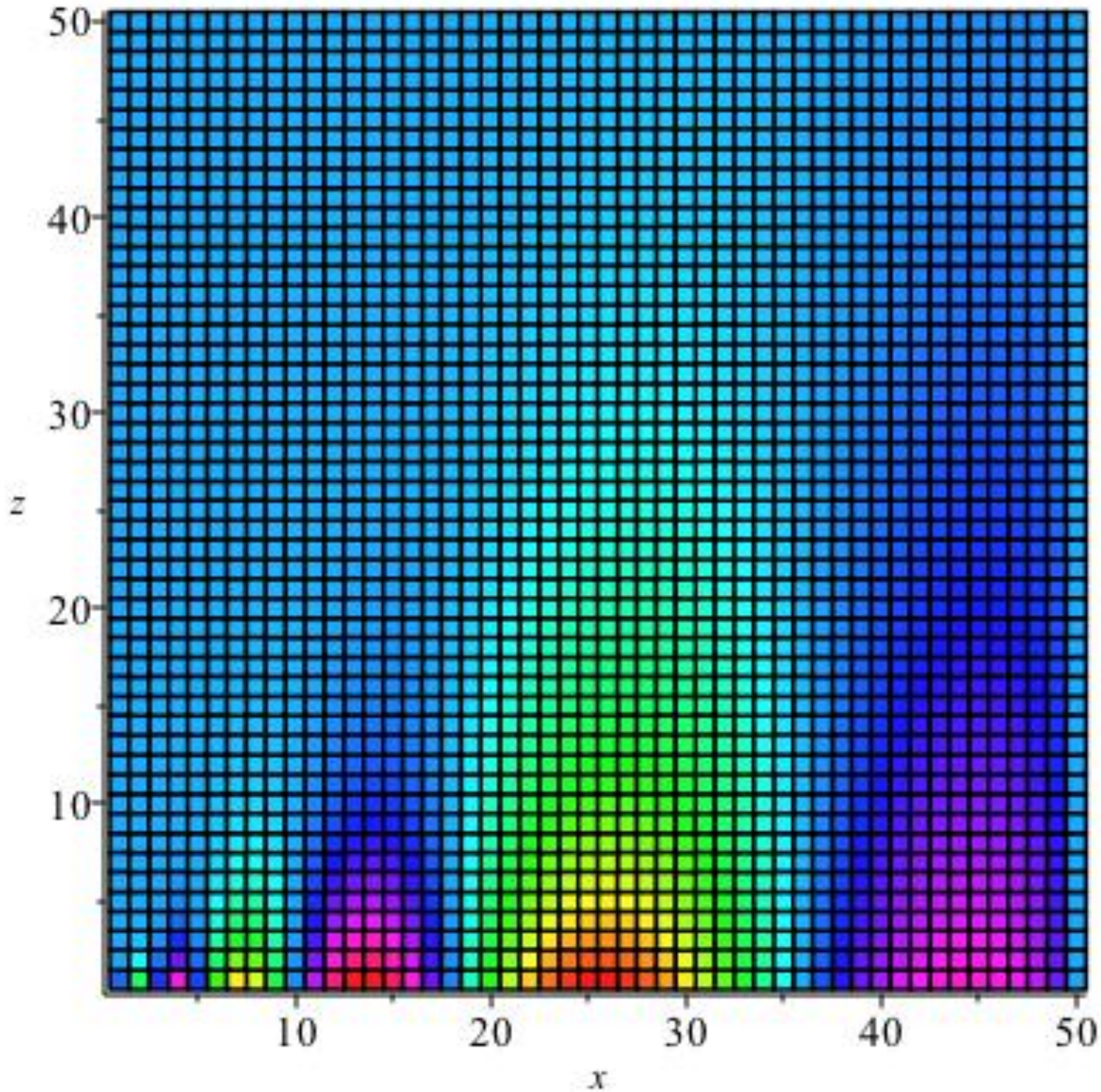
What has been achieved?

- CHANG-ES has achieved a systematic study of edge-on galaxies, their cosmic rays and magnetic fields.
- We have had to explain:
- 1. X type fields—we can now expect them to be associated with m=0 dynamo modes
- 2. Magnetic spiral arms- we know that these are produced by the classic dynamo due to Moffat, 1978. Rotating arms are the subject of today's report.

- 3. Parity inversion above the plane and sign changes across the galactic disc --check
- 4. spiral structure rising into the halo and spiral arms displaced relative to stellar arms—yes and possibly
- 5. importance of winds, rotation, radial motion are included ---I emphasize that the fields do not have to be ad hoc when fitting data. We have analytic solutions.
- How has this been possible? Generalized Scale Invariance—we do not have to think of scale invariance as an esoteric piece of the set of solutions—complex systems, undisturbed externally, evolve to scale invariant form.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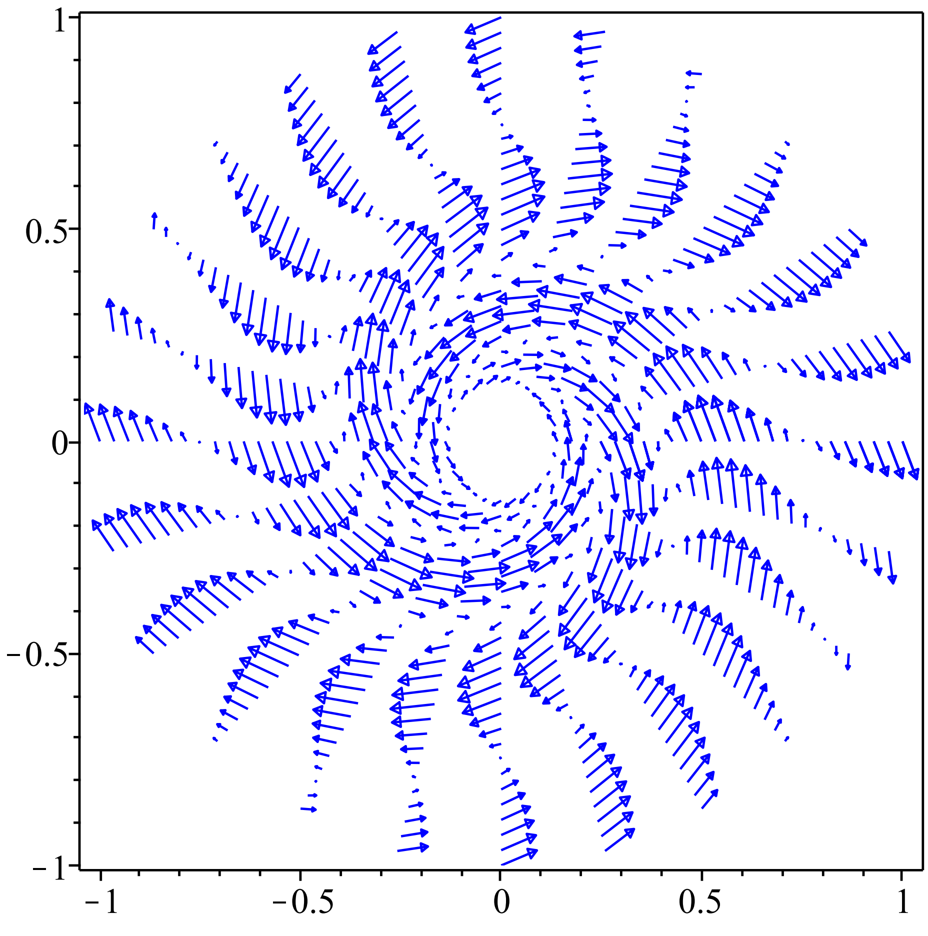

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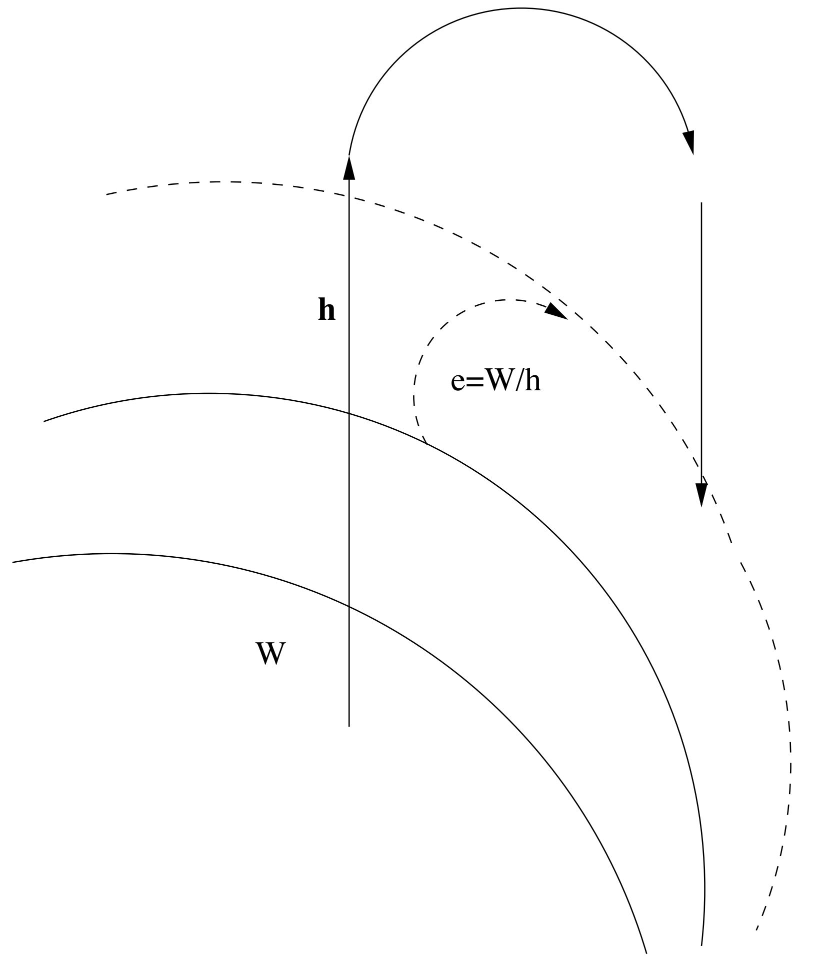
- So what is new? After Bochum 2016, we published in 2017 several papers on the scale invariant steady state. This year we introduced axially symmetric dynamo fields with amplitude time dependence. Now we introduce rotating spiral-symmetric fields also with varying amplitude. This is relevant to magnetic field evolution.
- Now we are ready to attempt real galaxies—see Alex and Jennifer

Rotating Dynamo Magnetic Fields

Hiding the magnetic field in Edge-on galaxies?

Magnetic Field Rotation



Classical Dynamo Theory

We refer to the classical mean-field dynamo equations Moffat (1978) in the form for the magnetic vector potential Henriksen (2017b)

$$\partial_t \mathbf{A} = \mathbf{v} \wedge \nabla \wedge \mathbf{A} - \eta \nabla \wedge \nabla \wedge \mathbf{A} + \alpha_d \nabla \wedge \mathbf{A}. \quad (2)$$

In these equations $\mathbf{v}$ is the mean velocity, η is the resistive diffusivity and α_d is the magnetic 'helicity' resulting from sub-scale magnetohydrodynamic turbulence and $\mathbf{A}$ is the potential. We proceed by setting $\eta = 0$ so that the medium is infinitely conducting.

Under the assumption of temporal scale invariance, the time dependence will simply be a power law or (in the limit of zero similarity class) an exponential factor. Hence the geometry of the magnetic field remains 'self-similar' over the time evolution, and we can therefore study the geometry without requiring a fixed epoch. Although we have no way of bringing

Scale Invariant Variables

The scale invariance is found following the technique advocated in Carter&Henriksen (1991) and Henriksen (2015). We first introduce a time variable T along the scale invariant direction according to

$$e^{\alpha T} = 1 + \tilde{\alpha}_d \alpha t, \qquad (3)$$

where $\tilde{\alpha}_d$ is a numerical constant that appears in the scale invariant form for the helicity, α_d, which form is to be given below. The constant numerical factor $\tilde{\alpha}_d$ in equation (3) is purely for subsequent notational convenience. The quantity α should not be confused with the helicity as it is an arbitrary scale used in the temporal scaling. The cylindrical coordinates $\{r, \phi, z\}$ are transformed into scale invariant variables $\{R, \Phi, Z\}$ according to (e.g. Henriksen (2015))[1]

