1997 CAP Undergraduate Prize Examination

Wednesday, February 5

from 2:00 to 5:00

Calculators are permitted.

Each question is to be written in a separate book with the number of the problem, the name of the candidate, and the name of the university indicated clearly on the first page.

The candidates may attempt as many questions as possible in whole or in part.

Each question holds the same value.

Please return the exams to:  Dr. J. Hardy
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1. Capacitors with dielectrics

In Fig. 1, \( A_1 \) is the area of the dielectric with dielectric constant \( K_1 \), \( A_2 \) is the area of the dielectric \( K_2 \), and \( x \) is the thickness of the dielectrics. In Fig. 2, \( A \) is the area of the dielectrics, \( x_1 \) is the thickness of the dielectric \( K_1 \), and \( x_2 \) is the thickness of the dielectric \( K_2 \).

(a) Find the capacitance for the arrangement shown in Fig. 1.

(b) Find the capacitance for the arrangement shown in Fig. 2.

(c) If a charge \( Q \) is applied to the capacitor shown in Fig. 1, what is the resulting energy densities in the regions with dielectric constant \( K_1 \) and \( K_2 \), respectively?

(Take \( x_1 + x_2 = x \), and \( A_1 + A_2 = A \).)

2. Interface plasmons.

We consider the plane \( z = 0 \) between metal 1 with \( z > 0 \) and metal 2 with \( z < 0 \). Metal 1 has bulk plasma frequency \( \omega_{p1} \); Metal 2 has bulk plasma frequency \( \omega_{p2} \). A solution of Laplace's equation \( \nabla^2 \phi = 0 \) in the plasma is

\[ \phi_1(x,z) = A \cos k_x e^{-k_z z} \quad \text{for} \quad z > 0 \]

\[ \phi_2(x,z) = A \cos k_x e^{k_z z} \quad \text{for} \quad z < 0. \]

(a) Calculate \( E_x \) and \( E_z \) on each side of the boundary.

(b) Show that \( E_x \) is continuous across the boundary.

(c) Next, from the continuity of the \( z \) component of \( D \) at the boundary, and the fact that \( \varepsilon_i(\omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} \), for \( i = 1 \) and 2, show that \( \omega = \sqrt{\left(\frac{\omega_{p1}^2 + \omega_{p2}^2}{2}\right)} \).
3. In this problem, assume it is known that the energy density \( u \) (in J/m\(^3\)) for blackbody radiation is a function of the temperature \( T \) only, and also that the pressure \( p = u/3 \). The problem is to determine how \( u \) depends on \( T \). This can be done as follows. Let the radiant energy in a cylinder be carried through a Carnot cycle, as shown in the diagram, consisting of an isothermal expansion at temperature \( T \), an infinitesimal adiabatic expansion in which the temperature drops to \( T - dT \), an isothermal compression at \( T - dT \), and an infinitesimal adiabatic compression to the original state.

(a) Plot the cycle in the \( p \)-\( V \) plane.
(b) Calculate the work done by the system during the cycle.
(c) Calculate the heat flowing into the system during the cycle.
(d) Show that the energy density \( u \) is proportional to \( T^4 \) by considering the efficiency of the cycle.

4. The magnetic moment of an ion of spin \( J \) can have \((2J+1)\) orientations with respect to an external field \( B \). The components of the moment along \( B \) can be \( Jm \), \((J-1)m \), \((J-2)m \), \ldots \((-J+1)m \), \(-Jm \). Consider a paramagnetic system of \( N \) distinguishable lattice sites, each occupied by one ion of spin \( J \). (The ions are identical, except for being "nailed down", one to each site.) The paramagnetic system is in equilibrium at temperature \( T = kT \), where \( k \) is the Boltzmann constant. Show that the magnetic moment \( M \) of the system is given by

\[
M = Nm \left( \frac{J}{2} \right) \coth \left( \frac{J + \frac{1}{2}}{2} \frac{mB}{kT} \right) - \frac{1}{2} \coth \left( \frac{mB}{2kT} \right)
\]

