A tactile sensor based on thin-plate deformation
R.E. Ellis,* † S.R. Ganeshan* and S.J. Lederman‡‡

(Received in final form: June 3, 1993)

SUMMARY
Traditionally, tactile sensors have been designed using compliant, rubber-like materials; when such a sensitized gripper grasps or otherwise manipulates an object, the normal strain deformation in the compliant material is sampled. The resulting information can be used to deduce simple local geometry of the contact, but the transduction process does not typically permit use of the individual strains in determining large-scale properties of the object (e.g., the inertia). Measurements of inertial parameters of grasped objects require accurate, low-hysteresis transduction that few tactile sensors currently provide.

An alternative is to work from the task specification, and determine the tactile information that is necessary to accomplish the task. Here, we consider how to sense the length and mass of a uniform object that is gripped in a gravitational field, and show the design and assessment of a new kind of tactile sensor that is based on the theory of the deformation of thin plates. Features of this design include its potentially rugged realization, and its high-accuracy measurement that is more typical of force sensors than of tactile sensors.

KEYWORDS: Tactile sensor; Thin-plate deformation; High accuracy.

1. INTRODUCTION
The principal design constraint driving the development of robotic tactile sensors has been the determination of fine-form geometry.1-7 A typical robotic sensor is composed of a rubber-like covering material, beneath which normal displacement is transduced by technology that may be: resistive,8-13 optical,14-20 photoelastic,21,22 capacitive,23-25 magnetic,26-28 piezoelectric,29,30 electrochemical,31 impedance,32 or dynamic vibration.33 Some sensors can also transduce shear components34-37 or thermal effects,38-40 but these have not been the major design considerations to date.

Most of these transduction designs have only a few bits of reliable information available from each sensing site, exhibit high hysteresis, and cannot be used at a high temporal frequency. To achieve these latter abilities, a few researchers have attempted to construct very small force-sensing fingertips41,42 or whisker-like probes.43 Such sensors are intriguing, but are very limited in their ability to detect object properties.

Our approach to the development of a new kind of tactile sensor has been to select a specific object property to be determined, and to use resulting constraints to drive the engineering process. This has led us to employ a novel form of skin—a thin metal plate—which allows deduction of the desired object property by means of a sturdy sensor that can easily fit inside a traditional planar-finger robot gripper.

Most robotic tactile sensors are built with the ultimate goal of an autonomous robot system, and do not consider the possibility of human interaction. In order to perform teleoperation, however, the human operator must have sufficient information available if the task of remote manipulation is to be performed successfully. Here, we consider an extremely simple task:

If the remote robot is grasping a tool, what are the approximate dimensions of the tool?

This task arises in an environment in which the human and robot are to use tools to accomplish some greater goal, but it is not desirable or possible to sense the tool in some other way (e.g., by hard automation or visual inspection of the tool to determine its type).

In order to render the task solvable, we model the tool as a simple uniform object with homogeneous material properties, and the grasp as a static equilibrium in a gravitational field. A mechanical analysis of the gripper-tool system shows that two independent and simultaneous force-torque measurements are needed just to detect tool length—readings that current transducers seem unable to provide with sufficient fidelity.

Given the demonstrated need for two independent measurements, we then consider how a tool would deform a thin plate composing part of the robot gripper. The deformation of this plate can be detected using strain gauges or other technology, and the local strain readings can be inverted to find the forces and torques at two independent sites. These can in turn be used to deduce the desired object properties.

We have constructed a prototype sensor, and have conducted an experimental analysis of its behavior that demonstrates the feasibility and limitations of this technique. In a related publication,44 we present the results of psychophysical investigation of the human ability to perform the same task.

* Department of Computing and Information Science, † Department of Mechanical Engineering, ‡ Department of Psychology, Queen's University, Kingston, Ontario (Canada) K7L 3N6.
2. MECHANICAL ANALYSIS OF GRASPING A ROD

An object held still in the hand or gripped by a robot gripper, without any rotary or translatory movement, constitutes a physical system at rest. This system can be effectively studied using the principles of statics: that branch of mechanics dealing with bodies in equilibrium, that is, bodies at rest or moving with constant velocity.