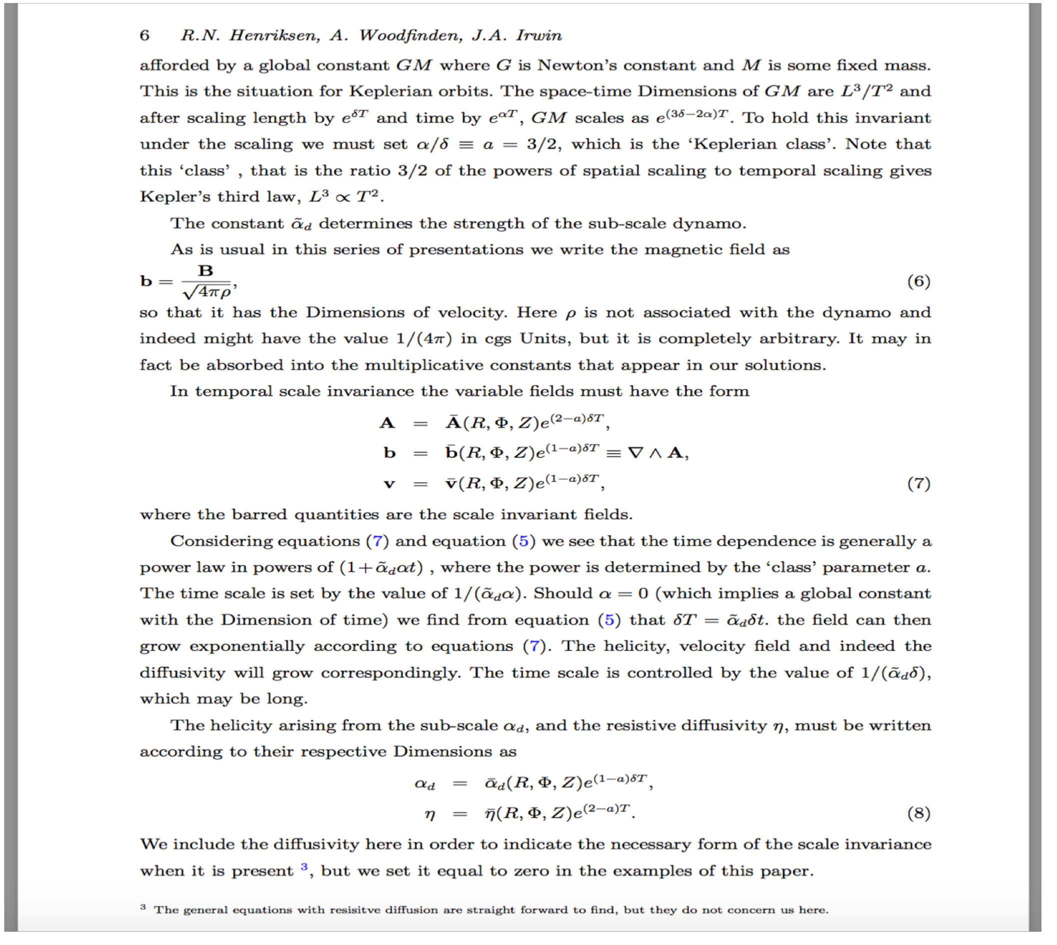
$$r = Re^{\delta T}, \quad \Phi = \phi + (\epsilon/\delta + q)\delta T, \quad z = Ze^{\delta T}, \qquad (4)$$

where δ is another arbitrary scale that appears in the spatial scaling, and ϵ is a number that fixes the rate of rotation of the magnetic field in time. We add q to the arbitrary ϵ for subsequent algebraic convenience (see equation (9) below). In our subsequent discussion $1/q > 0$ appears as the pitch angle of a spiral mode that is lagging relative to the sense of increasing angle ϕ. [2] We note that

$$e^{\delta T} = (1 + \tilde{\alpha}_d \alpha t)^{1/a}, \qquad (5)$$

where the 'similarity class' $a \equiv \alpha/\delta$ is a parameter of the model defined as the self-similar 'class' Carter&Henriksen (1991), which reflects the Dimensions of a global constant. This is discussed in some detail in Henriksen, Woodfinden &Irwin (2018), but a simple example is

Scale Invariant Fields



afforded by a global constant GM where G is Newton's constant and M is some fixed mass. This is the situation for Keplerian orbits. The space-time Dimensions of GM are L^3/T^2 and after scaling length by $e^{\delta T}$ and time by $e^{\alpha T}$, GM scales as $e^{(3\delta-2\alpha)T}$. To hold this invariant under the scaling we must set $\alpha/\delta \equiv a = 3/2$, which is the 'Keplerian class'. Note that this 'class' , that is the ratio 3/2 of the powers of spatial scaling to temporal scaling gives Kepler's third law, $L^3 \propto T^2$.

The constant $\tilde{\alpha}_d$ determines the strength of the sub-scale dynamo.

As is usual in this series of presentations we write the magnetic field as

$$\mathbf{b} = \frac{\mathbf{B}}{\sqrt{4\pi\rho}}, \tag{6}$$

so that it has the Dimensions of velocity. Here ρ is not associated with the dynamo and indeed might have the value $1/(4\pi)$ in cgs Units, but it is completely arbitrary. It may in fact be absorbed into the multiplicative constants that appear in our solutions.

In temporal scale invariance the variable fields must have the form

$$\begin{aligned} \mathbf{A} &= \bar{\mathbf{A}}(R,\Phi,Z)e^{(2-a)\delta T}, \\ \mathbf{b} &= \bar{\mathbf{b}}(R,\Phi,Z)e^{(1-a)\delta T} \equiv \nabla \wedge \mathbf{A}, \\ \mathbf{v} &= \bar{\mathbf{v}}(R,\Phi,Z)e^{(1-a)\delta T}, \end{aligned} \tag{7}$$

where the barred quantities are the scale invariant fields.

Considering equations (7) and equation (5) we see that the time dependence is generally a power law in powers of $(1+\tilde{\alpha}_d\alpha t)$, where the power is determined by the 'class' parameter a. The time scale is set by the value of $1/(\tilde{\alpha}_d\alpha)$. Should $\alpha = 0$ (which implies a global constant with the Dimension of time) we find from equation (5) that $\delta T = \tilde{\alpha}_d\delta t$. the field can then grow exponentially according to equations (7). The helicity, velocity field and indeed the diffusivity will grow correspondingly. The time scale is controlled by the value of $1/(\tilde{\alpha}_d\delta)$, which may be long.

The helicity arising from the sub-scale α_d, and the resistive diffusivity η, must be written according to their respective Dimensions as

$$\begin{aligned} \alpha_d &= \bar{\alpha}_d(R,\Phi,Z)e^{(1-a)\delta T}, \\ \eta &= \bar{\eta}(R,\Phi,Z)e^{(2-a)T}. \end{aligned} \tag{8}$$

We include the diffusivity here in order to indicate the necessary form of the scale invariance when it is present [3], but we set it equal to zero in the examples of this paper.

[3] The general equations with resisitve diffusion are straight forward to find, but they do not concern us here.

Rotational, Helicity, Velocity, Diffusivity

At this stage a substitution of the forms (7) into equations (2) yields three partial differential equations in the variables $\{R, \Phi, Z\}$. Thus, only the time dependence has been eliminated (e.g. Carter&Henriksen (1991)) through the assumption of temporal scale invariance. However we are seeking non axially symmetric spiral symmetry in the magnetic fields to match the observations summarized in Beck (2015) and Krause (2015). Any combination of the scale invariant quantities $\{R, \Phi, Z\}$ will render the barred quantities in equations (7) scale invariant, so we are free to seek a spiral symmetry by combining them.

We choose a combination inspired by our previous modal analysis Henriksen (2017b). We assume that the angular dependence may be combined with R in a *rotating* logarithmic spiral form as (recalling the definition of Φ from equation (4))

$$\kappa \equiv \Phi + q \ln R \equiv \phi + q \ln(r) + \epsilon T. \tag{9}$$

Moreover we combine the R and Z dependence into a dependence on the conical angle through

$$\zeta \equiv \frac{Z}{R}. \tag{10}$$

The linearity of equations (2) allows us to seek solutions in the complex form

$$\bar{\mathbf{A}}(R, \Phi, Z) = \tilde{\mathbf{A}}(\zeta) e^{im\kappa}. \tag{11}$$

On substituting these assumed forms into equation (2) one finds that a solution is possible in terms of κ and ζ, *provided that the ancillary quantities satisfy*

$$\begin{aligned} \bar{\alpha}_d &= \tilde{\alpha}_d \delta R, \\ \bar{\eta} &= \tilde{\eta} \delta R^2, \\ \bar{\mathbf{v}} &= \tilde{\alpha}_d \delta R \ \{u, v, w\}. \end{aligned} \tag{12}$$

The constant quantities denoted $\tilde{()}$ and the velocity components $\{u, v, w\}$ are Dimensionless.

Under these conditions the equations (2) become, *after setting the resistive diffusion equal to zero*,

$$\begin{aligned} K\tilde{A}_r - im\tilde{A}_z &= (1 + imq)v\tilde{A}_\phi - (1 + \zeta v)\tilde{A}'_\phi - w(\tilde{A}'_r + \zeta \tilde{A}'_z - imq\tilde{A}_z), \\ (K - imv)\tilde{A}_\phi &= -u(1 + imq)\tilde{A}_\phi + (\zeta u - w)\tilde{A}'_\phi + im(w\tilde{A}_z + u\tilde{A}_r) + (\tilde{A}'_r + \zeta \tilde{A}'_z - imq\tilde{A}_z), \\ im\tilde{A}_r + K\tilde{A}_z &= (1 + imq)\tilde{A}_\phi + (v - \zeta)\tilde{A}'_\phi + u(\tilde{A}'_r + \zeta \tilde{A}'_z - imq\tilde{A}_z), \end{aligned} \tag{13}$$

Where the prime indicates differentiation with respect to ζ and

$$K \equiv (2 - a) + im(\epsilon + v). \tag{14}$$

B field and reduced equations

The coherence of these equations (assuming physical solutions) in spirally symmetric, scale invariant form, *already predicts the existence of rotating spiral magnetic dynamo fields.*

The magnetic field that follows from the curl of the potential takes the form

$$\bar{\mathbf{b}} = \frac{\tilde{\mathbf{b}}}{R} e^{(im\kappa)}, \tag{15}$$

where

$$\begin{aligned} \tilde{\mathbf{b}} &= \{im\tilde{A}_z - \tilde{A}'_\phi,\ \ \tilde{A}'_r + \zeta\tilde{A}'_z - imq\tilde{A}_z,\ \ (1+imq)\tilde{A}_\phi - \zeta\tilde{A}'_\phi - im\tilde{A}_r\}, \qquad (16) \\ &\equiv \{\tilde{b}_r, \tilde{b}_\phi, \tilde{b}_z\} \end{aligned}$$

Equations (16),(15), and the second of equations (7) together *give the complete time dependent magnetic field.*

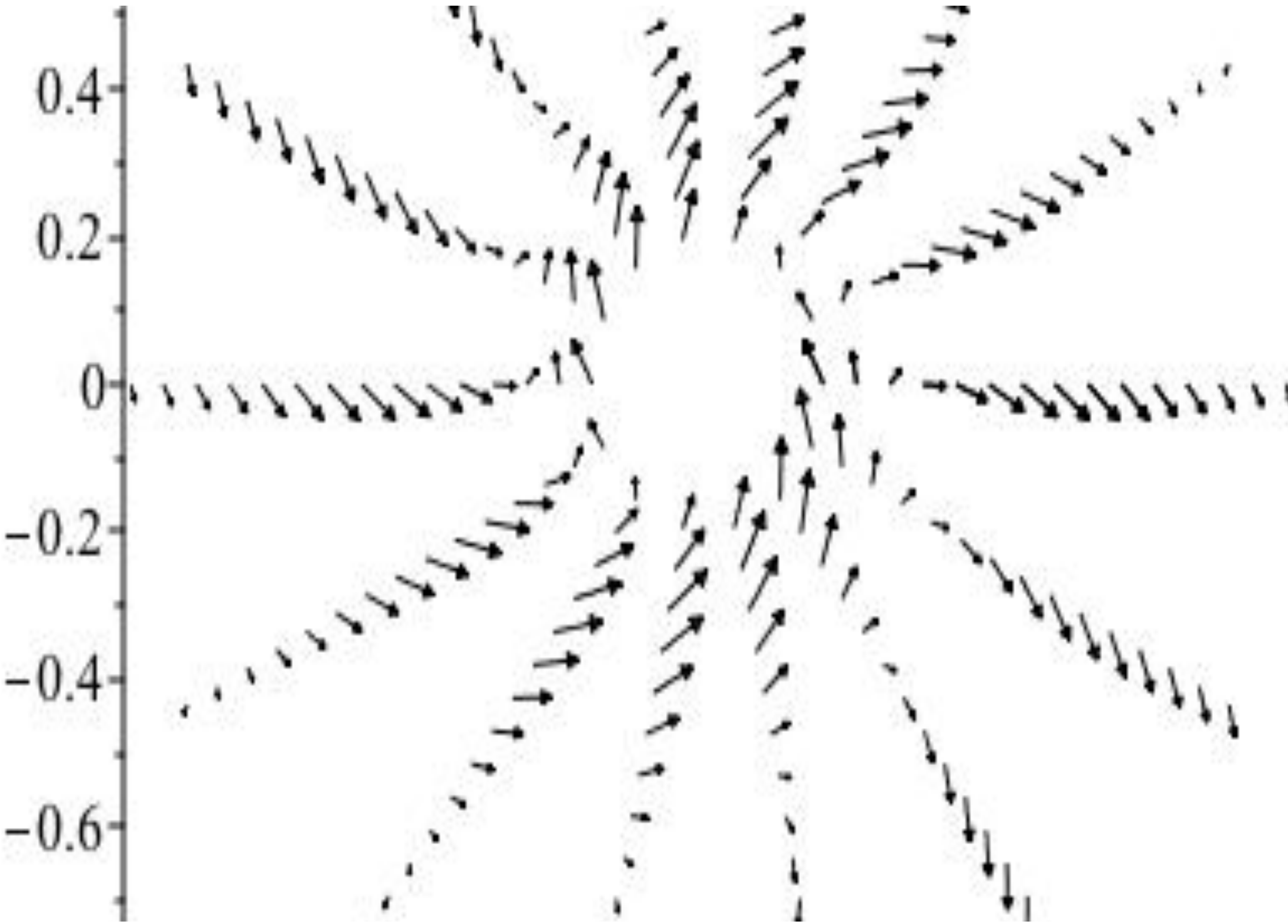
An examination of equations (13) indicates that one can rewrite equations (13) as one second order equation for $\tilde{A}_\phi$. The algebra is however formidable. One effective procedure is to replace with $\tilde{b}_\phi$ the combination $\tilde{A}'_r + \zeta\tilde{A}'_z - imq\tilde{A}_z$ everywhere in equations (13). We emphasize that the resulting equations do not 'know' that this combination of potentials is in fact the azimuthal field. We might have called the combination X.

