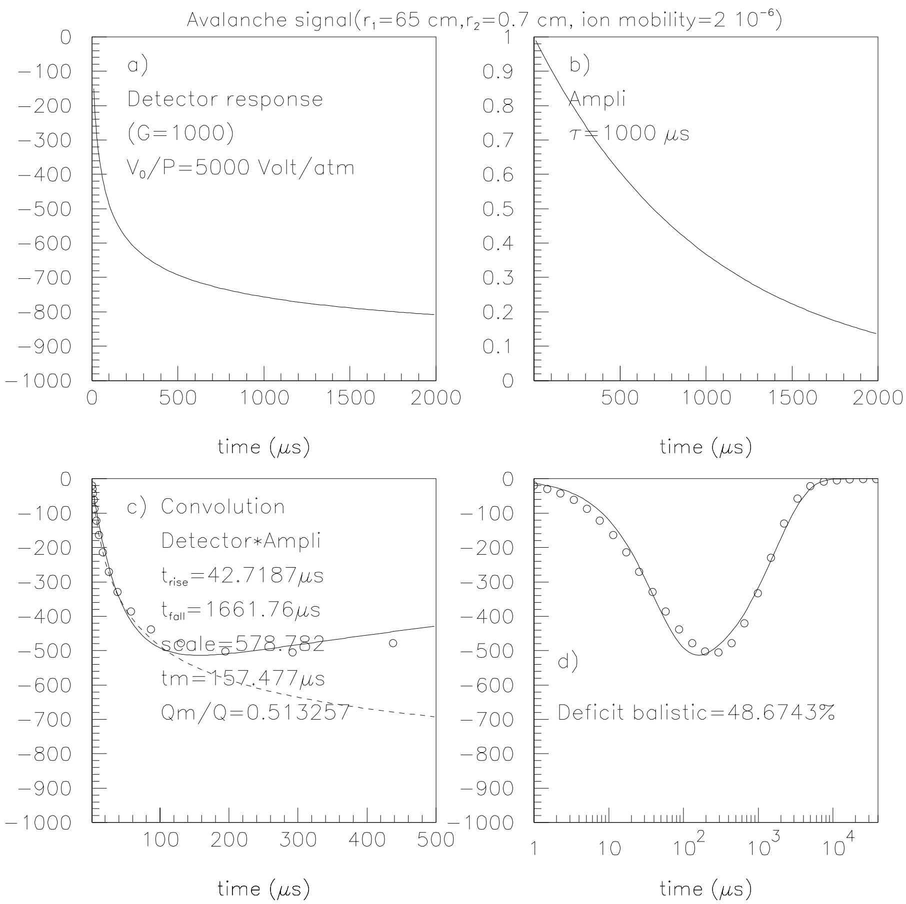


Figure 4: Simulation of the signal given by an avalanche(t=0)

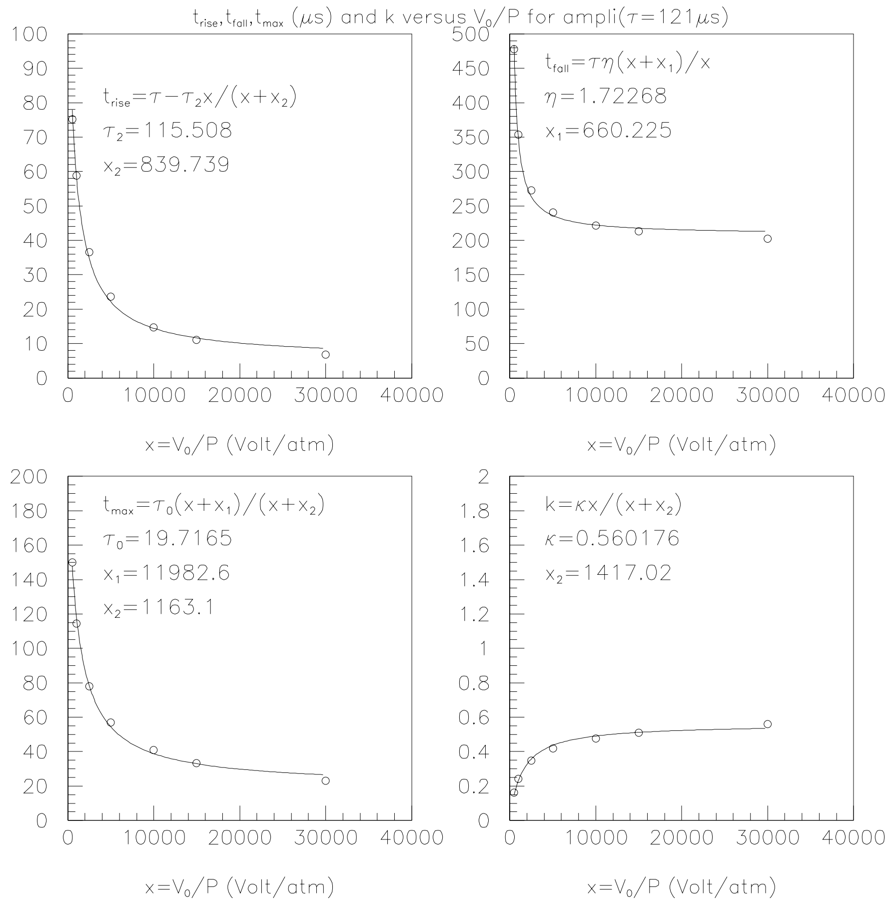
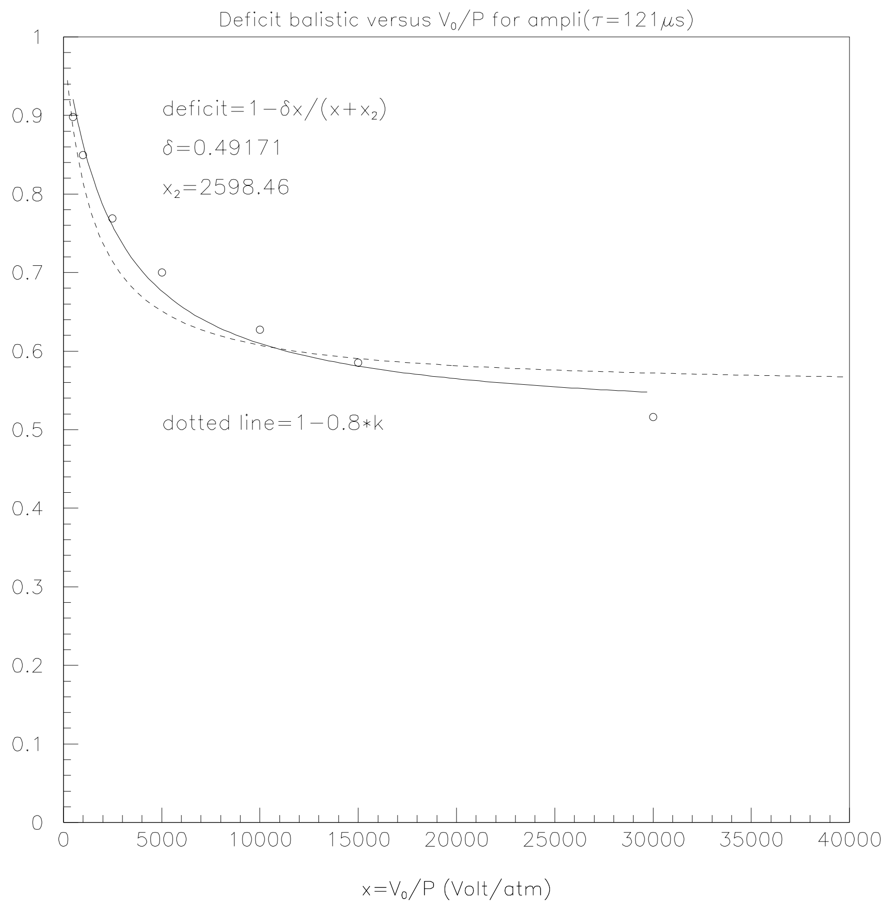