5. A point mass \( m_1 \) rests alone in space, while a distant second point mass \( m_2 \) moves with constant small velocity \( v_0 \). In the absence of gravity, \( m_2 \) would pass by \( m_1 \) with impact parameter (distance of closest approach) \( b_0 \). Taking gravity into account,

a) Find the energy and the angular momentum in the center of mass frame.

b) Find the actual impact parameter \( b \) in terms of \( b_0 \), \( v_0 \), \( G \), and the two masses.
6. A rare mode of inverse \( \beta \) -decay involves resonant capture of an electron antineutrino  \( \bar{\nu}_e \) (assumed massless) by a hydrogen atom, producing only a recoil neutron. If the hydrogen atom is in its ground state and at rest, what would be the speed of the recoiling neutron?
\[ m_H = 1.007825 \, u, \quad m_n = 1.008665 \, u, \quad 1 \, u = 931.502 \, MeV \]

7. The existence of neutrino masses, and of oscillations between the three generations of neutrinos, remains an intriguing possibility. A toy model, which exhibits vacuum oscillations between two mass eigenstates, has the simple mass Hamiltonian

\[
H = \begin{pmatrix} M & m \\ m & M \end{pmatrix} \mathbf{c}^2, \text{ where } M \geq m,
\]

and the two physical states represented by \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) are linear combinations of the mass eigenstates.

a) Find the eigenvalues of \( H \), and the corresponding mass eigenvectors.

b) By solving the time dependent Schrödinger equation

\[ i\hbar \partial_t \Psi(t) = H\Psi(t), \]

find the shortest time \( \tau \) necessary for a system, initially in the state represented by \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \), to transform completely into the state represented by \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).

c) If \( Mc^2 = 17 \, keV \), what would be the minimum value of \( \tau \) in seconds?
8. One example of the Mossbauer effect is given by the decay of the $^{57}\text{Co}$ nucleus to $^{57}\text{Fe}$ by electron capture where the final nucleus (of mass M) subsequently returns to the ground state by emitting a photon. The energy of that photon depends on the total energy available, 14.4 keV in this case and labelled as $E_\gamma$, and upon the division of energy between the photon and the recoiling nucleus. The heavier the nucleus, the less energy is required to satisfy momentum conservation, and the smaller is the possible spread of energies of the emitted photon. If the nucleus is part of a macroscopic crystal then the recoiling mass can be very large and consequently the photon energy approaches that of a monoenergetic, monochromatic source - hence the label $E_\gamma$, the energy a photon would have if recoiling from an infinitely large mass.

a) Considering the emission of a photon from one single $^{57}\text{Fe}$ nucleus, use non-relativistic arguments (justified as the velocity of the recoil nucleus is small) applied to energy and momentum conservation to calculate the difference between $E_\gamma$ and the actual energy $E_\gamma$ of the photon and show that the following approximate relation holds:

$$E_\gamma - E_\gamma = (E_\gamma)^2/(2Mc^2)$$

b) Calculate the numerical value of the resulting frequency shift in the case of a photon recoiling against a large crystal of mass 1g.

c) Using this as a source of monoenergetic, monochromatic X rays, what is the expected frequency shift when the photon falls 20m under gravity? Take the original frequency as $3.48 \times 10^{18}$ Hz.

Constants:
\[
e = 1.6 \times 10^{-19} \text{ C}
\]
\[
c = 3 \times 10^8 \text{ m/s}
\]
\[
h = 6.63 \times 10^{-34} \text{ J s}
\]
\[
g = 9.8 \text{ m/s}^2
\]
9. a) Material with a uniform resistivity $\rho$ is formed into the curved shape shown below. The two curved surfaces are circular with radii of $a$ and $b$ and the thickness of the slab is uniform and equal to $t$. Find an expression for the electrical resistance between the two faces of the slab labelled A and B.

b) If the material is aluminum with conductivity $0.355 \times 10^3 \text{ (}\Omega \cdot \text{m})^{-1}$ and the object is machined such that $a = 20 \text{ cm}$, $b = 10\text{ cm}$ and $t = 1 \text{ cm}$, calculate numerically the resistance between those same two surfaces.