The assumptions made in the analysis of the system are:

1. The material of the object is homogeneous.
2. The object is uniform.
3. The angle or orientation of the object with respect to the ground plane is known.
4. The gripper-object system is at rest and in a state of static equilibrium.

A body is homogeneous if matter is continuously distributed over the volume of the object, so that any element cut from the body possesses the same specific properties of the body. A body is uniform if it has the same geometric properties throughout its length, for example, a rod having a constant cross-section throughout its length. (For a homogeneous body, uniformity also implies that the body has the same mass per unit length throughout.)

The gripper-object system is considered to be in static equilibrium when there is no macroscopic motion; in this situation, a balance of both force and torque is maintained. This assumption excludes any situation where damages or permanent deformations may occur to either the object or the gripper due to excessive forces and torques, or to any micro-motion (e.g., creep).

2.2 Free-body analysis

Suppose that the object is gripped at two independent locations, simultaneously and from opposing sides as in Figure 1. For simplicity, assume that the object is parallel to the ground plane. The segment from the object base to the first grip, of length \( c \), is subject to a gravitational force \( \mathbf{F}_1 \); the segment between the grips, of length \( d \), is subject to \( \mathbf{F}_2 \); and the segment from the second grip to the tip, of length \( L - c - d \), is subject to \( \mathbf{F}_3 \).

The free body diagram representing all the forces and torques is shown in Figure 1(a). Let the distance separating the two grip locations be \( d \), where \( d \) is given. Let \( \mathbf{F}_{R1} \) and \( \mathbf{F}_{R2} \) represent the reactive forces developed at the two grip locations, and let \( \mathbf{T}_{R1} \) and \( \mathbf{T}_{R2} \) represent the reactive torques. This case can be analyzed by splitting the system into three independent sub-systems as shown in Figure 1(b), (c) and (d), and then superimposing the solutions obtained from the three.
independent analyses. The first and the third subsystem are simple to analyse; the second sub-system shown in Figure 1(b) is an example of an indeterminate structure, which can be solved by making use of the standard solutions available in the literature.\(^4\) Thus, the reactive forces developed at the two grip locations are given by:

\[
F_{R1} = -F_1 - \frac{1}{2}F_2 \\
F_{R2} = -F_3 - \frac{1}{2}F_2
\]

and the reactive torques are given by

\[
T_{R1} = F_1 \frac{c}{2} - F_2 \frac{d}{12} \\
T_{R2} = F_2 \frac{d}{12} - F_3 \frac{L - c - d}{2}
\]

where

\[
F_1 = \rho_A \cdot c \cdot g \\
F_2 = \rho_A \cdot d \cdot g \\
F_3 = \rho_A \cdot (L - c - d) \cdot g
\]

Given that the reactive forces and torques are known quantities, using any three of the above equations (equations (1) to (4)), the mass per unit length \(\rho_A\) and the total length \(L\) of the object can be determined. Thus, if two independent and simultaneous force-torque measurements are made, then the length of the object becomes statically determinate irrespective of the grip location.

Note, however, that if a single measurement is made then only certain relations can be deduced. Setting \(d = 0\), with the consequences of

\[
F_2 = 0 \\
F_3 = \rho_A \cdot L \cdot g \\
T_R = -\frac{1}{2} \rho_A (L - c)^2 - c^2 g
\]

entails that the grip location cannot be determined: a single force-torque sensor cannot be used to accomplish this task. The three unknowns—mass per unit length, total rod length, and grip distance from the rod tip—cannot be determined from the two equations that relate unknowns to sensor measurements.

The generalization to the situation in which the rod is held at some angle \(\theta\) to the ground plane (X axis) is straightforward. The gravitational acceleration \(g\) must be replaced by \(g \cos(\theta)\); clearly, this approaches zero as \(\theta \to \pm 90^\circ\), so the maximum torque sensitivity is achieved in a horizontal grip.

