MECHANICS

**Elementary Approach:**

1. Describe the kinematics (a) Co-ordinates  
   (b) Constraints

2. Diagram with all of the forces

3. Statics: $\sum F_x = 0; \quad \sum F_y = 0; \quad \sum \tau = 0$

4. Rigid Body-Free  
   (a) Motion of C of M: $\mathbf{F} = m \ddot{\mathbf{a}}$
   
   (b) Rotation about C of M: $\tau = I \ddot{\alpha}$

5. Rigid Body-Fixed Point 0  
   Rotation about 0 $\tau_0 = I_0 \alpha_0$

6. Use energy if "complicated"

7. Rotating System - use Euler's Equation  
   $\tau = \frac{d}{dt} \mathbf{L}$ in the rest frame

   becomes $\tau = \frac{d}{dt} \mathbf{L} + \omega \times \mathbf{L}$ in the rotating frame.

In the rotating frame, use principal axes as co-ordinates.

**Lagrangian Mechanics:**

1. Choose co-ordinates $q_1, q_2$ etc.

2. Express kinetic energy ($T$) in terms of the co-ordinates and their time derivatives $q_1, q_2$ etc.

3. Express the potential energy ($V$) in terms of the co-ordinates.
4. Write down the Lagrangian $L = T - V$.

5. Use the Lagrange equation

$$\frac{d}{dt} \left( \frac{dL}{dq_i} \right) - \frac{dL}{dq_i} = 0$$

for each of the co-ordinates $q_i$ as the equations of motion.

**ELECTRICITY AND MAGNETISM**

1. Gauss' Law: $\oint_{\Sigma} \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$ where $Q$ is the charge enclosed within the closed surface. In a dielectric medium $\oint_{\Sigma} \mathbf{D} \cdot d\mathbf{A} = Q/\varepsilon_0$ ($\mathbf{D} = \varepsilon\mathbf{E}$)

2. Potentials:

   Point charge: $V = Q/4\pi\varepsilon_0 r$

   Charged sphere, radius $a$

   Surface potential: $V = Q/4\pi\varepsilon_0 a$

   $r > a$: $V = Q/4\pi\varepsilon_0 r$

   Electric field:

   $$\begin{cases} 
   E = -\nabla V \\
   E_x = -dV/dx \text{ (one dimension)}
   \end{cases}$$

3. Magnetic Field:

   Amperes Law $\oint B \cdot dl = \mu_0 I$

   Long straight wire $B = \frac{\mu_0 I}{2\pi r}$ (tangential)

   Lorentz Force $d\mathbf{F} = dq \ \mathbf{v} \times \mathbf{B}$

   or $d\mathbf{F} = I \ dI \times \mathbf{B}$

   Lenz's Law $V = -\frac{d\phi}{dt}$

   Maxwell's Eqns. $\text{div} \ \mathbf{D} = \rho; \ \text{div} \ \mathbf{B} = 0$

   $\text{curl} \ \mathbf{B} = \mu_0 \mathbf{J}; \ \text{curl} \ \mathbf{E} = -\frac{d \mathbf{B}}{dt}$
THERMODYNAMICS

Ideal Gas:

\[ PV = nRT \quad \text{always} \]
\[ PV' = \text{const. for an adiabatic change} \]
\[ U = f_n(T) \text{ only} \]
\[ dU = C_VdT; \quad dH = C_pdT \]

Central Equation:

with 1st Law: \[ dU = dQ - dW \]
with 2nd law: \[ dU = TdS - PdV \left( + HdM + Ed\phi + FdL + EdZ \right) \]

Maxwell’s Relations:

\[ \frac{\partial T}{\partial V}_s = -(\frac{\partial P}{\partial S})_v; \quad \frac{\partial T}{\partial P}_s = (\frac{\partial V}{\partial S})_p \]
\[ \frac{\partial S}{\partial V}_T = (\frac{\partial P}{\partial T})_v; \quad \frac{\partial S}{\partial P}_T = -(\frac{\partial V}{\partial T})_p \]

Carnot Engine:

\[ W = |Q_H| - |Q_C|; \quad \eta = \frac{W}{|Q_H|} \]

Carnot theorem:- reversible heat engines

\[ \frac{|Q_H|}{|Q_C|} = \frac{T_H}{T_C} \]
\[ \therefore \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0 \]

General reversible cycle:

\[ \oint \frac{dQ}{T} = 0 \quad \text{or} \quad \oint dS = 0 \]

Note - For the reversible (Carnot) heat engine, the entropy of the total system is conserved.
SPECIAL THEORY OF RELATIVITY

Lorentz-Einstein Transformations

S is a frame at rest. S' has velocity v in the x direction

\[
\begin{align*}
x' &= \gamma(x - vt) \\
x &= \gamma(x' + vt') \\
t' &= \gamma(t - xv/c^2) \\
t &= \gamma(t' + x'v/c^2) \\
y' &= y \\
z' &= z
\end{align*}
\]

\[\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}\]

Lorentz contraction

L is the length of a ruler at rest in S

\[
L = x_2 - x_1
\]

L' is the length measured in S' ; it is \(x_2' - x_1'\) with \(x_1'\) and \(x_2'\) measured at the same time t'. Use,

\[
\begin{align*}
x_2 &= \gamma(x_2' + vt') \\
x_1 &= \gamma(x_1' + vt') \\
\therefore L' &= L/\gamma
\end{align*}
\]

the moving ruler is measured to be shorter.