10. Explain briefly, in one paragraph per topic, the physics principles involved in the operation of any FOUR of the following devices:

   i) a laser

   ii) an Electric Field Meter

   iii) a MOSFET transistor

   iv) a proportional counter (for measuring radiation fields)

   v) a thermocouple
(a) For Fig 1, let the surface charge densities be \( \sigma_1, \sigma_2 \).

Gauss' Law: \( D_1 = \sigma_1 \) \( D_2 = \sigma_2 \).

\[ E_1 = \frac{\sigma_1}{K_1}, \quad E_2 = \frac{\sigma_2}{K_2} \]

But \( V = -\int E \cdot dx \).

\[ E_1 = E_2 = \frac{V}{x} \]

\[ \sigma_1 = \frac{K_1 V}{x}, \quad \sigma_2 = \frac{K_2 V}{x} \]

\[ Q = \sigma_1 A_1 + \sigma_2 A_1 \]

\[ = \frac{K_1 V}{x} (A_1 K_1 + A_2 K_2) \]

\[ C - \frac{Q}{V} = \left[ \frac{A_1 K_1 + A_2 K_2}{x} \right] \]

(b) For Fig 2, let the surface charge density be \( \sigma \).

Gauss' Law: \( D_1 = D_2 = \sigma \)

\[ E_1 = \frac{\sigma}{K_1}, \quad E_2 = \frac{\sigma}{K_2} \]

\[ V = E_1 x_1 + E_2 x_2 \]

\[ = \sigma \left( \frac{x_1}{K_1} + \frac{x_2}{K_2} \right) \]

\[ = \frac{Q}{A} \left( \frac{1}{K_1} + \frac{1}{K_2} \right) \]

\[ C = \frac{Q}{V} = \frac{A}{(x_1/K_1 + x_2/K_2)} \]
{\textbf{Solution from LDF}}

\( \mathbf{E} = -\partial \mathbf{\Phi} = -\frac{\partial \mathbf{\Phi}}{\partial x} - \frac{\partial \mathbf{\Phi}}{\partial t} \)

\( \begin{align*}
\text{If } & \ z > 0 \quad \mathbf{\Phi} = A \cos k x e^{-k z} \quad \implies \quad \mathbf{E} = A \sin k x e^{-k z} \frac{1}{k} \mathbf{x} + A k \cos k x e^{-k z} \frac{1}{k} \mathbf{x} \\
\text{If } & \ z < 0 \quad \mathbf{\Phi} = A \cos k x e^{k z} \quad \implies \quad \mathbf{E} = A \sin k x e^{k z} \frac{1}{k} \mathbf{x} - A k \cos k x e^{k z} \frac{1}{k} \mathbf{x} \\
\end{align*} \)

\( E_x(1) = A \sin k x e^{-k z} \)
\( = A \sin k x k \quad \text{at } \ z = 0 \)

\( E_x(2) = A \sin k x e^{k z} \)
\( = A \sin k x k \quad \text{at } \ z = 0 \)

\( \therefore \quad E_x(1) = E_x(2) \text{ at } \ z = 0 \)

\( \text{Continuity of } D_z \text{ at } \ z = 0 \)

\( E_1 E_z(1) = E_2 E_z(1) \text{ at } \ z = 0 \)

\( \varepsilon_0 \left( 1 - \frac{\omega_r^2}{\omega^2} \right) A k \cos k x \mathbf{e} = \varepsilon_0 \left( 1 - \frac{\omega_r^2}{\omega^2} \right) (-A k \cos k x) \)

\( 1 - \frac{\omega_r^2}{\omega^2} = -1 + \frac{\omega_r^2}{\omega^2} \)

\( 2 = \frac{\omega_r^2}{\omega^2} + \frac{\omega_r^2}{\omega^2} \)

\( \omega = \sqrt{\left( \frac{\omega_1^2}{\omega^2} + \frac{\omega_2^2}{\omega^2} \right)} / 2 \)
(a) Along the isentropes, \( u = \text{constant} \) \( \Rightarrow p = u/3 = \text{constant} \).