Subsequently we use the first and third equations of the set (13) to solve for $\tilde{A}_r$ and $\tilde{A}_z$ in terms of $\tilde{b}_\phi$, $\tilde{A}_\phi$ and $\tilde{A}'_\phi$. Substituting these expressions for $\tilde{A}_r$ and $\tilde{A}_z$ into both the expression for $\tilde{b}_\phi$ following from the second equation of equations (13) and into the form of $\tilde{b}_\phi$ in terms of the potentials from equation (16), yields after tedious algebra two equations for $\tilde{b}_\phi$ and $\tilde{A}_\phi$ in the form

$$\begin{aligned} \tilde{b}_\phi(K^2 - m^2(1+u^2+w^2)) &= (K - imv)\big[(K^2 - m^2 + (Ku - imw)(1+imq))\tilde{A}_\phi \\ &+ ((w - u\zeta)K + im(u + w\zeta))\tilde{A}'_\phi\big], \qquad (17) \end{aligned}$$

$$\begin{aligned} \tilde{b}_\phi(K^2 &- m^2(1+qw) + imqKu) + \tilde{b}'_\phi((K - im\zeta)w - (K\zeta + im)u) = \\ &- (K - imv)((1+\zeta^2)\tilde{A}''_\phi - 2imq\zeta\tilde{A}'_\phi + imq(1+imq)\tilde{A}_\phi) \qquad (18) \end{aligned}$$

One must exercise caution in using these two equations. Rather than treating them as two equations for the quantities $\tilde{A}_\phi$ and $\tilde{b}_\phi$, the correct procedure is to substitute the first into the second in order to obtain a second order differential equation for $\tilde{A}_\phi$. The resulting equation is rather elaborate, given a general velocity field, so that it is more convenient to make the substitution after a particular velocity field has been chosen. Hence writing the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−0.4
−0.6

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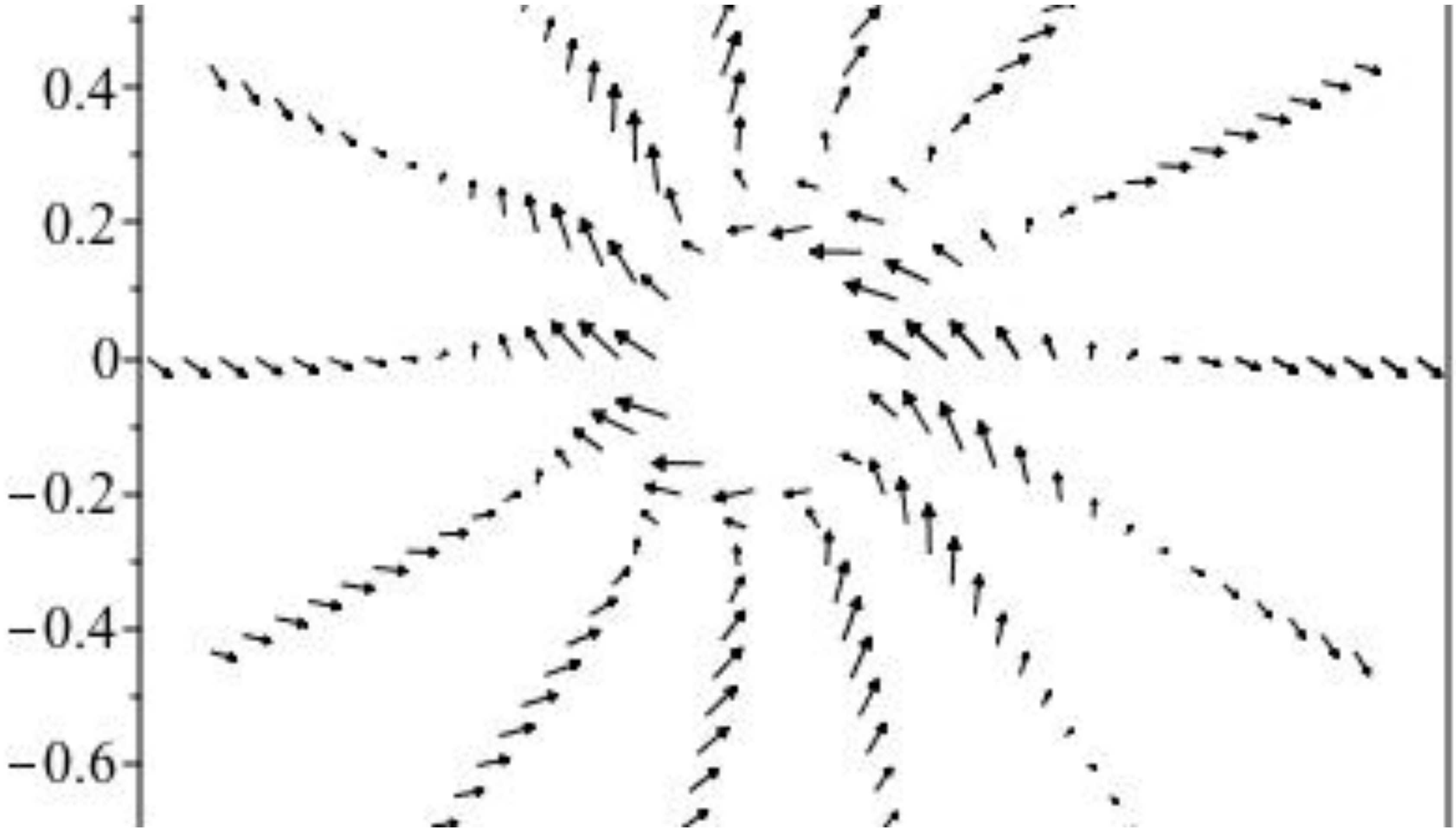
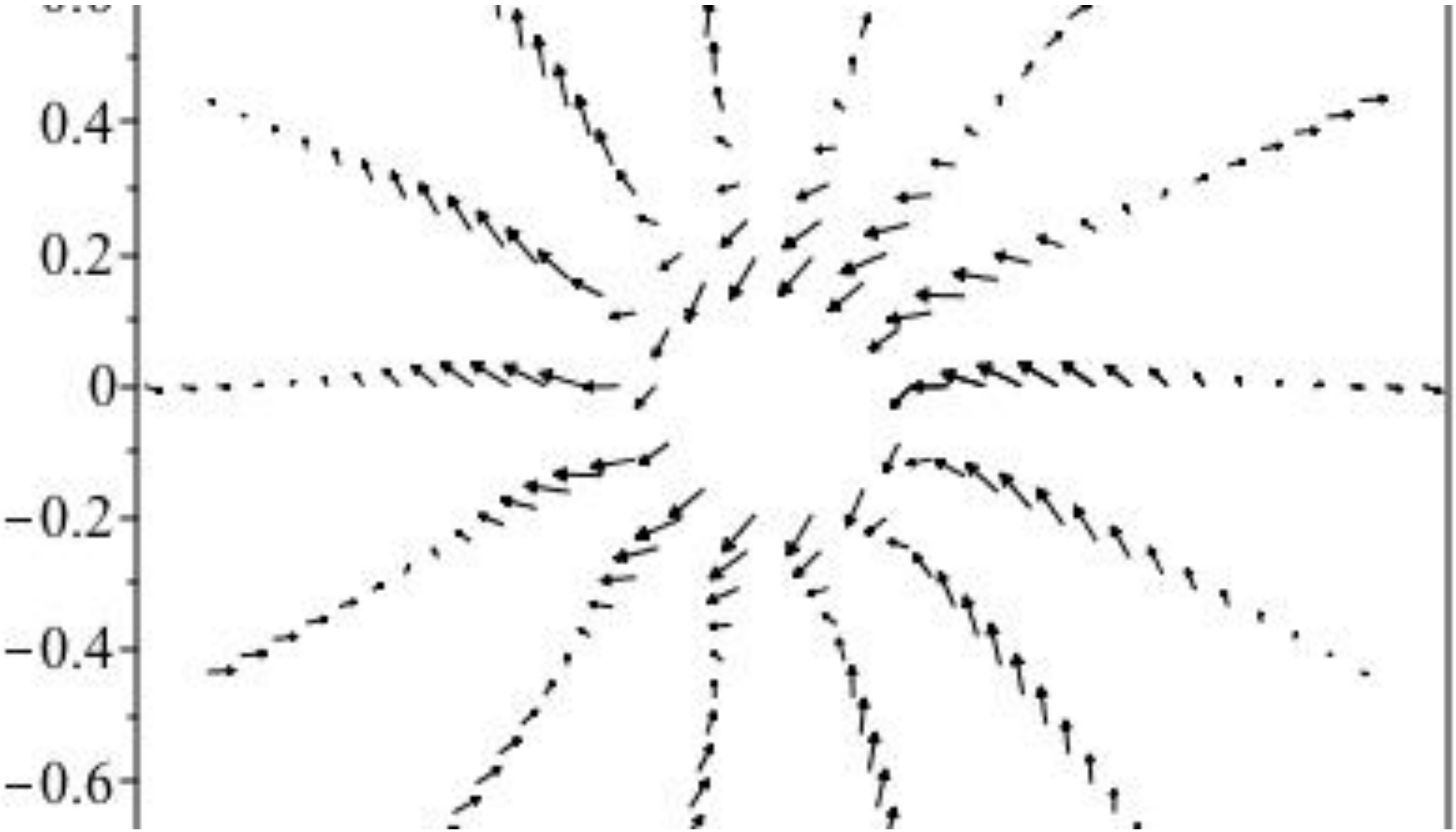
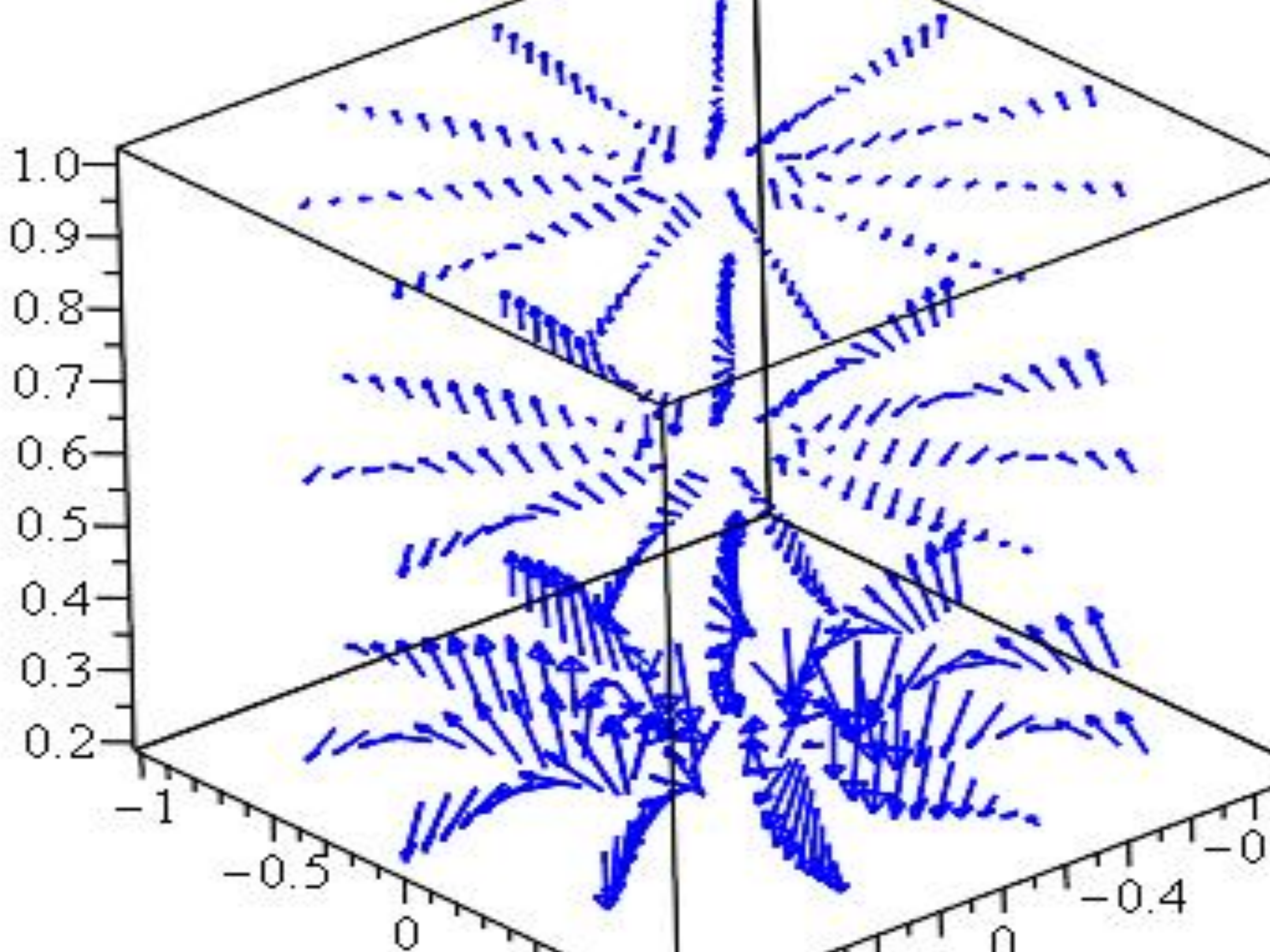