Figure 5: Simulation of the avalanche signal parameters versus V_0/P



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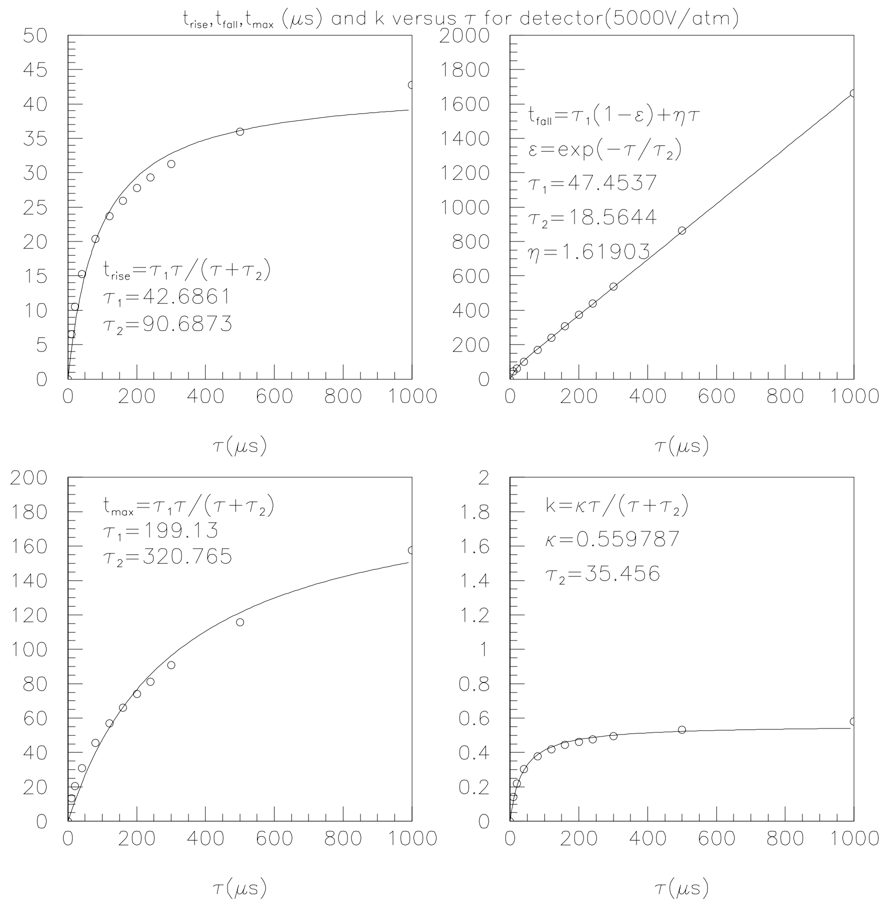