(b) \[ \Delta W_{12} = P(V_2 - V_1) \]
\[ \Delta W_{23} = \text{infinite small} \]
\[ \Delta W_{34} = -(P - \Delta P)(V_2 - V_1) \]
\[ \Delta W_{41} = \text{infinite small} \]
\[ \therefore \Delta W = \Delta P(V_2 - V_1) = \frac{\Delta u}{3}(V_2 - V_1). \]

(c) \[ \Delta Q_{12} = \Delta U_{12} + \Delta W_{12} \]
\[ = \Delta u(V_2 - V_1) + P(V_2 - V_1) \]
\[ = \frac{4u}{3}(V_2 - V_1). \]

There is also heat flowing out along 3-4.

(d) \[ \eta = \frac{\Delta W}{\Delta Q_{12}} = \frac{\Delta u/3}{4u/3} = \frac{1}{4} \frac{\Delta u}{u}. \]

Best for the Carnot cycle
\[ \eta = \frac{\Delta T}{1} \]
\[ \therefore \frac{\Delta u}{u} = 4 \frac{\Delta T}{T} \text{ and } u \propto T^4. \]
\[ M = N \bar{m} = N \mu \]

(Use \( m \) for the magnetic q.m.)

\[ \mu^j_s = -g_j m_j / \mu_B \]

\[ \Delta E_j = g_j m_j \mu_B B = m_j (g_j \mu_B) B \]

\[ m_j \mu_B \] for this problem.

\[ P_{\Delta j} = e^{-\Delta E_j / z} = P_j \]

\[ \bar{\mu} = \sum_j \frac{m_j m_j P_j}{\sum P_j} = e \left( \frac{1}{\alpha} \right) + \frac{1}{2} \frac{d \mu_B}{dB} \]

\[ z = \sum e^{-\frac{\Delta E_j}{z}} = \sum e^{-\frac{m_j x}{z}} \] where \( \alpha = \frac{g_B \mu_B}{z} \)

\[ = e^{-Jx} + \ldots + e^{-x} \]

\[ = \frac{e^{-Jx} - e^{-x}}{1 - e^{-x}} \]

\[ = \frac{\sinh(x(J+\frac{1}{2}))}{\sinh(x/2)} \]

\[ \frac{dz}{dx} = \frac{\sinh(\frac{1}{2}z) \cosh(\frac{1}{2}z) x - \sinh(\frac{1}{2}z) \cosh(\frac{1}{2}z) \sinh(\frac{1}{2}z)}{\cosh^2(x/2)} \]

\[ \frac{1}{2} \frac{d^2}{d\alpha} = (J+\frac{1}{2}) \coth(J+\frac{1}{2}) x - \frac{1}{2} \coth(\frac{x}{2}) \]

\[ \frac{1}{2} \frac{d^2}{dR} = \frac{\mu}{c} \left[ \begin{array}{c} \end{array} \right] \]

\[ \bar{\mu} = \mu \left[ \begin{array}{c} \end{array} \right] \]

\[ \text{and} \]

\[ M = N m \left[ (J+\frac{1}{2}) \coth(J+\frac{1}{2}) \frac{\mu_B^2}{e^2} - \frac{1}{2} \coth \right] \]