90 degrees counter clockwise





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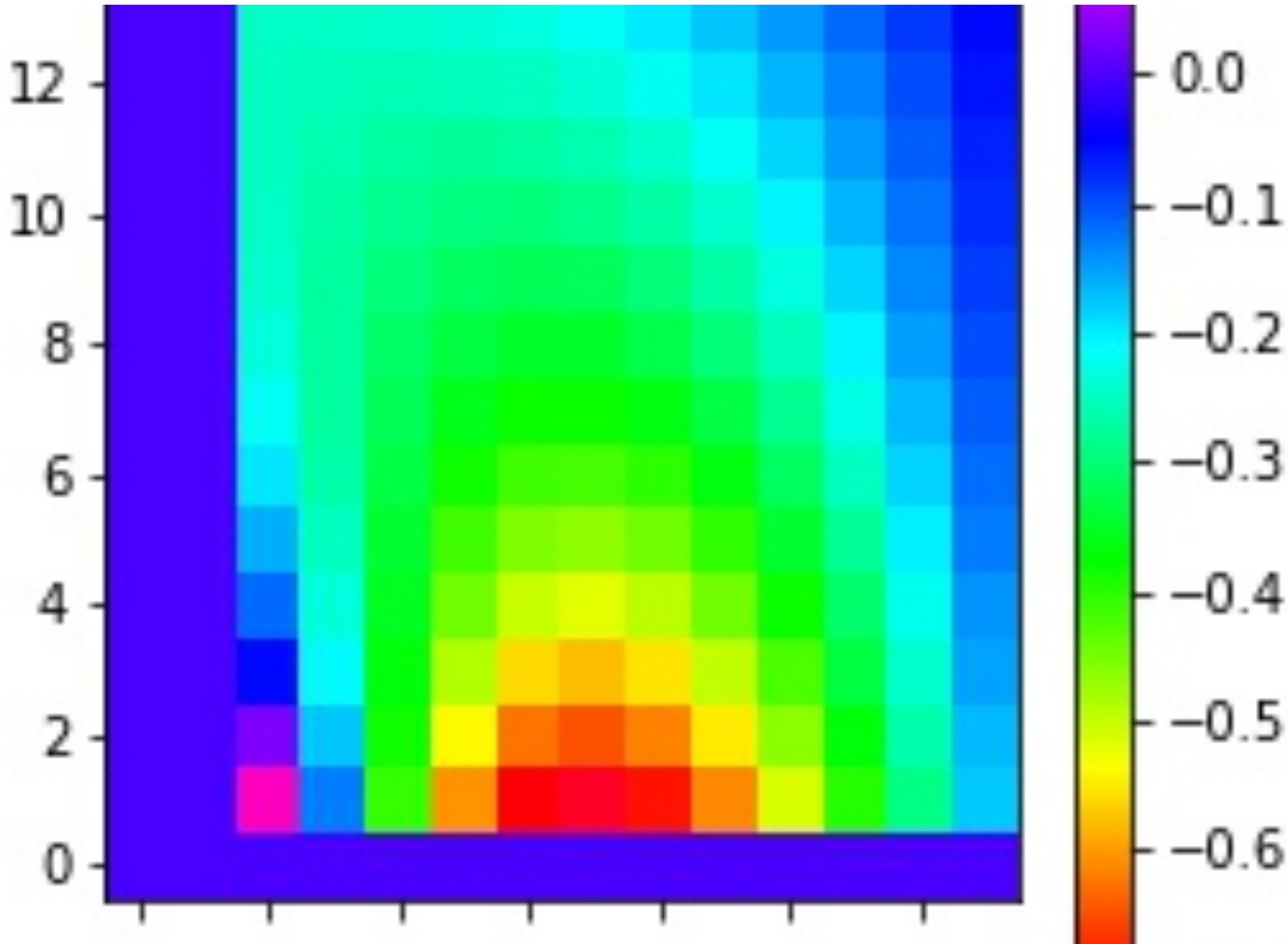

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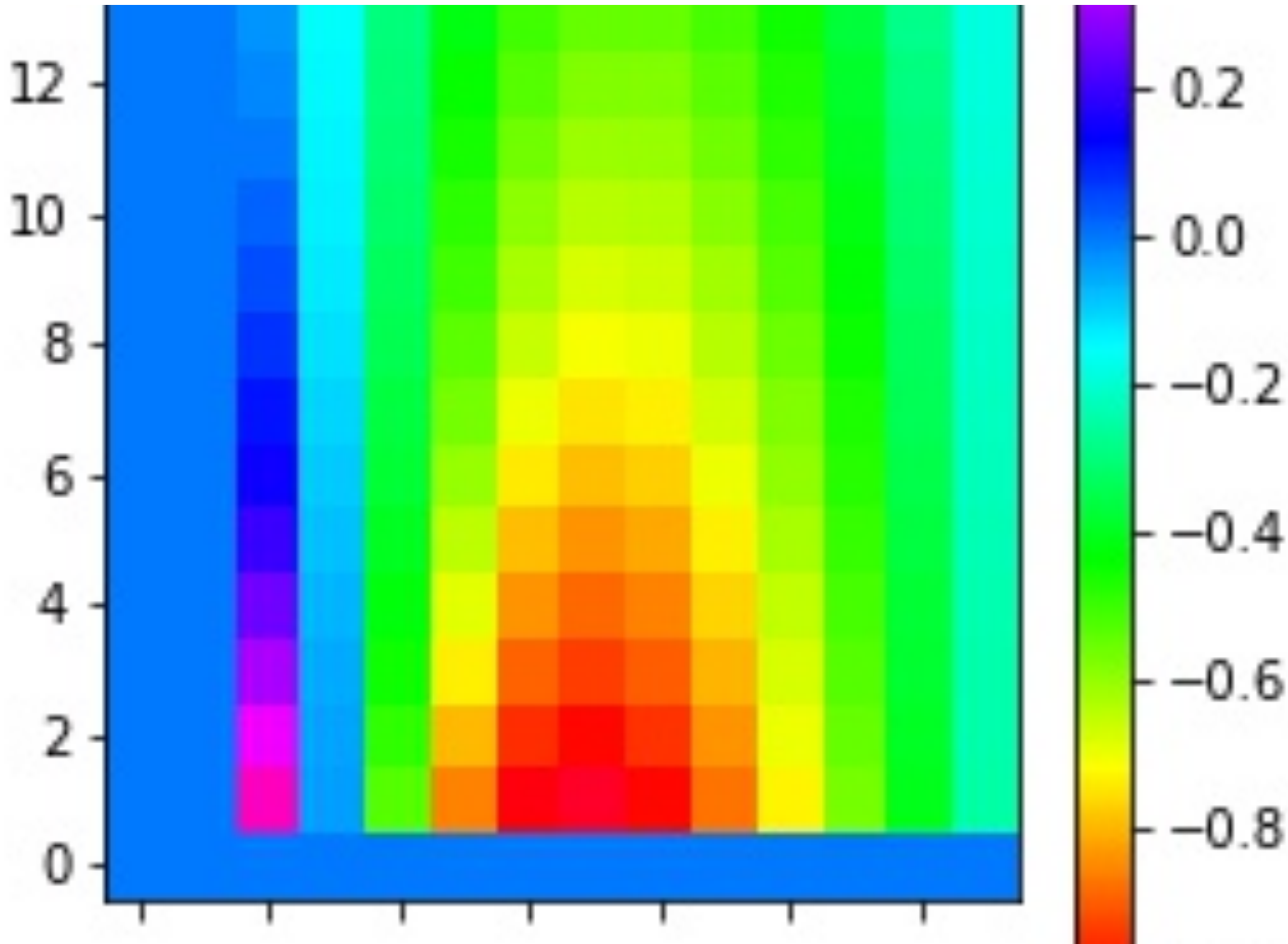

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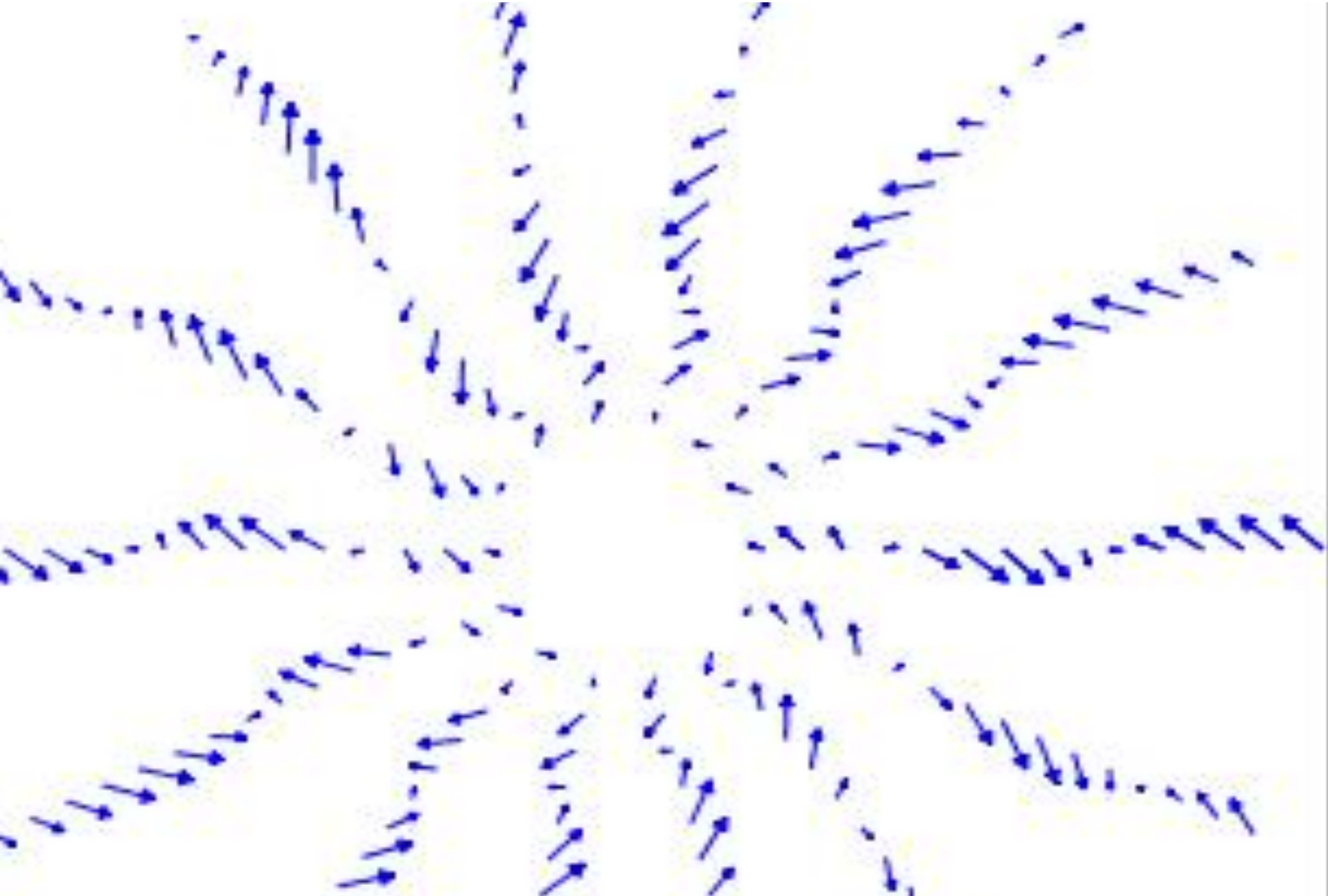

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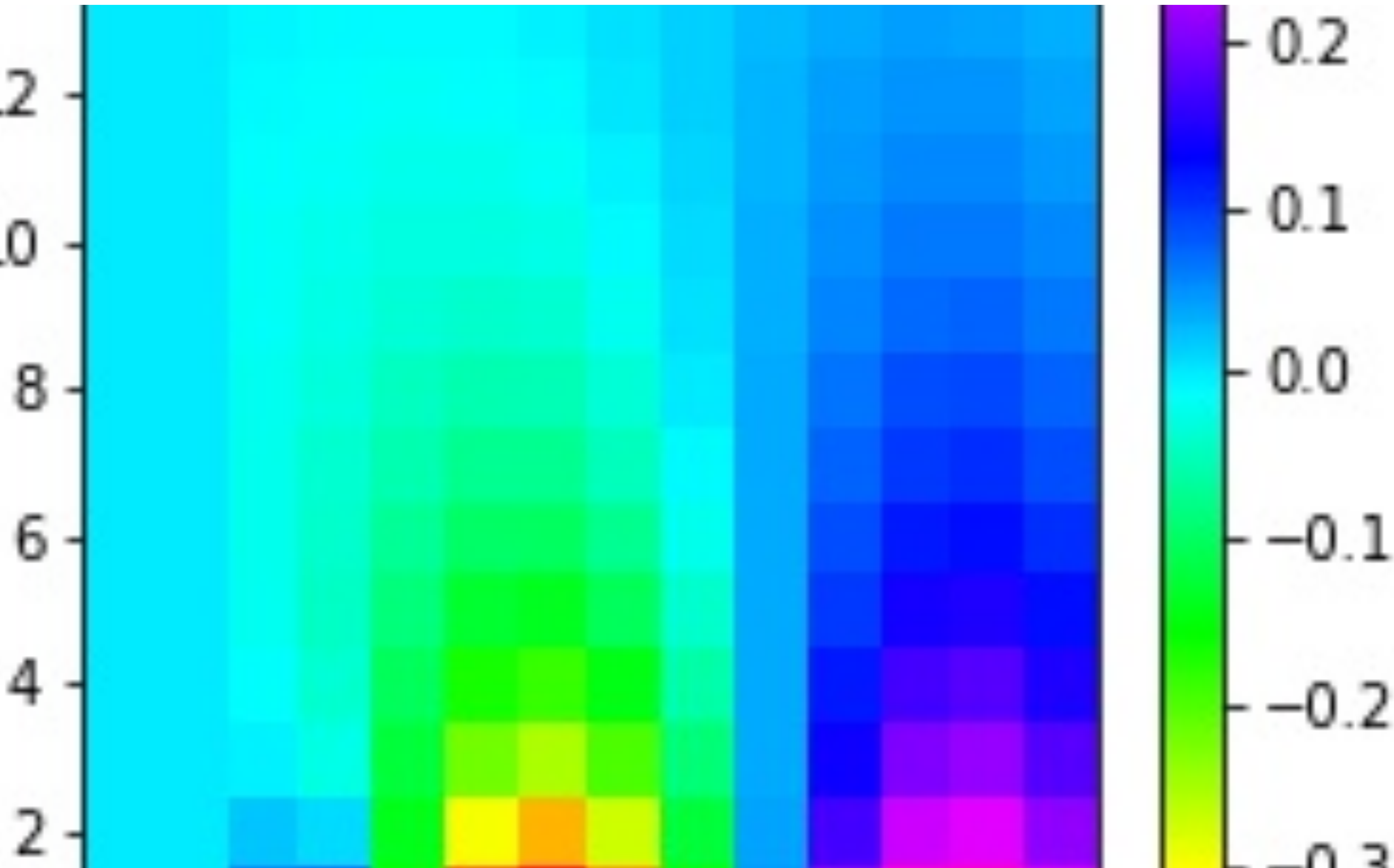