Figure 7: Simulation of the avalanche signal parameters versus τ

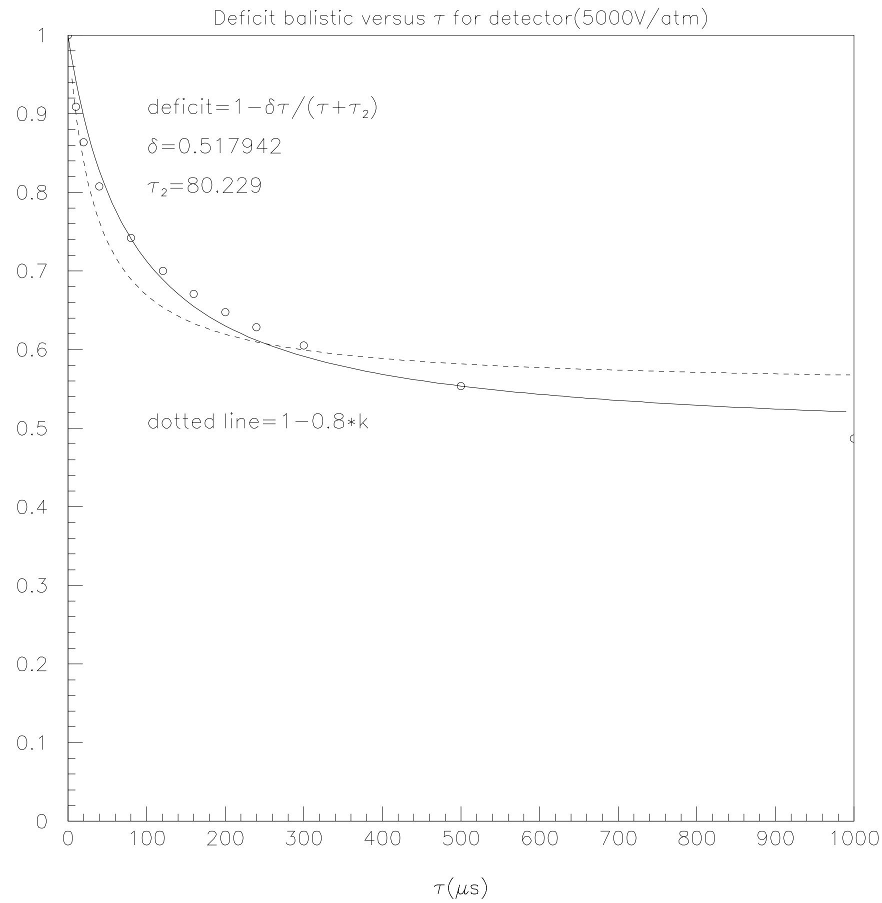


Figure 8: Simulation of the deficit balistic versus τ

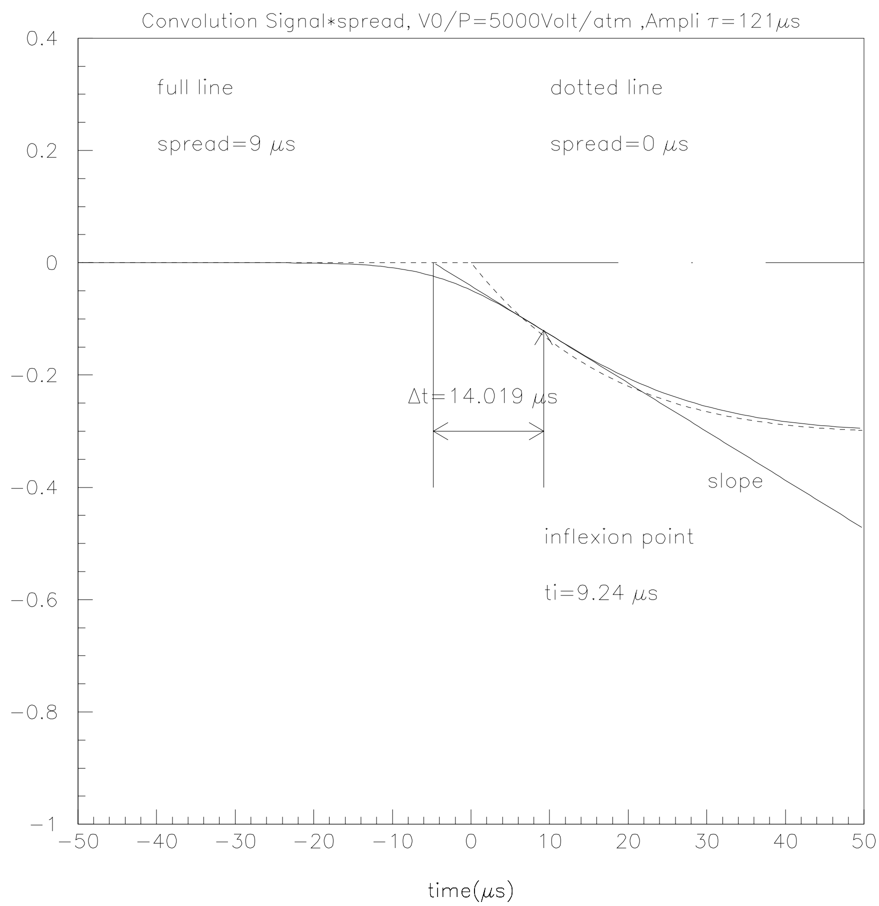
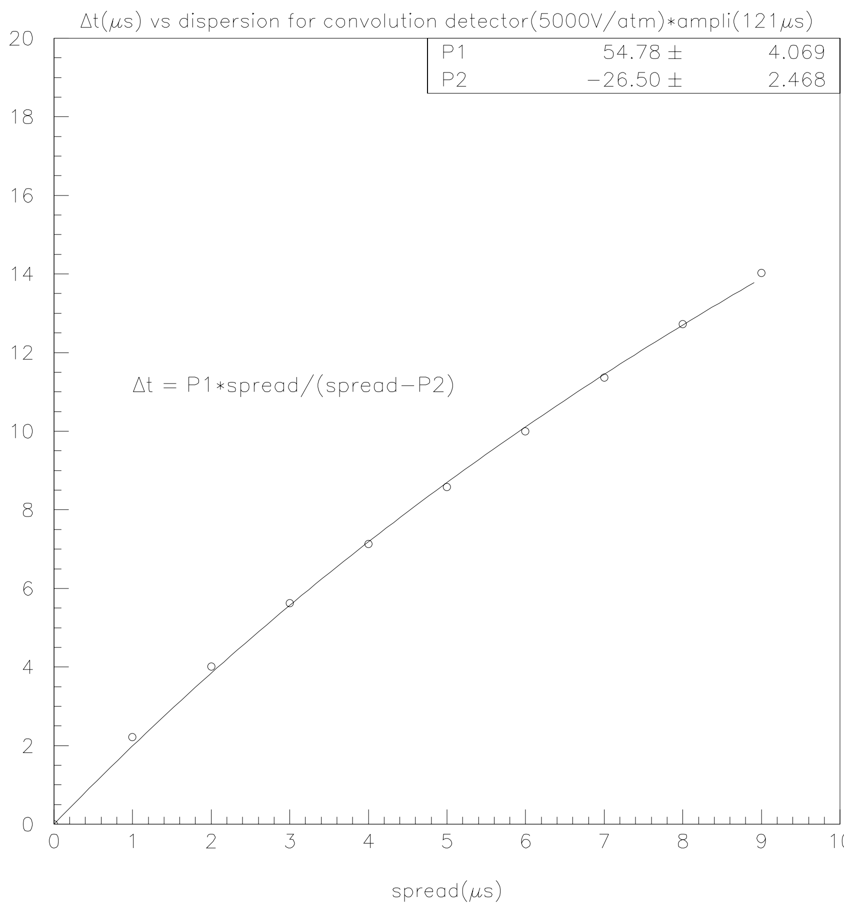