\( \checkmark \)
5. (a) In the C.M. frame,

\[ m_1 v_1 = m_2 v_2 \quad (\text{definition of C.M. frame}) \]

also \[ v_0 = v_1 + v_2 \quad (\text{non-relativistic}) \]

\[ \therefore m_1 (v_0 - v_2) = m_2 v_2 \]

\[ \therefore v_0 = \left( \frac{m_1 + m_2}{m_1} \right) v_2 \]

\[ E_{\text{cm}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]

\[ = \frac{1}{2} m_1 v_0^2 \left( \frac{m_2}{m_1 + m_2} \right)^2 + \frac{1}{2} m_2 v_0^2 \left( \frac{m_1}{m_1 + m_2} \right)^2 \]

\[ = \frac{1}{2} \mu v_0^2 \]

where \[ \mu = \frac{m_1 m_2}{m_1 + m_2} \quad (\text{reduced mass}) \]
Angular momentum about the center of mass:

\[ L_{cm} = -m_1 v_1 (y_1 - y_{cm}) + m_2 v_2 (y_2 - y_{cm}) \]

\[ (y = \text{vertical coordinate in picture}) \]

\[ y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \quad (\text{definition of cm}) \]

Also \( y_2 - y_1 = b_0 \)

Let's define our coordinates so that \( y_{cm} = 0 \)

Then \( m_1 y_1 = -m_2 y_2 \)

\[ m_1 (y_2 - b_0) = -m_2 y_2 \]

\[ y_2 = \frac{m_1 b_0}{m_1 + m_2} \]

\[ y_1 = -\frac{m_2 b_0}{m_1 + m_2} \]

\[ L_{cm} = \frac{m_1 m_2 v_1 b_0}{m_1 + m_2} + \frac{m_1 m_2 v_2 b_0}{m_1 + m_2} \]

\[ = \mu v_0 b_0 \]

\[ \Rightarrow \text{the motion is equivalent to one-body motion for a body of mass } \mu, \text{ velocity } v_0, \text{ and position } \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \]
(b) We can picture the equivalent 1-body motion:

The angular momentum about $r = 0$ is conserved (central force).

$$\mathbf{L} = \mu v_0 b_0 = \mu r v \mathbf{\dot{r}}$$

$$= \mu r^2 \mathbf{\dot{r}}$$ where $\mathbf{\dot{r}} = \frac{d\mathbf{r}}{dt}$

By applying $F = ma$ to the radial motion, we can obtain the whole trajectory $(r, \phi)$. In this case, we just want $b = r_{\text{min}}$, so we can make use of the symmetry of the problem to take a short cut.

At $r = r_{\text{min}}$, $\dot{r} = 0$ and $v_\phi = V$

$$\mu v_0 b_0 = \mu v b$$ where $v$ is the speed at closest approach.
Conservation of energy gives:
\[
\frac{1}{2} \mu V_0^2 = - \frac{G m_1 m_2}{b} + \frac{1}{2} \mu V^2
\]
\[
= - \frac{G m_1 m_2}{b} + \frac{1}{2} \mu \left(\frac{V_0 b_0}{b}\right)^2
\]
\[
\therefore 1 = - \frac{G m_1 m_2}{\frac{1}{2} \mu V_0^2 b} + \left(\frac{b_0}{b}\right)^2
\]
\[
\frac{b^2}{b_0^2} + \frac{G m_1 m_2}{\frac{1}{2} \mu V_0^2 b} - 1 = 0
\]
\[
\therefore b = - \frac{G m_1 m_2}{\frac{1}{2} \mu V_0^2 b} + \sqrt{\left(\frac{G m_1 m_2}{\frac{1}{2} \mu V_0^2 b_0}\right)^2 + \frac{4}{b_0^2}}
\]
\[
\frac{2}{b_0^2}
\]