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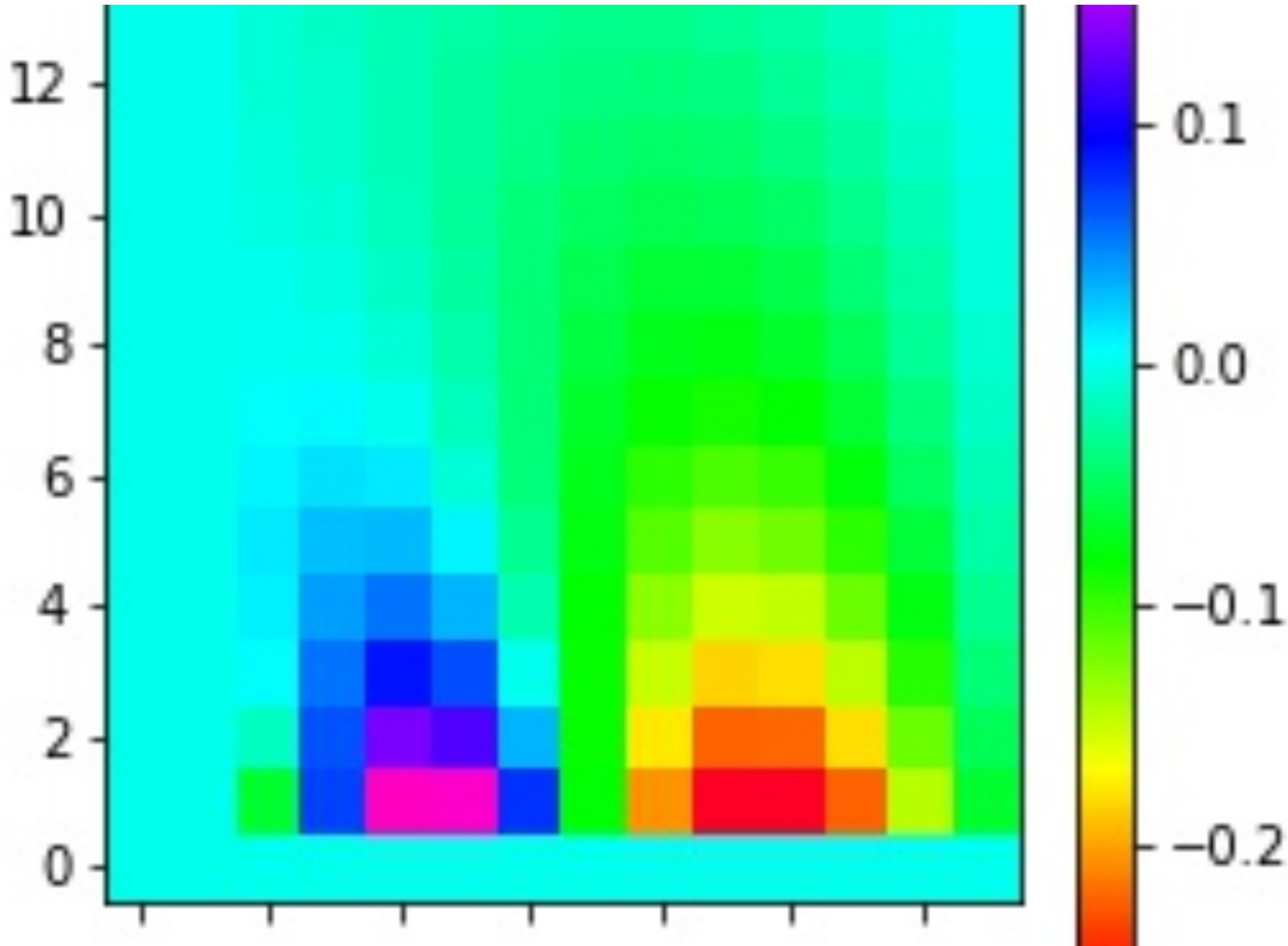

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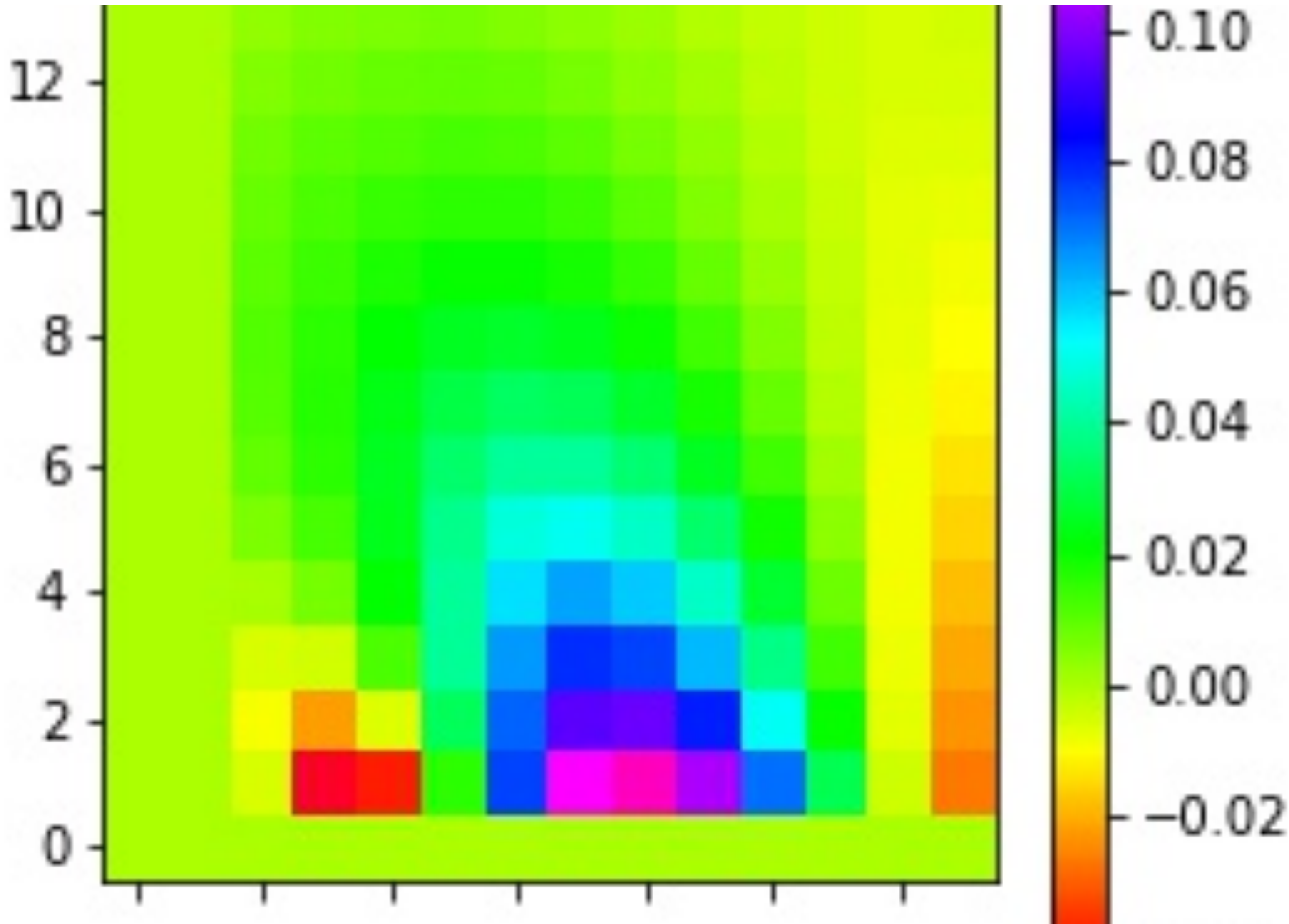

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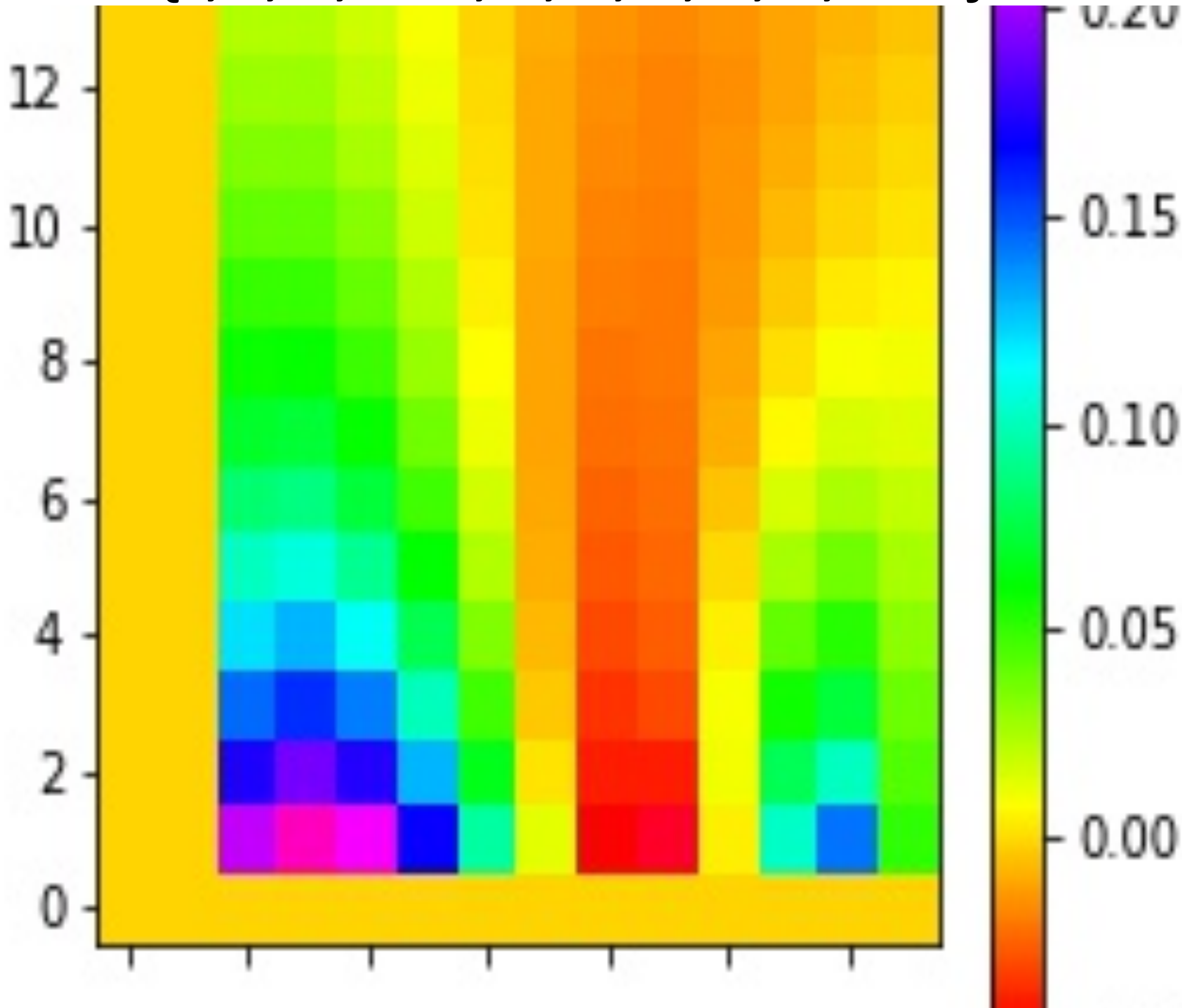