3. ANALYSIS OF THIN-PLATE DEFORMATION

In order to identify the object parameters that were described in Section 2, accurate measurements of multiple forces and torques must be made within a robot gripper. We propose that studying the manner in which thin plates are deformed by the object during the grasp is a novel and feasible way of acquiring the necessary information. That is, we propose that by attaching thin, flexible plates to the inner surface of a robot gripper, it is possible to analyse the deformation and deduce the length and mass of a uniform object grasped by the robot.

A plate is a planar (flat, noncurved) structure whose thickness is very small when compared to its other two dimensions, and thus can be approximated as a two-dimensional object. This section describes the mechanics of plate deformation, and how to invert strain readings to find the object parameters of interest.

One of the chief considerations in the design of a sensor using thin plates is the specification of the boundary conditions at the edges of the sensor plate. The discussion here is restricted to plates that are bound geometrically by straight lines, because of the difficulty of analysing boundary conditions of plates that have curved edges. For simplicity, we analyse only square plates.

3.1 Design considerations

If the primary consideration is the ease with which the analytical solutions are obtained, then the chosen design will be a plate bounded by simple supports on all four sides. The solutions for this case are well known.\(^4\) But if we are to mount a sensor on an inner side of a gripper, then we can see that this is highly impractical: it is essential that at least one edge of the sensor plate be fixed, or the plate could fall off the gripper.

The most sensitive design will involve a plate sensor fixed at one edge and free at the other three edges, but this is clearly not the most rugged design: the risk of damage to the plate in the form of deformations or buckling is very high. Free edges offer high sensitivity to the plate sensor but have the serious drawback of rendering it less robust mechanically. On the other hand, the most robust design will involve a plate sensor bound by fixed edges on all four sides. Fixed edges lend a high degree of robustness but have the drawback of reduced sensitivity. Therefore, one pair of fixed edges and one pair of simply supported edges are considered a reasonable compromise, since the degree of sensitivity is not greatly affected, and the structure is also rendered resistant to damage by this choice. Our design has fixed supports at two opposite edges, with the other two edges simply supported (see Figure 2).

Once the boundary conditions have been determined,
the next important aspect to be considered is the type of loading pattern that will be applied on the plate sensor. The load patterns that may be applied on the plate sensor can be broadly classified into uniformly distributed loads, uniformly varying loads and non-uniform loads. These load patterns can be point loads, line loads, partial loads, or they may be present on the entire surface of the plate. The load pattern can be categorized by considering how a simple gripper would grasp a rod.

To obtain two force-torque measurements simultaneously, two similar sets of plate sensors each consisting of an upper plate (U) and a bottom plate (B) must be used; all four plate arrangements can have the same boundary conditions. The object grasped by both sets of plate sensors, which could be mounted inside a parallel-jaw gripper, is shown in Figure 3. For an object that is grasped with its longitudinal axis coincident with the X axis of the plate, the predicted loading pattern is in the non-uniform load category. The loads may be present on the entire surface of the plate (when the width of the object is equal to, or greater than the width of the plate), or may cover only a part of the sensor surface (when the width of the object is less than that of the plate).

The analysis for uniformly distributed loads and uniformly varying loads are fairly well-established in the literature, but there has been little work done in the area of non-uniform loads because so general a problem cannot be stated precisely. In Section 3.2, we present the forward solution for a plate sensor subjected to a non-uniform loading pattern; this solution corresponds to the situation where the strains are determined from the known loads (forces) acting on the plate. In Section 3.3, we present the inverse solution, which determines the loads applied on the plate from strains that are measured at desired locations.

3.2 Forward solution of the proposed plate configuration
Consider a square plate with sides of length b. The sides x = 0 and x = b are simply supported, with the plate fixed at y = ±b/2. (See Figure 2). If the loading pattern varies in both the x and y directions, then the analysis becomes very complicated; also, the plate-deformation strains would have to be measured in both x and y directions. For ease of analytical and experimental analysis, it is assumed that the load varies in the x direction only. This type of loading corresponds to objects that have non-varying cross-sections orthogonal to some direction, for example, a rectangular parallelepiped or a rod of circular cross-section.