Take positive solution for \( b \):
\[
b = - \frac{G m_1 m_2}{\mu V_0^2} + b_0 \sqrt{1 + \left(\frac{G m_1 m_2}{\mu V_0^2 b_0}\right)^2}
\]
\[
= b_0 \left\{ \sqrt{1 + \left(\frac{G m_1 m_2}{\mu V_0^2 b_0}\right)^2} - \frac{G m_1 m_2}{\mu V_0^2 b_0} \right\}
\]

Substituting for \( \mu = \frac{m_1 m_2}{m_1 + m_2} \):
\[
b = b_0 \left\{ \sqrt{1 + \left[\frac{G (m_1 + m_2)}{m_1 + m_2}\right]^2} - \frac{G (m_1 + m_2)}{m_1 + m_2} \right\}
\]
\[ \bar{\nu}_e + p \rightarrow n + e^+ \quad \text{normally} \]

Here we have \[ \bar{\nu}_e + H \rightarrow n \]

i.e. in terms of weak interaction, its \[ \bar{\nu}_e + e^- + p \rightarrow n \]

Neutron beta decay in reverse, except the \( e^- \) state is

As far as the kinematics goes, we only need to consider

\[ \bar{\nu}_e + H \rightarrow n \]

\[ \begin{array}{c|c}
\text{Initial State} & \text{Final State} \\
\hline
\bar{\nu}_e & \nu \\
C & H \\
n & \nu \\
\end{array} \]

\[ \text{Momentum:} \quad E_{\bar{\nu}} = P_{\bar{\nu}} c = P_n c = E_n - m_n c^2 \]

\[ \text{Energy:} \quad E_{\bar{\nu}} + m_n c^2 = E_n \]

\[ \Rightarrow \quad (E_n - m_n c^2)^2 = E_n^2 - m_n^2 c^4 \]

\[ E_n^2 - 2m_n c^2 E_n + m_n^2 c^4 = E_n^2 - m_n^2 c^4 \]

\[ E_n = \frac{m_R^2 c^4 + m_n^2 c^4}{2 m_n c^2} = \frac{(m_R c^2 + m_n c^2)^2}{2 m_n c^2} \]
\[ E_v = E_n - m_n c^2 = \frac{(m_n c^2)^2 - (m_H c^2)^2}{2 m_H c^2} \]

On the momentum sign

\[ E_v = p_n c = \gamma m_n V c \]

\[ E_n = \gamma m_n c^2 \]

\[ \frac{E_v}{E_n} = \frac{V}{c} \]

\[ \frac{V}{c} = \frac{E_v}{E_n} = \frac{m_n^2 - m_H^2}{m_n^2 + m_H^2} = \left( \frac{m_n + m_H}{m_n^2 + m_H^2} \right) \left( \frac{m_n - m_H}{m_n^2 + m_H^2} \right) \]

\[ = 8.33 \times 10^{-4} \]

\[ V = 2.5 \times 10^5 \text{ m/s} \]

**NB** The resonance energy, \( E_v \), is determined entirely by the masses, except for the "width" of the reaction state width

\[ \Gamma = 900 \text{ sec} \]

width \( \Gamma = \frac{h}{\Gamma} = \frac{6.6 \times 10^{-16} \text{ ev sec}}{900 \text{ sec}} = 7 \times 10^{-19} \text{ ev} \]

Basically, to see this reaction, you have to fine tune your neutrino beam energy to within \( 7 \times 10^{-15} \text{ ev} \). I doubt this reaction has ever been observed!
\[ H = \begin{pmatrix} M & m \\ m & M \end{pmatrix} c^2, \quad M \geq m \]

(a) Let the eigenvalues be \( E_n \), and the eigenvectors be \( |n\rangle \) (\( n = 1, 2 \)).