Time Dilation

A clock in S is at rest. It measures a time interval \(T = t_2 - t_1\). Its position x is constant. The equivalent interval in S' is

\[
T' = t_2' - t_1' = \gamma T \text{ from above.}
\]

That is \(T < T'\). The time interval registered by the moving clock is less than the time interval in the frame S'. The moving clock runs slowly.

Velocities

\[
\begin{align*}
u^x_x &= x'/t' = (u_x - v)/(1 - u_xv/c^2) \\
u^y_y &= y'/t' = u_y/\gamma(1 - u_xv/c^2)
\end{align*}
\]
Doppler Shift

\[ \omega' = \omega \sqrt{(1 - v/c)(1 + v/c)} \] receding

\[ \omega' = \omega \sqrt{(1 + v/c)(1 - v/c)} \] approaching

Dynamics

Particles: \[ E^2 = E_0^2 + p^2 c^2 \]

with \( E = mc^2 = \gamma m_0 c^2 \); \( E_0 = m_0 c^2 \); \( p = mv \)

Photon, Neutrino \( m_0 = 0; E_0 = 0 \); \( E = pc \)

Conserve momentum and energy.

STATISTICAL MECHANICS

Microstates: Defined by giving the state of each particle in the system. Every accessible microstate is equally probable.

For a system composed of two parts, with numbers of accessible microstates \( \Omega_1 \) and \( \Omega_2 \), for the system \( \Omega = \Omega_1 \Omega_2 \). The temperature of a system is \( \frac{1}{k_B T} = \frac{d \ln \Omega(E)}{dE} \).

Boltzmann Factor: The probability that a system is in a microstate \( r \) when in thermal equilibrium with a reservoir at temperature \( T \) is,

\[ P_r = e^{-\beta E_r} / \sum_r e^{-\beta E_r} \]

Alternatively,

\[ dP(E) = g(E) dE e^{-\beta E} / \int g(E) dE e^{-\beta E} \]

where \( \beta = 1/k_B T \), \( E_r \) is the energy of state \( r \) and \( g(E) \) is the density of states.
**Mean Value:** For a system in equilibrium at temperature $T$

$$
\bar{a} = \sum_r a_r P_r.
$$

**Partition Function:**

$$
Z = \sum_r e^{-\beta E_r}
$$

Then,

$$
\bar{E} = -\frac{d \ln Z}{d \beta}
$$

$$
P = kT\left(\frac{d \ln Z}{d V}\right)
$$

$$
F = -kT \ln Z
$$

$$
C_V = -(d \beta / d T)(d^2 \ln Z / d \beta^2)
$$

Chemical Potential $\mu = -kT(d \ln Z / d N)$

**Classical and Quantum Gases:**

Classical: All microstates are allowed. Bose-Einstein only one of (0110), (1010), (0011).

Fermi-Dirac: (0012) is forbidden
(1347) is good.

Mean occupation: Classical:

$$
f_s \propto e^{-\beta \varepsilon_s}
$$

Bose:

$$
f_s = \frac{1}{e^{\beta (\varepsilon - \mu)} - 1}
$$

Fermi:

$$
f_s = \frac{1}{e^{\beta (\varepsilon - \mu)} + 1}
$$
QUANTUM MECHANICS

Schrödinger’s Equation:

\[ \hat{H} u = E u \quad \text{(time-independent potential)} \]

\[ u(r,t) = u(r) e^{-iEt/\hbar} \]

\[ \psi = \sum c_n u_n; \quad \sum c_n^2 = 1 \]

\[ c_n = \int u_n^* \psi d\tau \]

New Basis

\[ v_n = \sum c_m u_m \]

\[ \int v_m^* v_n \, d\tau = \delta_{m,n} \]

Operators

\[ \hat{x} = x; \quad \hat{p}_x = -i\hbar \frac{d}{dx}, \quad \hat{H} = \frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V \]

Boundary Conditions

\[ V = \infty \quad u_1 = u_2 \]

\[ V = \text{step} \quad u_1 = u_2 \quad \text{and} \quad \frac{du_1}{dx} = \frac{du_2}{dx} \]

Time Dependence

\[ \text{e.g. } \psi(x,t) = \frac{1}{\sqrt{2}} (u_\alpha e^{-iEt/\hbar} + u_\beta e^{-iEt/\hbar}) \]

Hence interference.
Perturbation Theory

\[ E_m = E_m^0 + \sum \frac{|H_{nk}|^2}{E_m - E_k^0} \]

where \[ H_{nk}' = \int u_m^* H' u_k \, d\tau \]

\[ u_m = u_m^0 + \sum \frac{H_{km} u_k^0}{E_m - E_k^0} \]