The first step in obtaining the forward solution for a given plate is the proper choice of a function to represent the loading pattern. In the case of uniformly distributed loads or uniformly varying loads, the assumed functions are straightforward. For a load distribution produced by a rod grasped parallel to the X axis (thus producing uniform loading in Y and non-uniform loading in X) the loading function can be represented by the infinite series

\[ F_x(x, y) = \sum_{i=1}^{\infty} F_i \sin \left( \frac{i\pi x}{b} \right) \]

(5)

where \( F_i \) is the load acting at a point (x, y) and in the direction of the surface normal.

The deflection equation for a normally loaded plate, with an arbitrary loading function, is

\[ \frac{\partial^4 W(x, y)}{\partial x^4} + 2 \frac{\partial^4 W(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x, y)}{\partial y^4} = \frac{F_x(x, y)}{D} \]

or by common convention

\[ \nabla^4 W = \frac{F_x}{D} \]

for a plate deflection W(x, y) and flexural rigidity D.

This can be solved by representing W as the sum of the component \( W_1 \) that is due to the loading, and the component \( W_2 \) that is conserved:

\[ W = W_1 + W_2 \]

where

\[ \nabla^4 W_1 = \frac{F_x}{D} \]

(6)

\[ \nabla^4 W_2 = 0 \]

(7)

Expanding equation (5) into the right-hand side of equation (6) and integrating, we find that

\[ W_1 = \sum_{i=1}^{\infty} \frac{b^4 F_i}{i^4 \pi^4 D} \sin \left( \frac{i\pi x}{b} \right) \]

(8)

Equation (7) can be solved, using standard methods and the simple-support boundary conditions at x = 0 and x = b, to obtain

\[ W_2 = \sum_{i=1}^{\infty} \left[ A_i \cosh \left( \frac{i\pi y}{b} \right) + B_i \frac{\sinh \left( \frac{i\pi y}{b} \right)}{\pi f} \right] \]

(9)

so the governing equation we now express as

\[ W = \sum_{i=1}^{\infty} \left[ \frac{b^4 F_i}{i^4 \pi^4 D} + A_i \cosh \left( \frac{i\pi y}{b} \right) + B_i \frac{\sinh \left( \frac{i\pi y}{b} \right)}{\pi f} \right] \]

(10)
Note that the integral coefficients \( A_i \) and \( B_i \) must satisfy the fixed-support boundary conditions at \( y = \pm b/2 \). We abbreviate a frequently occurring expression as

\[
\alpha_i \overset{\text{def}}{=} \left( \frac{i\pi}{2} \right)
\]

and we observe that, because \( W \) must vanish at the boundary points,

\[
\frac{b^4F_i}{i^4\pi^4D} + A_i\cosh(\alpha_i) + B_i\alpha_i\sinh(\alpha_i) = 0
\]

\[
A_i\sinh(\alpha_i) + B_i\sinh(\alpha_i) + B_i\alpha_i\cosh(\alpha_i) = 0
\]

(11)

(12)

Using the abbreviations

\[
\beta_i \overset{\text{def}}{=} \frac{\tanh(\alpha_i)}{\sinh(\alpha_i) + \alpha_i\cosh(\alpha_i) - \alpha_i\tanh(\alpha_i)\sinh(\alpha_i)}
\]

\[
\alpha_i \overset{\text{def}}{=} \frac{b^4F_i}{i^4\pi^4D}
\]

we obtain from Equations 11 and 12 the solutions

\[
A_i = -\alpha_i \left[ \frac{1}{\cosh(\alpha_i)} + \beta_i\alpha_i\tanh(\alpha_i) \right]
\]

\[
B_i = -\alpha_i\beta_i
\]

(13)

(14)