Figure 9: Effect of the electron drift diffusion on the signal



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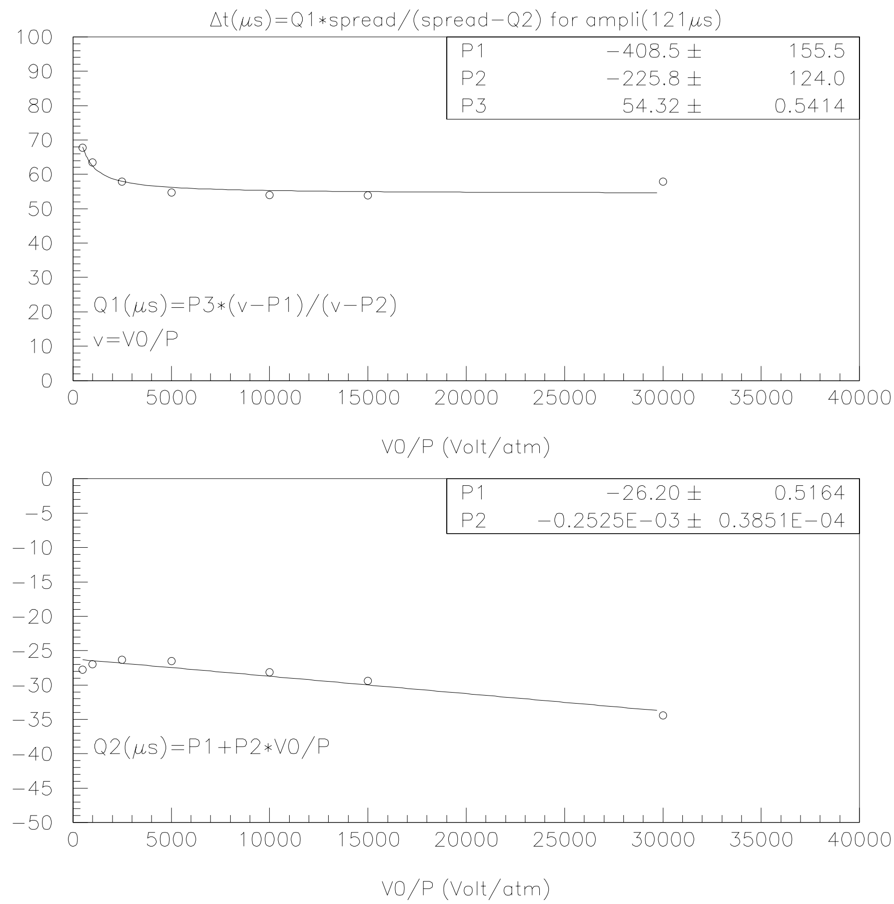