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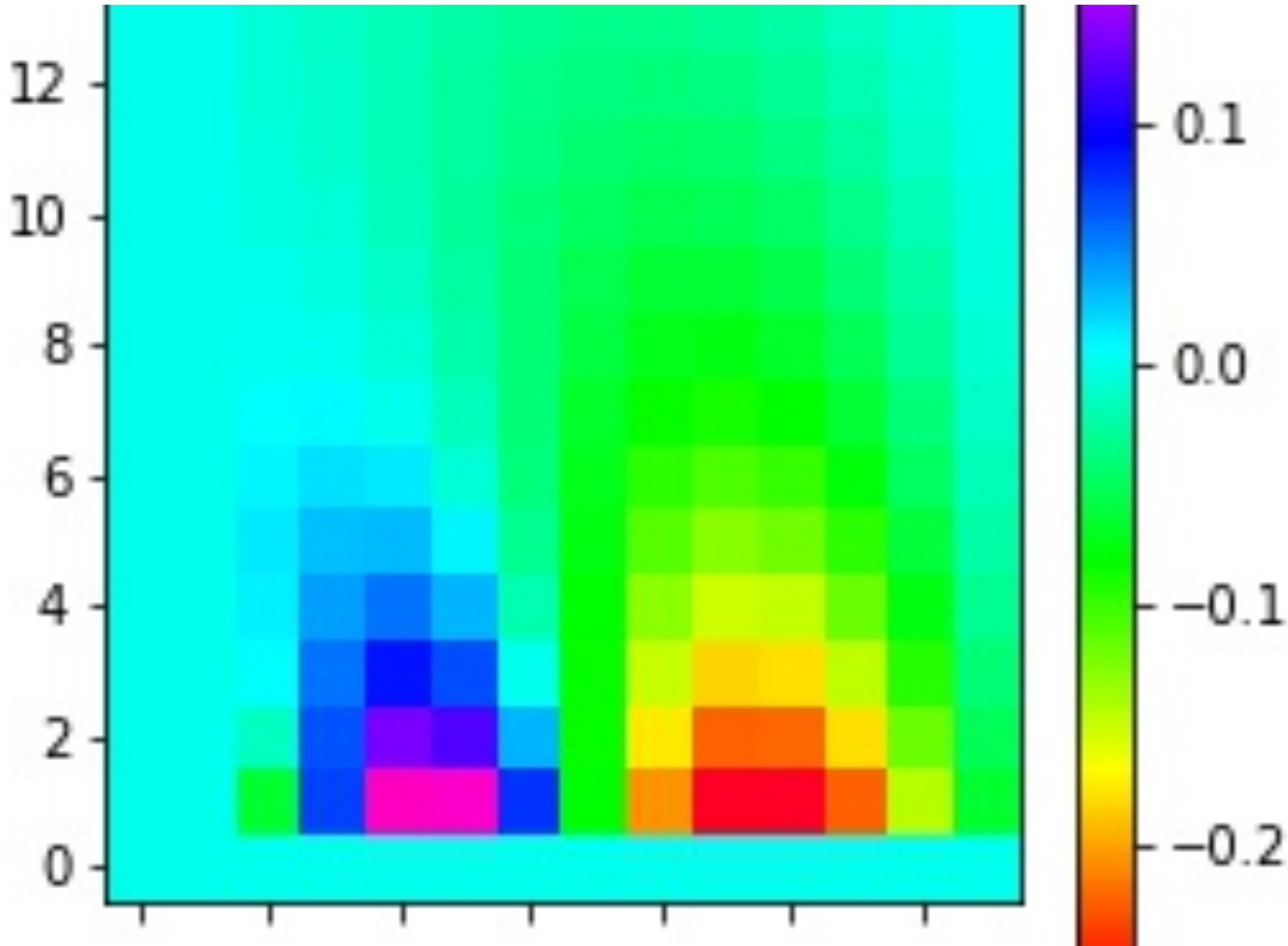

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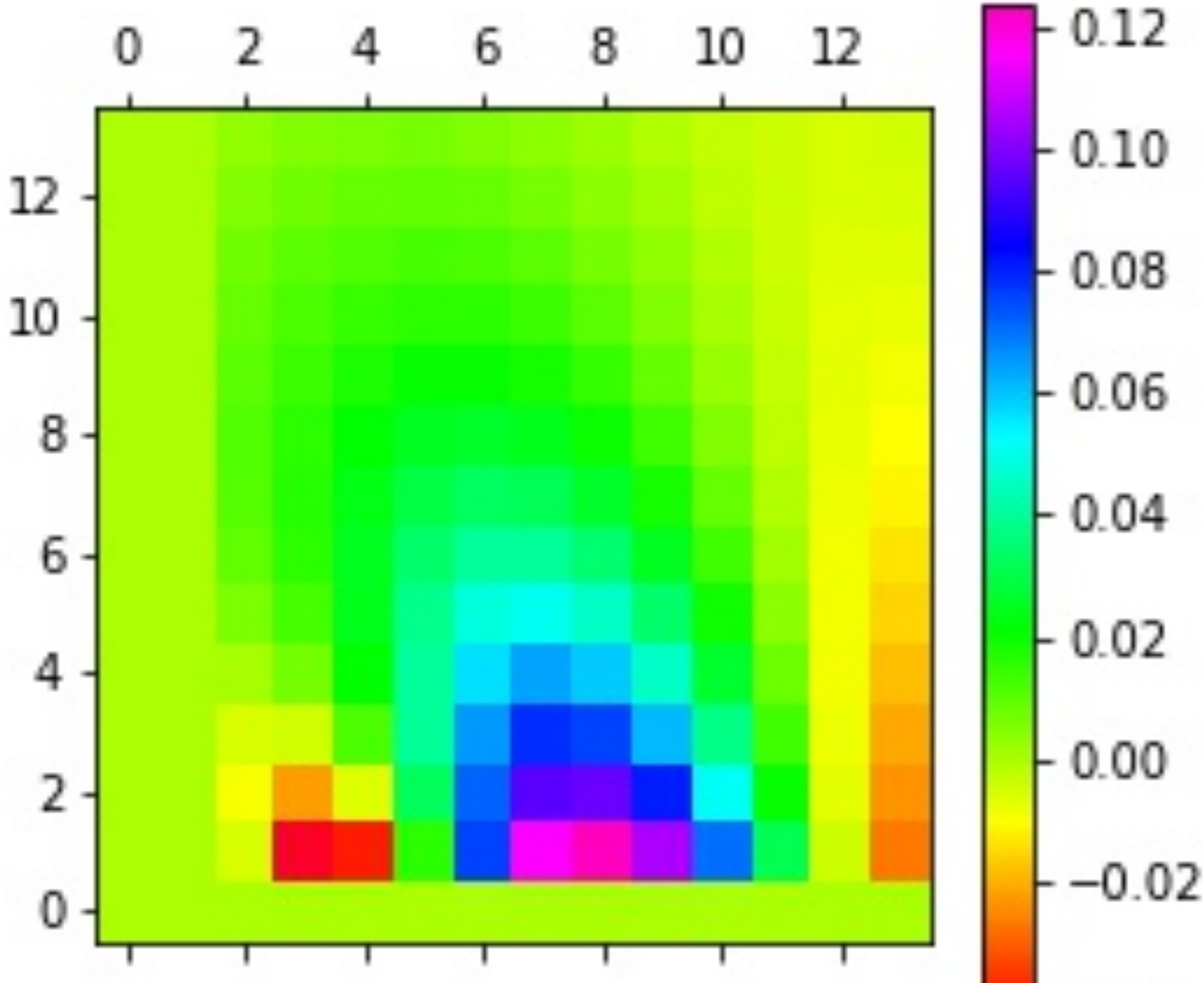

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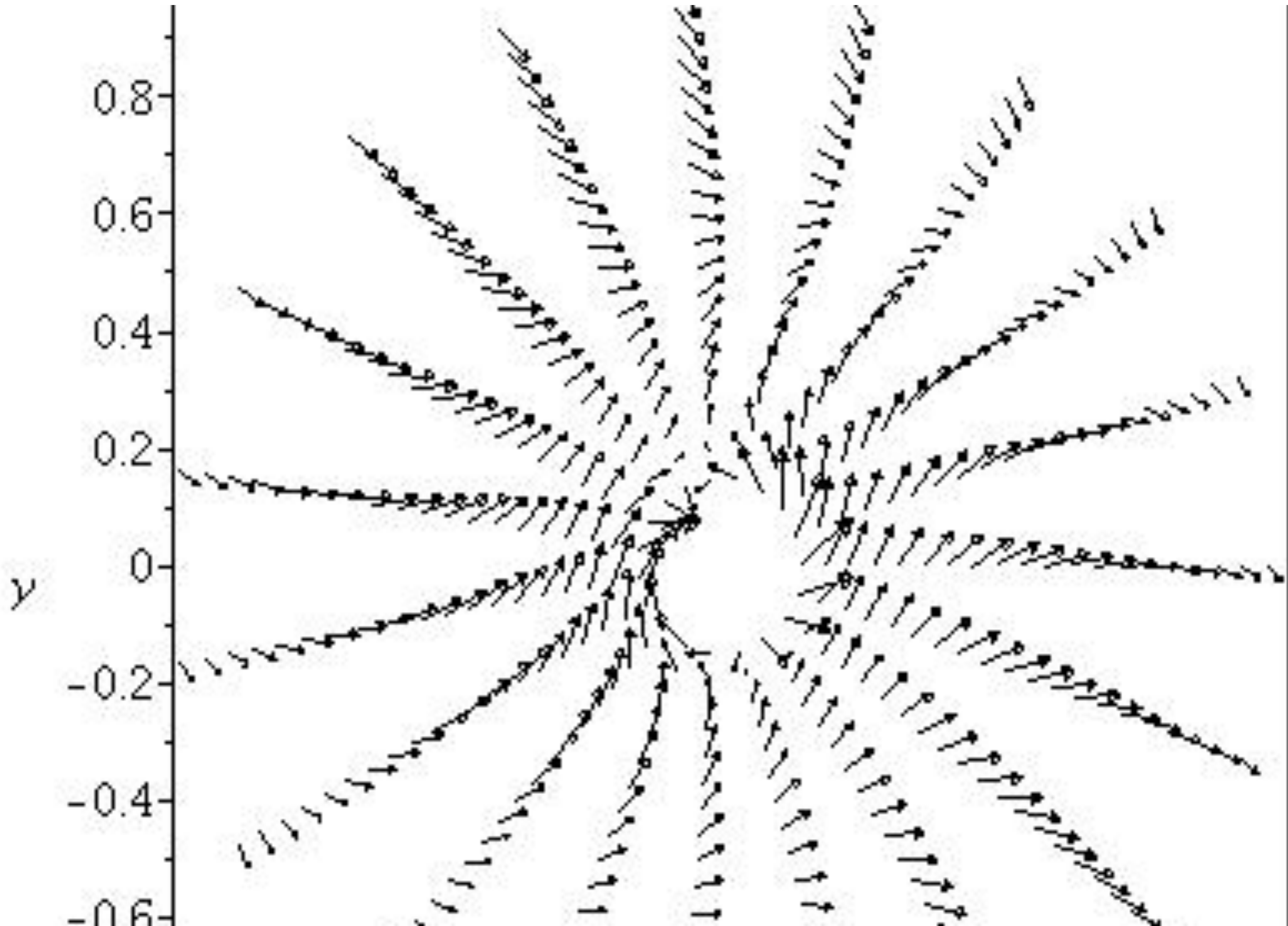
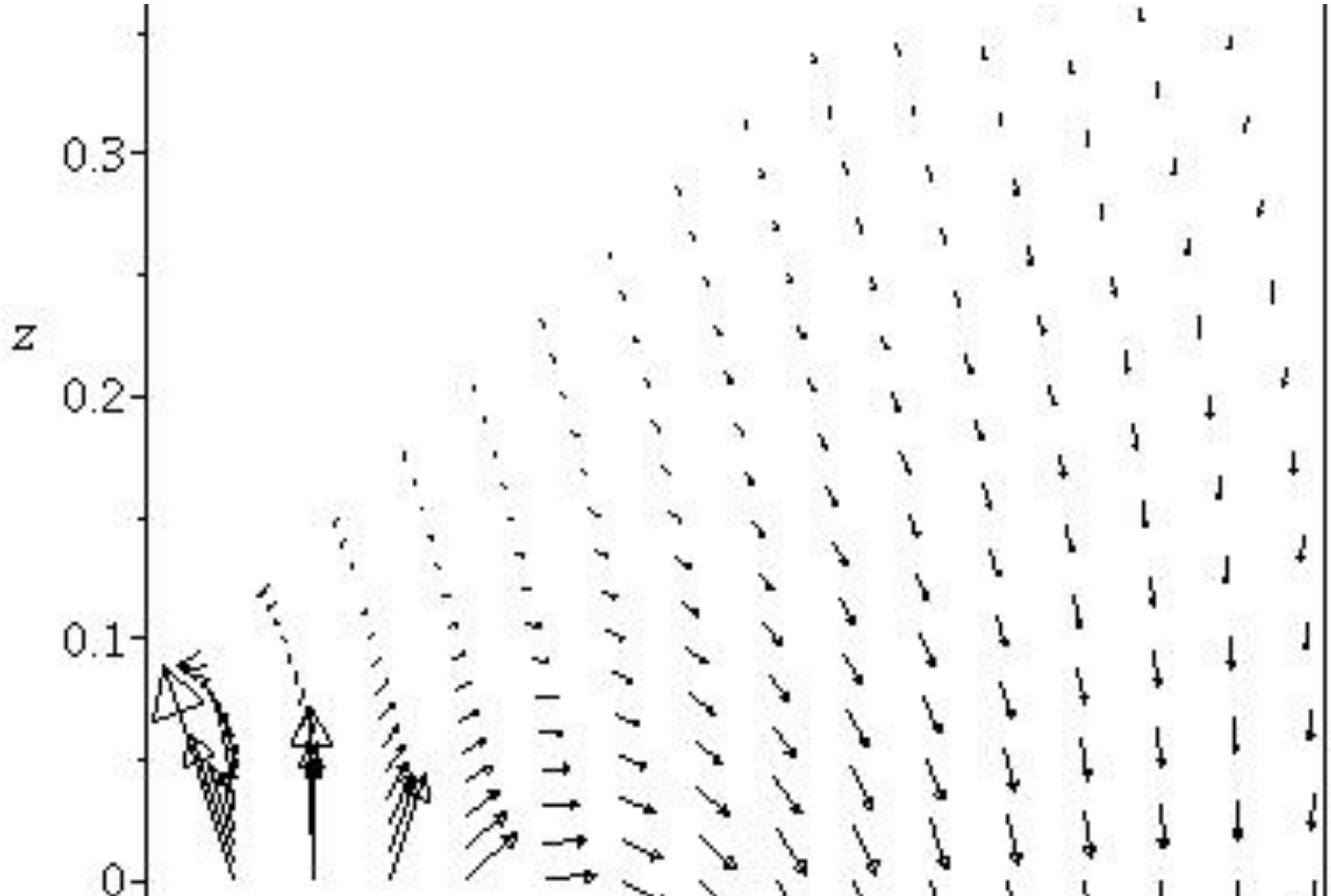


