Quantifying Euler-Bernoulli and Timoshenko Beam Theory Accuracy for Estimating Flexural Rigidity of a Bone Surrogate in Four-Point Bending with Finite Element Analysis

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Introduction: Bone mechanical properties are estimated by Euler-Bernoulli (EB) and Timoshenko (TI) beam theories. Mechanical testing and image-based finite element analysis (FEA) are commonly used for bone mechanical assessment; their accuracy is therefore crucial. This study quantified some of the errors in long-bone mechanical assessments.

Methodology: Four-point bend tests on a bone surrogate were performed with three replicates and six repeats on each. Deformation was measured at the crosshead (CH), midpoint (mid) and with digital image correlation (DIC). The experimental bending stiffness (K_{CH} and K_{Mid}) was calculated as the slope of the linear portion of the force-(F) deflection (δ) data. The same bending test was simulated with FEA (Abaqus CAE/2017) on CAD and CT-based models. The computational bending stiffness (K_{FEA}) was calculated by dividing the total reaction force at bottom supports over the vertical deflection due to bending at the midpoint. TI and EB beam theories were used to calculate the bone surrogate flexural rigidity (EI).

Results: Relative to DIC results, EB and TI beam theories underestimated the K_{CH}, and K_{Mid} by -39% and -29%, respectively. While K_{FEA} of the CAD-FEA agreed with the DIC results, K_{FEA} of CT images showed a 5% overestimation. EB theory underestimated the EI from all methods by -60% to -27%; whereas, TI theory prediction of EI was more accurate with -40% to +5% error. The DIC method with TI theory estimated EI with a +1% error.

Discussion: Accurate estimation of K and EI is crucial for investigations of bone disease prevention, treatment and diagnosis. Limitations of this study were: small sample sizes; and testing was performed on a bone surrogate with simplified geometry and materials. The results of this study confirmed that using TI beam theory instead of EB beam theory can improve the accuracy of bone health research.

References

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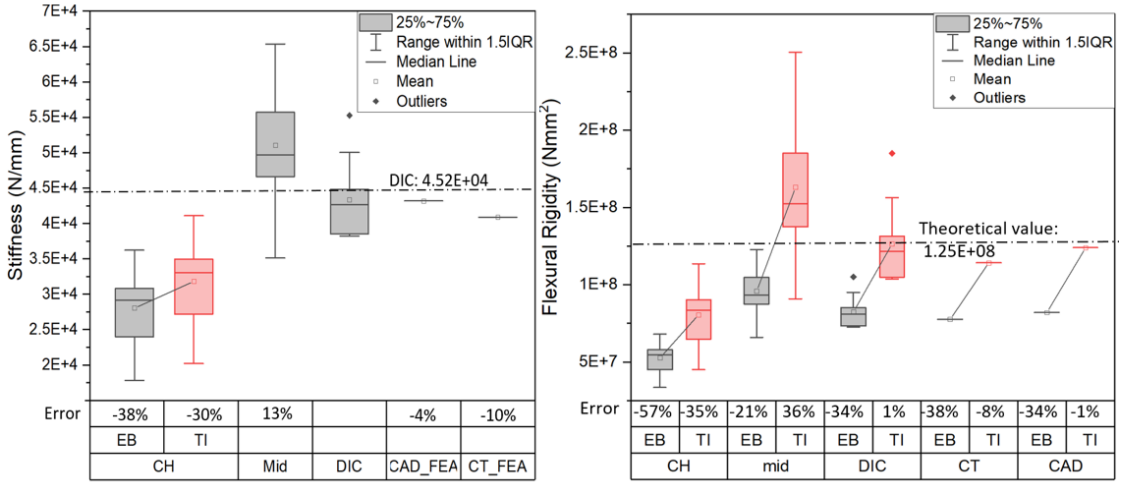

Figure 1. Left: Bone surrogate bending stiffness; Right: Bone surrogate flexural rigidity (CH: crosshead displacement, mid: midpoint deflection, CT: FEA on CT-model, CAD: FEA on CAD-model, DIC: Digital Image Correlation, TI: Timoshenko beam theory, EB: Euler-Bernoulli beam theory)