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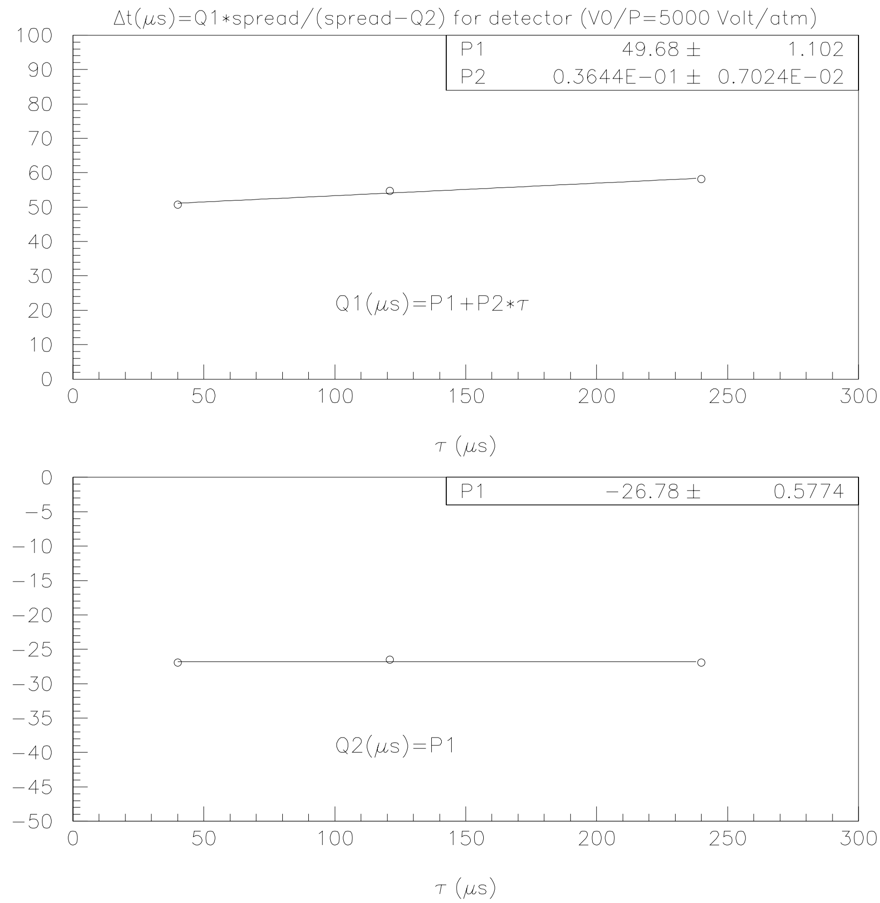

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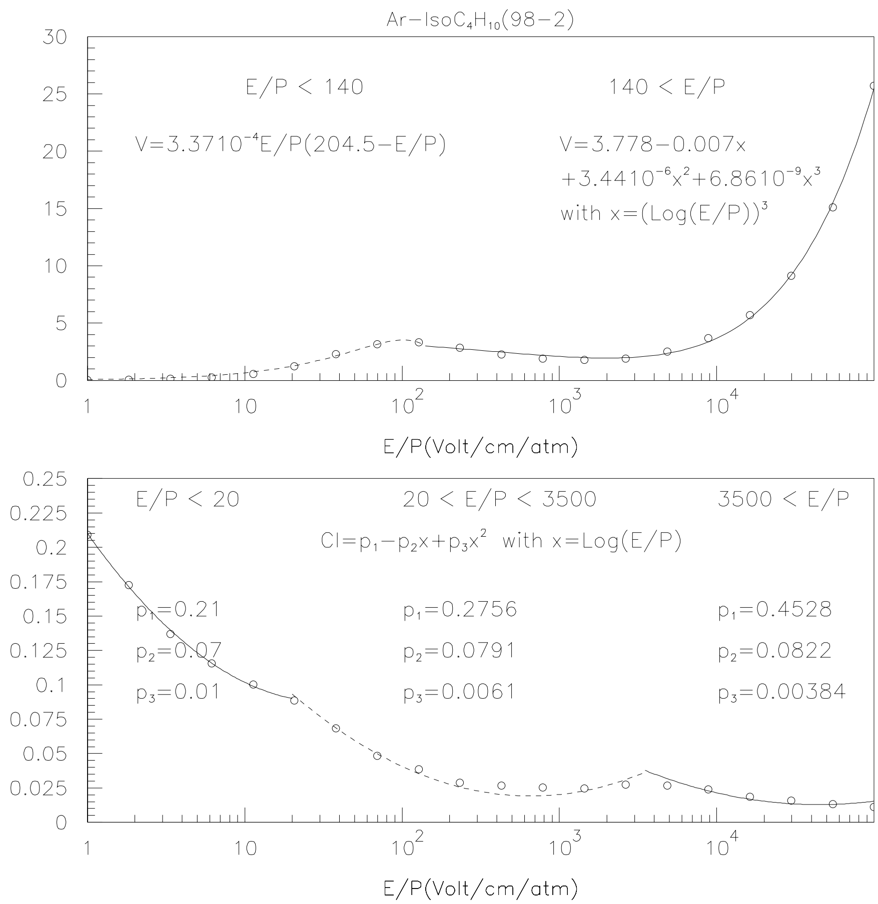

Figure 13: Gas mixture Ar-IsoC4H10(98-2), top: electron drift velocity (cm/μs), bottom: longitudinal diffusion coefficient (cm/$\sqrt{cm}$)