Poloidal cut over previous arm



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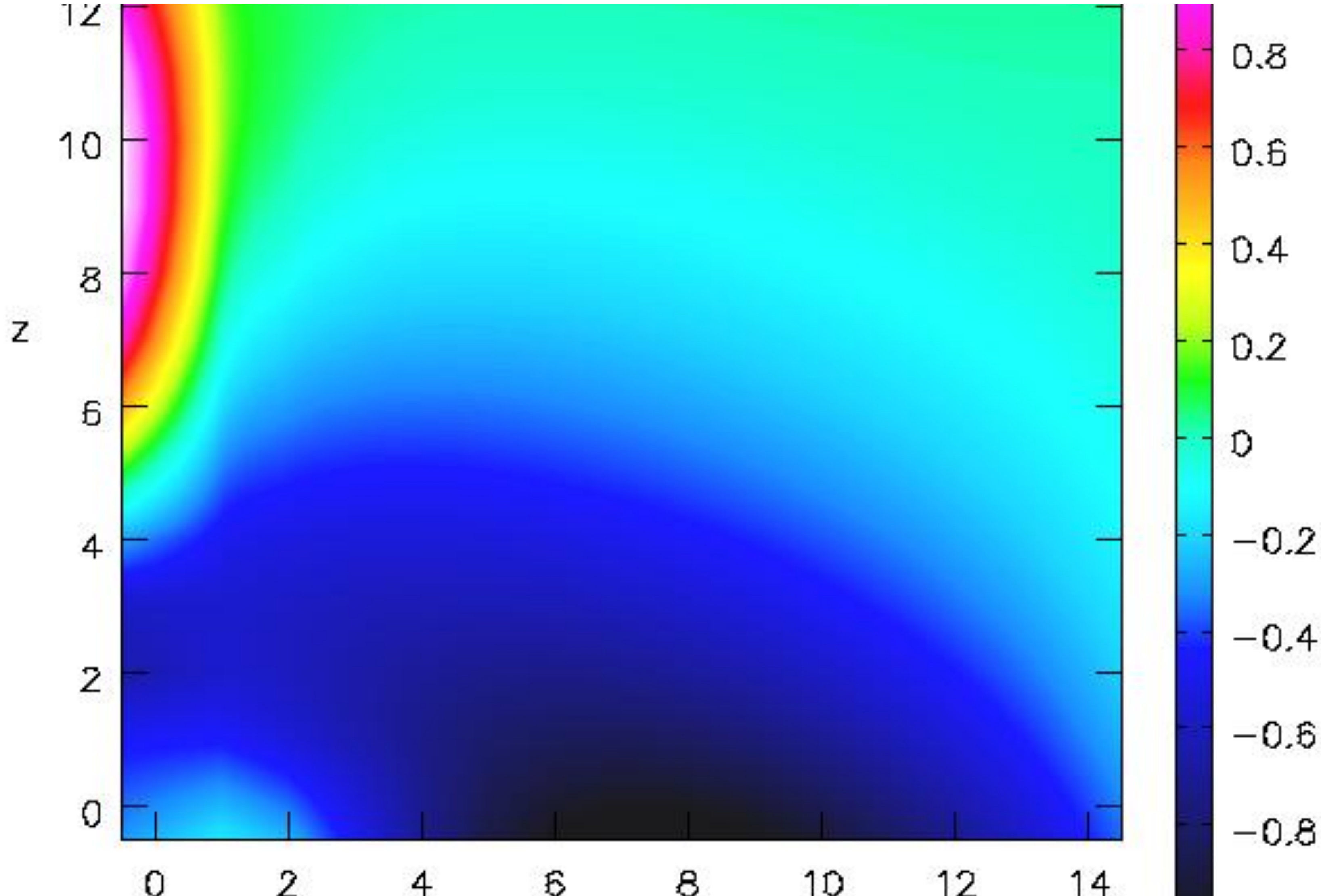

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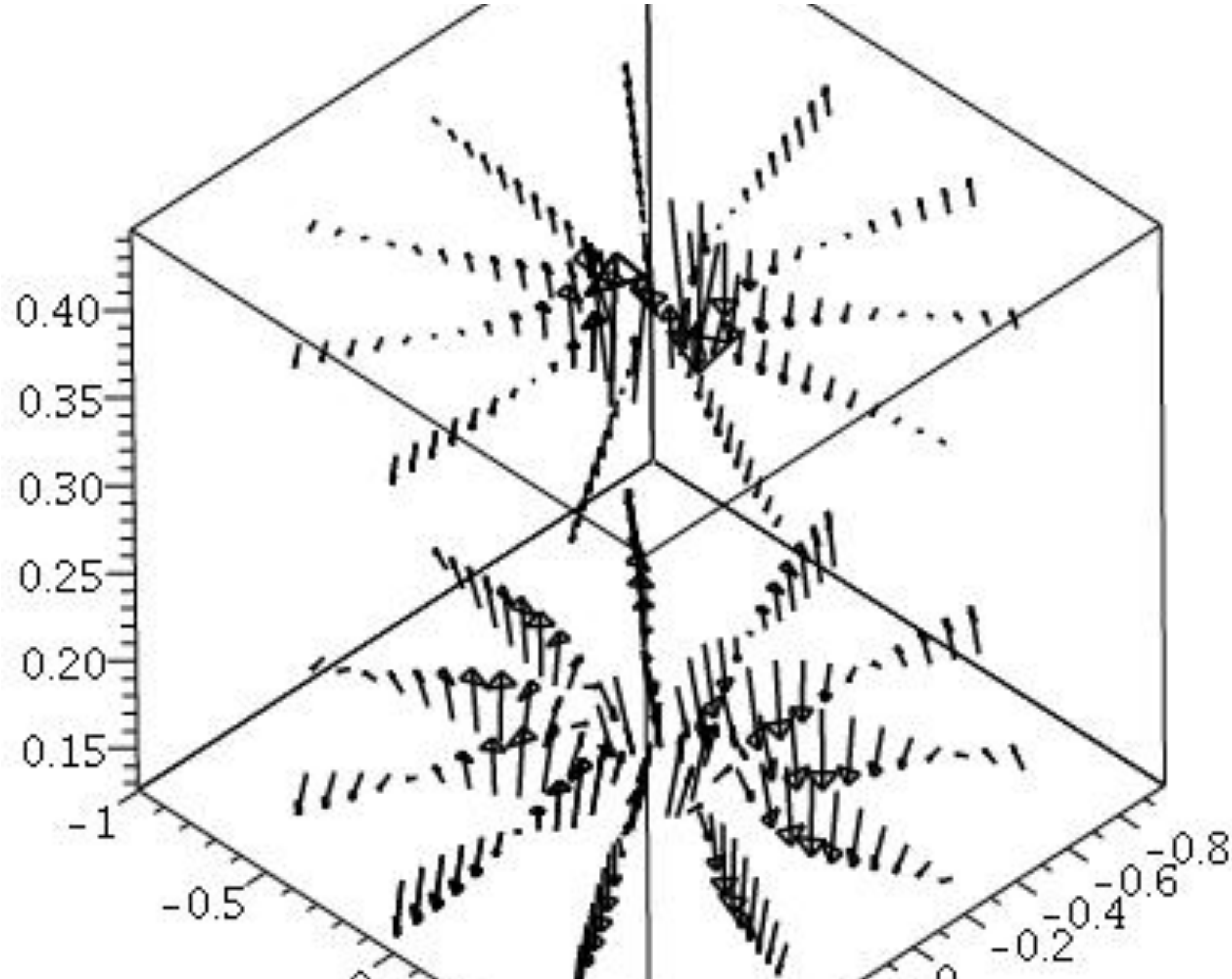