Then \[ H |n\rangle = E_n |n\rangle \]

\( (H - IE_n) |n\rangle = |0\rangle \)

where \( I = \) identity matrix = \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) and \( |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)

For nontrivial solutions, \( \det (H - IE_n) = 0 \)

ie, \[ \begin{vmatrix} M - E_n & mc^2 \\ mc^2 & M - E_n \end{vmatrix} = 0 \]

\[ (M - E_n)^2 - (mc^2)^2 = 0 \]

\[ M - E_n = \pm mc^2 \]

\[ E_n = Mc^2 \pm mc^2 \]

Let \( E_1 = Mc^2 + mc^2 \), \( E_2 = Mc^2 - mc^2 \)

Plug each into \( \otimes \) to obtain corresponding eigenvectors.
\( n = 1, \quad \begin{pmatrix} -mc^2 & mc^2 \\ mc^2 & -mc^2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \)

\(- \alpha + \beta = 0, \quad \alpha = \beta.\)

Pick \( \alpha = 1, \beta = 1 \) \( \Rightarrow \) \( |1\rangle = (1) \)

\[ n = 2, \quad \begin{pmatrix} mc^2 & mc^2 \\ mc^2 & -mc^2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

\(\alpha + \beta = 0\)

Pick \( \alpha = 1, \beta = -1 \)

\( \Rightarrow \) \( |2\rangle = (1 \quad -1) \)

(2) Solution to Schrödinger's equation is

\[ \psi(t) = A|1\rangle e^{-iE_1t/\hbar} + B|2\rangle e^{-iE_2t/\hbar} \]

The initial condition is \( \psi(0) = (1) \)

Hence \( (1) = A(1) + B(-1) \)

So \( A + B = 1 \)

\( A - B = 0 \)

\( \therefore A = B = \frac{1}{2} \)
\( M_i^2 = 17 \text{ keV}, \quad m \leq M \)

\[
\tau = \frac{\pi \hbar}{2mc^2} \geq \frac{\pi \hbar}{2Mc^2} \\
= \frac{\pi (\hbar c)}{2Mc^2} \cdot c \\
= \frac{\pi (197.3 \text{ MeV} \cdot \text{fm})}{2(0.017 \text{ MeV})(3 \times 10^{23} \text{ fm/s})} \\
= 6 \times 10^{-20} \text{ seconds} .
\]

Let the recoil velocity be $v$.
The unshifted $Y$ energy is $E_0$.
The shifted $Y$ energy is $E_Y$.

Conservation of energy: \[ \frac{1}{2} M u^2 + E_Y = E_0 \]
Conservation of momentum: \[ M u = E_Y/c = E_0/c \]

\[ E_0 - E_Y = \left( \frac{E_0^2}{2Mc^2} \right) \]

(b). If $M = 1g$, \[ E_0 - E_Y = \left( \frac{14.4 \times 10^{16}}{2 \times 10^{-3} \times 9 \times 10^6} \right)^{\frac{1}{2}} \]
\[ \therefore \Delta V = 4.4 \times 10^{-11} \text{ m/s} \]

(c). Photon falls 20m under gravity:
\[ \Delta E = Mgh \]
\[ E = mc^2 \]
\[ \Delta E = \frac{\Delta V}{c} = \frac{gh}{c^2} \]

With $V = 3.48 \times 10^{18} \text{ Hz}$, \[ \Delta V = 7.73 \text{ kHz} \]
\[ I = \sigma E = \sigma \frac{V_{appl}}{(2\pi r/4)} \]

\[ dI = j dr \]

\[ I = \int_{a}^{b} j dr = \sigma V_{appl} t \frac{2}{\pi} \int \frac{dr}{r} \approx \text{Effudent} \]

\[ R = \frac{V_{appl}}{I} = \frac{\pi}{\sigma t} 2 \frac{1}{\ln{(b/a)}} \]

\[ R = \frac{\pi}{2} \frac{1}{0.355 \times 10^8} \frac{1}{1/100} \frac{1}{\ln{(20/10)}} \]

\[ = \frac{T}{2 \ln{2} \times 0.355} \frac{1}{10^6} \Omega \approx 20 \mu\Omega \]