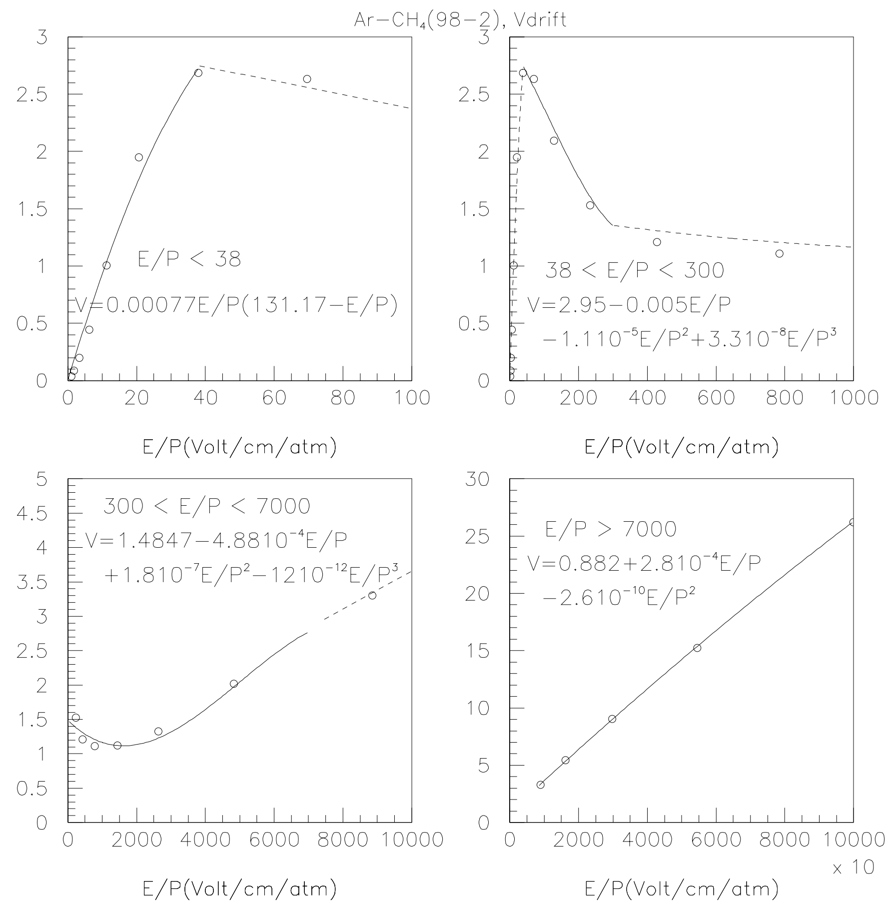

Figure 14: Gas mixture Ar-CH4(98-2), electron drift velocity (cm/μs)

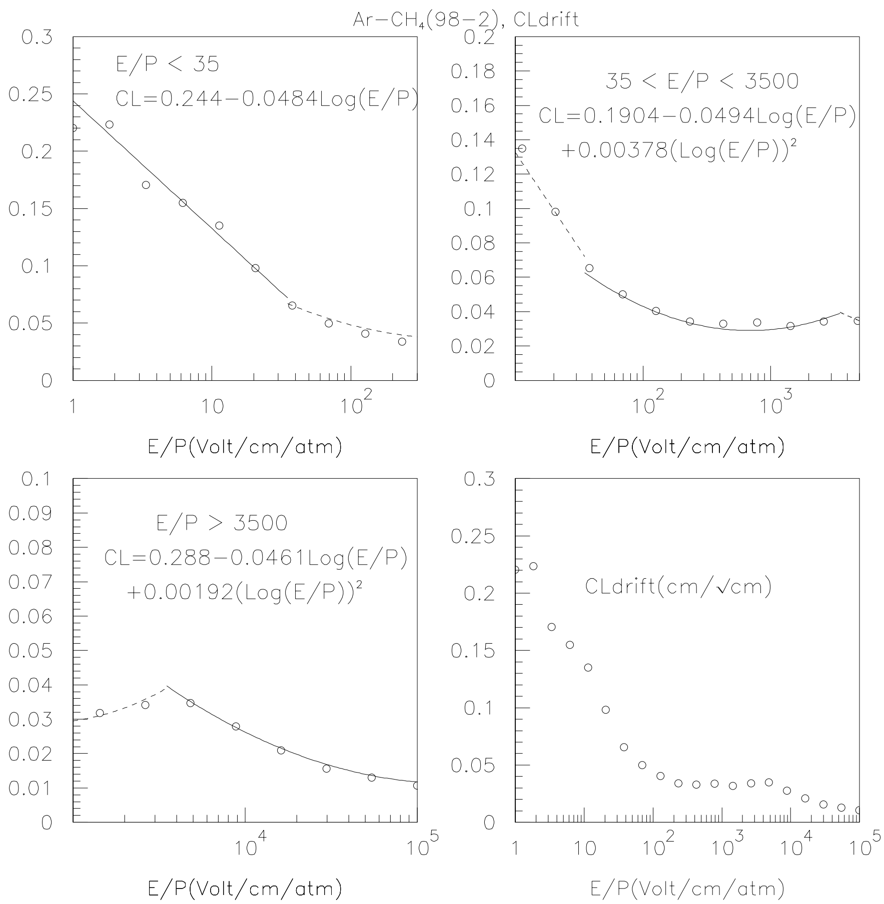

Figure 15: Gas mixture Ar-CH4(98-2), longitudinal diffusion coefficient (cm/$\sqrt{cm}$)