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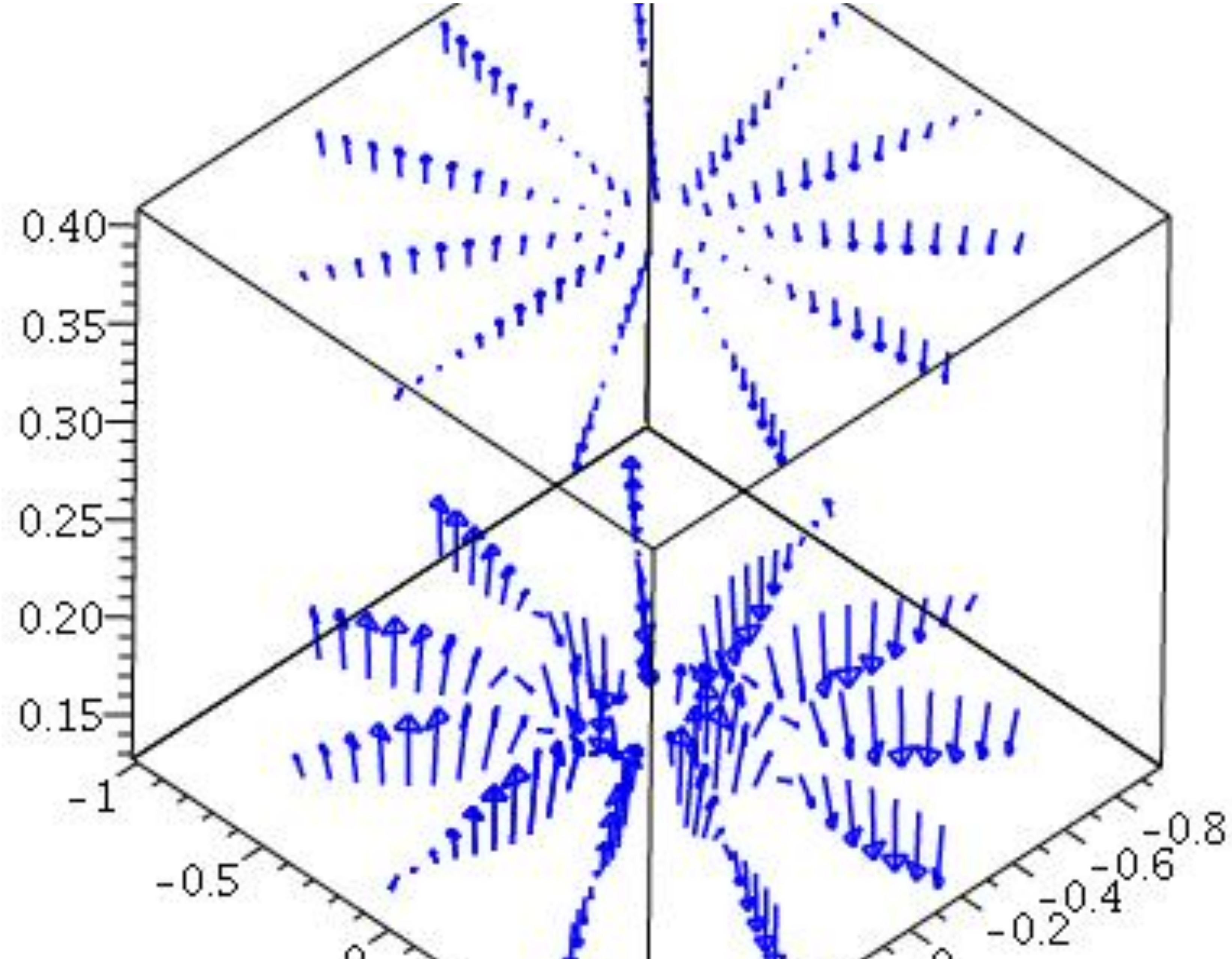
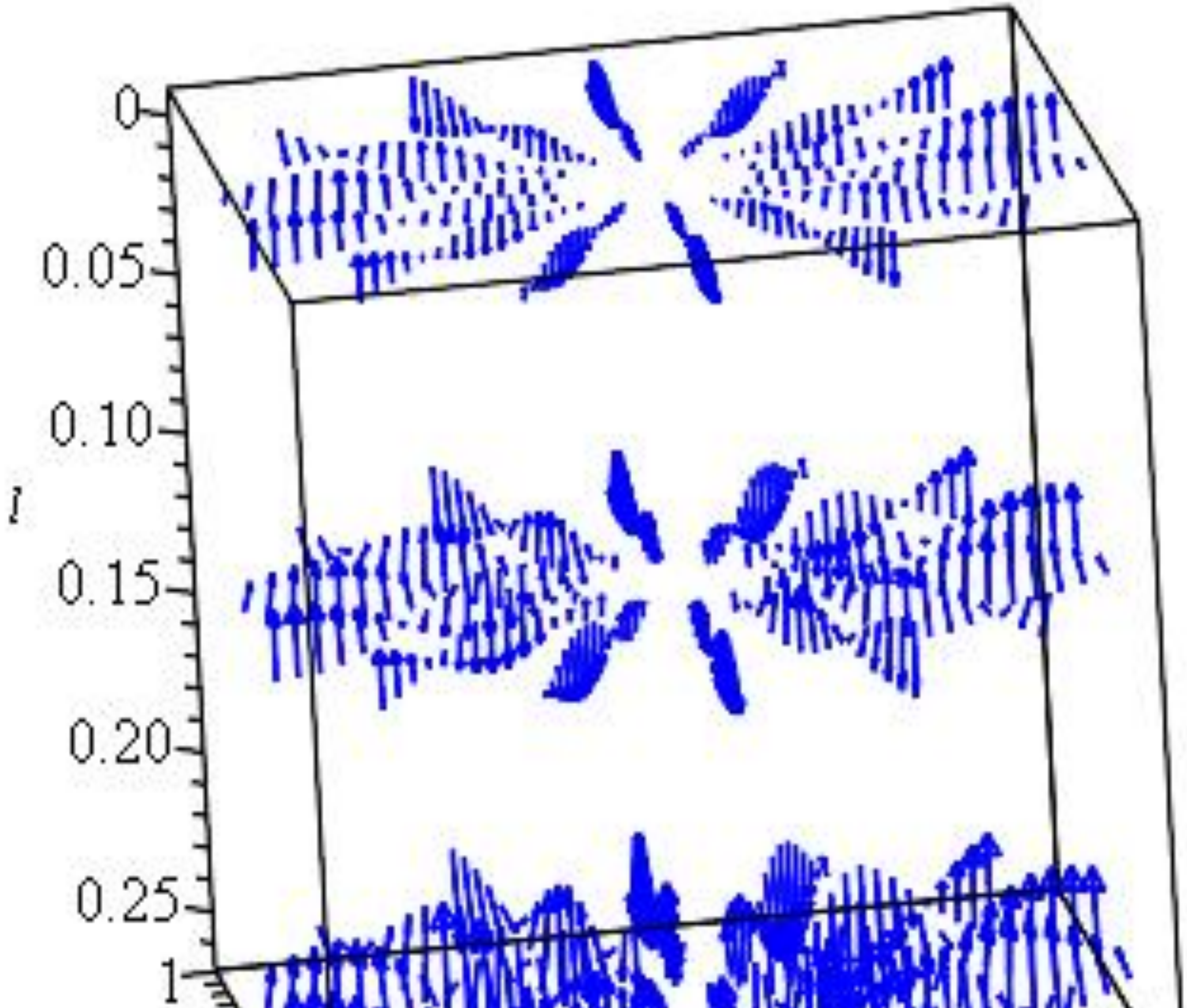
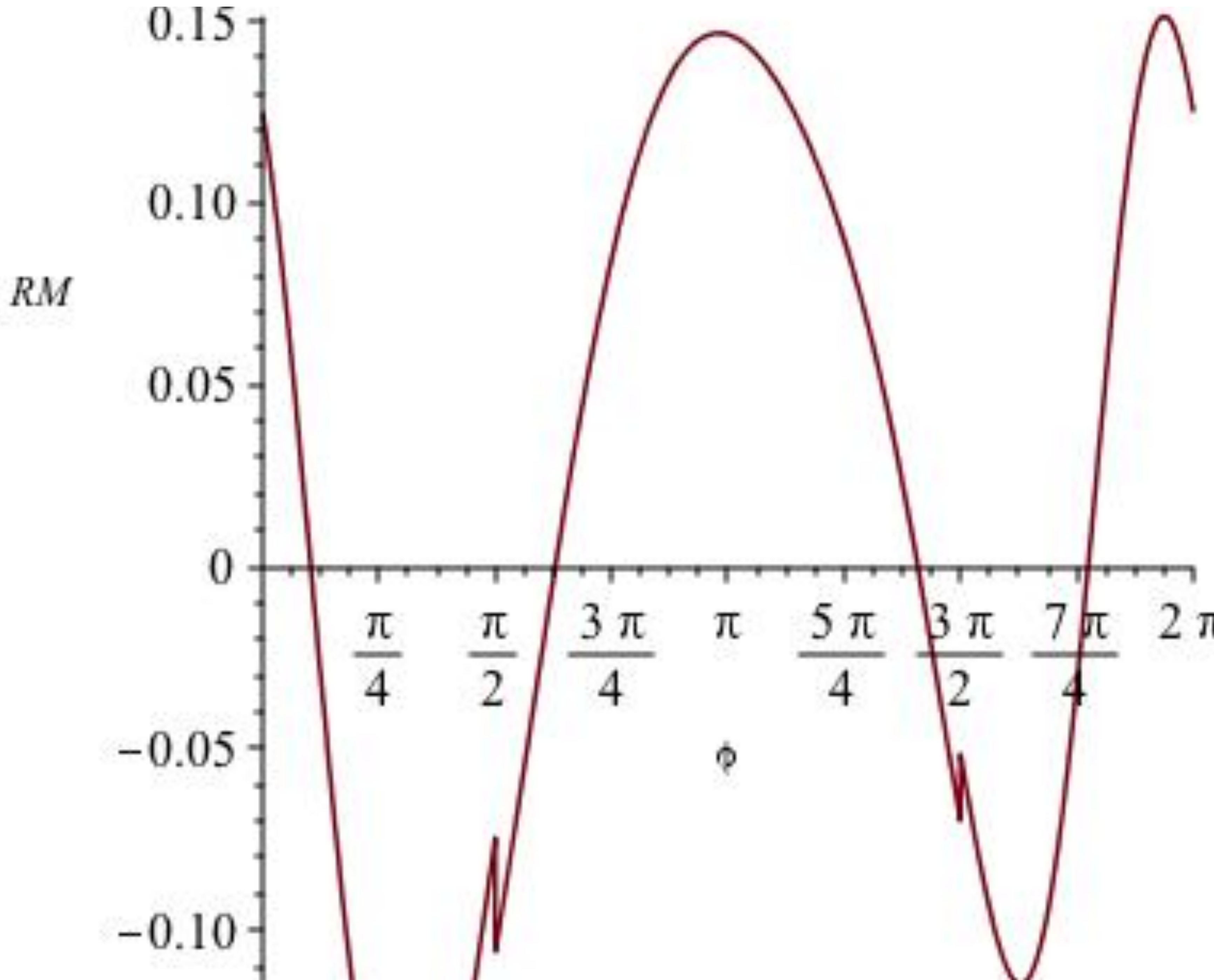


Face on Galaxies





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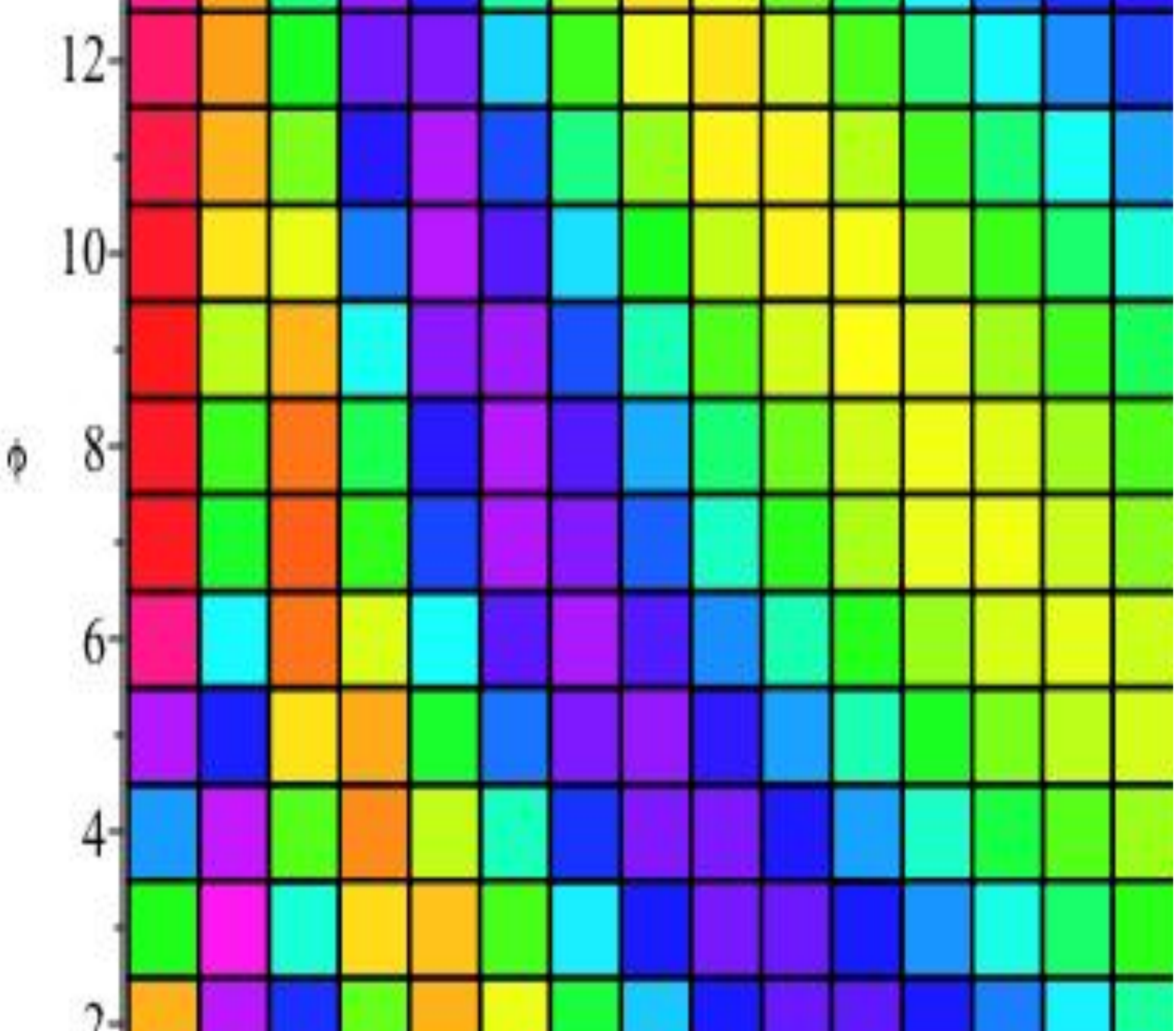
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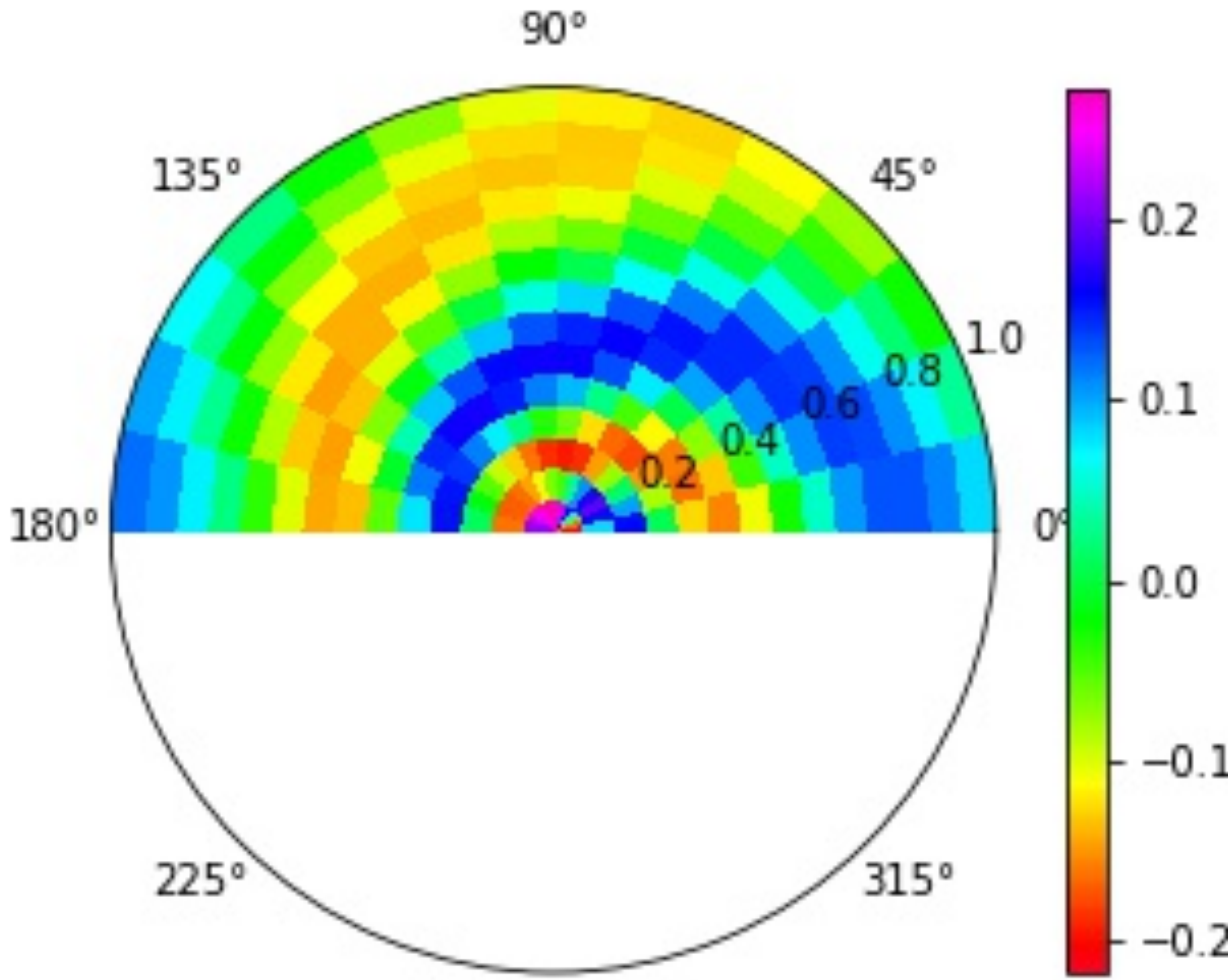
10
φ
8
6
4
2



90°
135°
45°
180°
0°
225°
315°
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0.4
0.6
0.8
1.0
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0.0
−0.1
−0.2