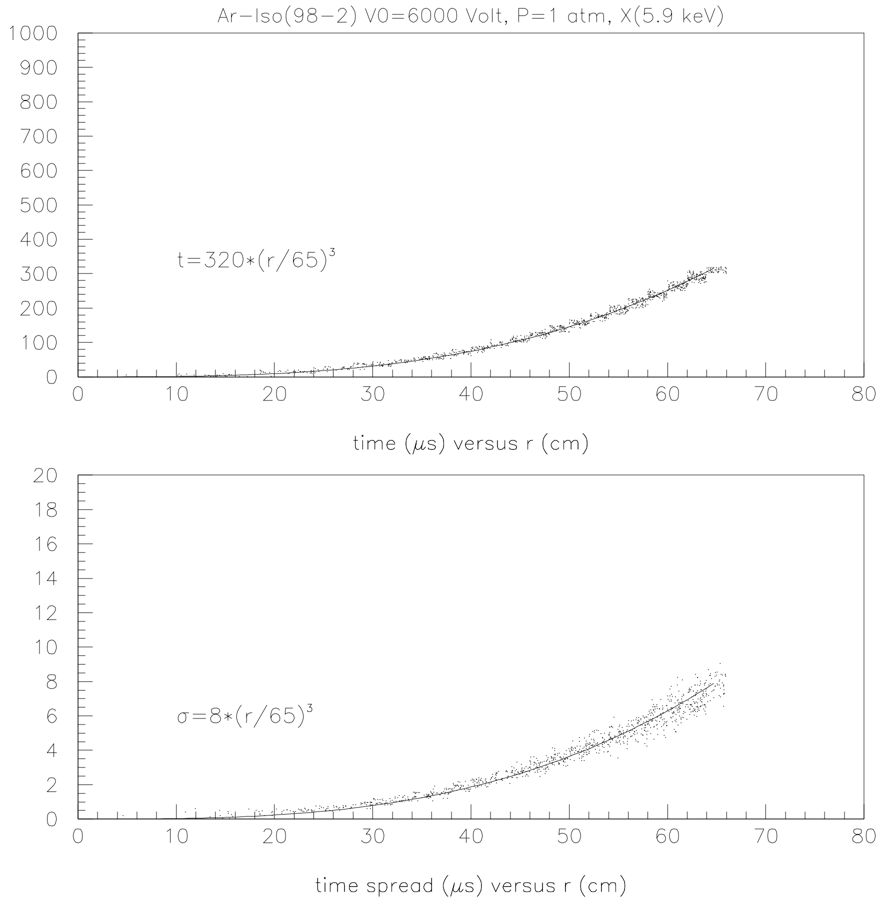

Figure 16: Electron drift time and time spread into Ar-IsoC4H10(98-2) ,$V_0 =$ 6000 Volt,$P = 1$ atm.

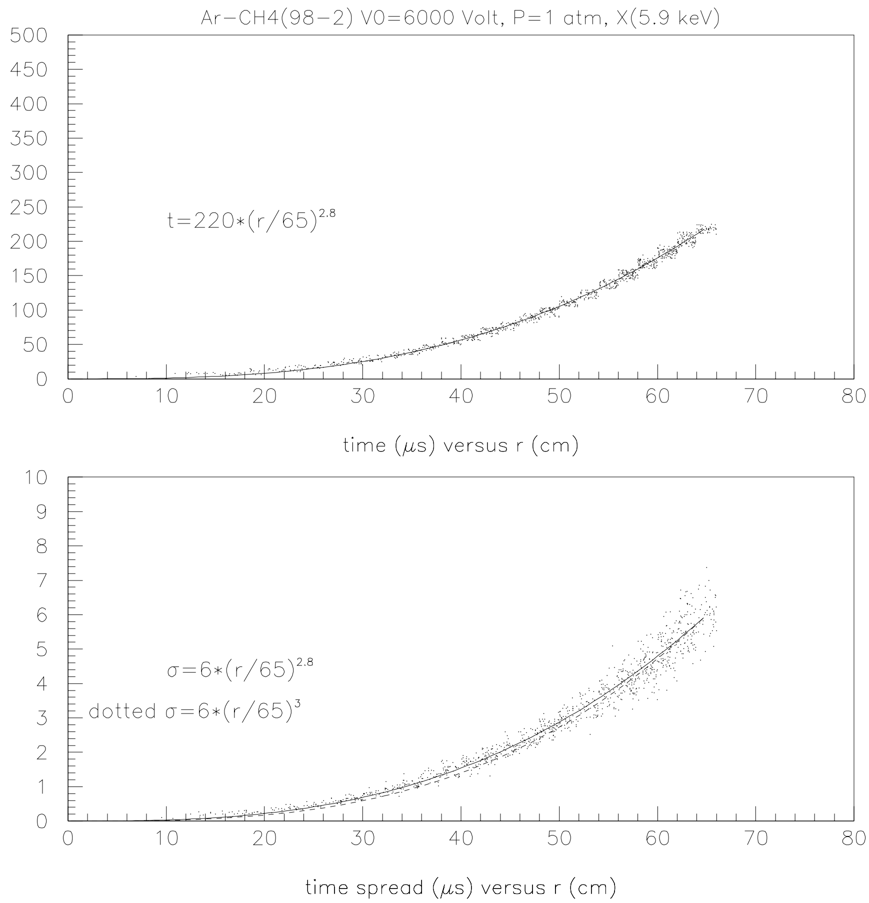

Figure 17: Electron drift time and time spread into Ar-CH4(98-2) ,$V_0 = 6000$ Volt,$P = 1$ atm.

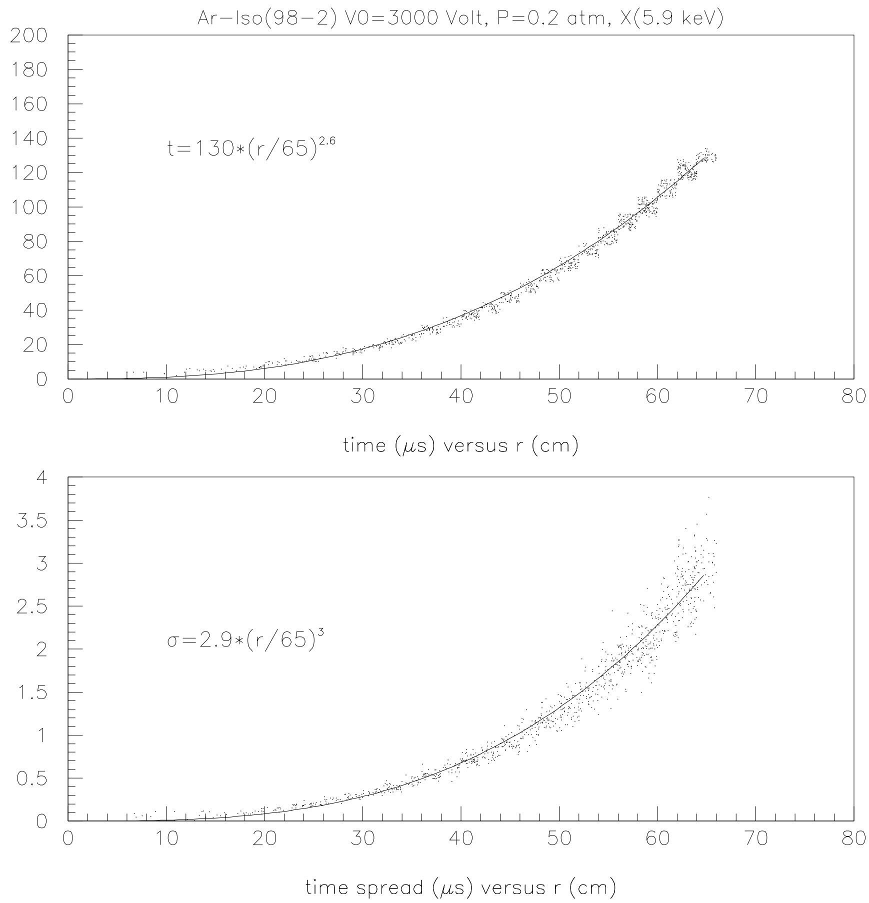

Figure 18: Electron drift time and time spread into Ar-IsoC4H10(98-2) ,$V_0 =$ 3000 Volt,P=0.2 atm.

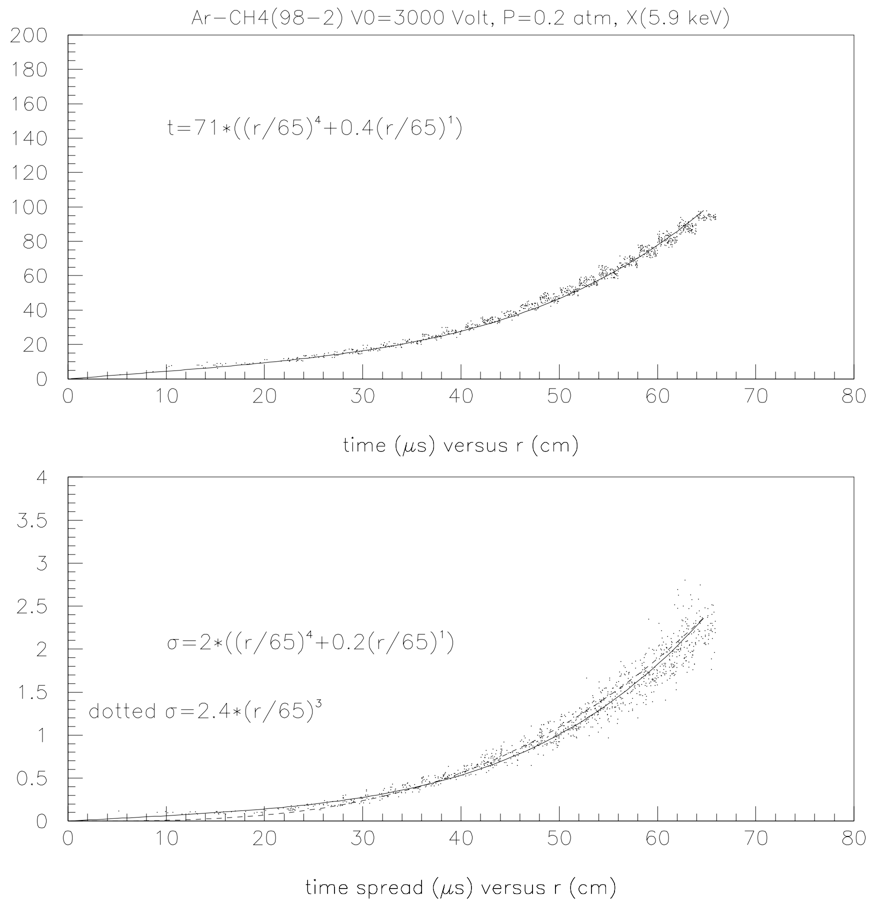

Figure 19: Electron drift time and time spread into Ar-CH4(98-2) ,$V_0 = 3000$ Volt,P=0.2 atm.