Using the equation of strain in the \( X \) direction \( \varepsilon_x(x, y) \) at a point \((x, y)\) for a plate of thickness \( h \) loaded nonuniformly in \( X \)

\[
\varepsilon_x(x, y) = -\frac{h}{2\pi} \frac{\partial^2W}{\partial x^2}
\]

we can express the strain at a point of the plate explicitly as

\[
\varepsilon_x(x, y) = \frac{h}{2\pi} \sum_{i=1}^{\infty} \left[ \alpha_i + A_i\cosh\left(\frac{i\pi y}{b}\right) + B_i\left(\frac{i\pi y}{b}\right)\sinh\left(\frac{i\pi y}{b}\right) \right] \sin\left(\frac{i\pi x}{b}\right) i^2
\]

(15)

This solution is useful only if the plate is loaded uniformly in the \( Y \) direction. A reasonable approximation is to consider a uniformly loaded region and an unloaded region, as in Figure 2, and the width of the load is equal to the width of the sensor plate. The solution for partial loading, where the load width is less than the plate width, can be obtained by solving for the loaded region bounded by \( y = \pm \Delta/2 \), and making suitable modifications to the constants of integration.\(^{47}\)

Using the abbreviations

\[
a_i \overset{\text{def}}{=} \frac{b^4F_i}{i^4\pi^4D}
\]

\[
\alpha \overset{\text{def}}{=} \frac{i\pi}{2}
\]

we finally have

\[
A_i = A'_i + a_i \left( \gamma_i\sinh(2\gamma_i) - \cosh(2\gamma_i) \right)
\]

\[
B_i = B'_i + \frac{a_i}{2} \cosh(2\gamma_i)
\]

(16)

(17)

Substituting the values of equations (16) and (17) into equation (15) gives an approximation of the strain in a thin deformed plate arising from the grasp of a uniform rod.

It is possible to analyze more complicated loading patterns,\(^{47}\) but the ones presented here suffice for the grasping task that is to be addressed.

3.3 Inverse solution of the proposed plate configuration

If a plate is to be used as a sensor to detect the presence or absence of loads, or to sense the magnitude of the loads acting on the plate surface, then an inverse solution is essential. The inverse solution is used to determine the lateral load distribution on the surface of the plate given the strain measurements obtained from the plate, from which the rod properties can be deduced.

Observe that the forward-deformation solution, equation (15), expresses the strain at a point as an infinite series. Because of the way \( F_i \) occurs in \( A_i \) and \( B_i \), terms can be collected to re-express the relation as

\[
\varepsilon_x(x, y) = \sum_{i=1}^{\infty} C_i F_i
\]

(18)

A fourth-order approximation is accurate to about 98% in a related loading situation,\(^{46}\) which we take to be sufficient for our purposes. Representing the individual strain readings taken at points \((x_i, y_i)\) as

\[
\varepsilon_i \overset{\text{def}}{=} \varepsilon_x(x_i, y_i)
\]

the four equations

\[
\varepsilon = C_{11}F_1 + C_{12}F_2 + C_{13}F_3 + C_{14}F_4
\]

produce the linear system

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} \\
C_{21} & C_{22} & C_{23} & C_{24} \\
C_{31} & C_{32} & C_{33} & C_{34} \\
C_{41} & C_{42} & C_{43} & C_{44}
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4
\end{bmatrix}
\]

(19)

or, more tersely,

\[
\bar{\varepsilon} = \bar{C}\bar{F}
\]

(20)

The compliance matrix \( C \) is invertible (except for
3.4 Calculating rod length from plate sensor data

Consider a rod of length $L$, with mass per unit length $\rho_A$, that is gripped above and below by two plate sensors; the free-body diagram is shown in Figure 4. From conditions of static equilibrium, we have

$$ F_{B_1} - F_{U_1} = \rho_A L_1 g \quad \text{(25)} $$

$$ \rho_A(L_1 - b)g \left[ \frac{L_1 - b}{2} + (b - \bar{x}_{U_1}) \right] = F_{B_1}(\bar{x}_{U_1} - \bar{x}_{B_1}) \quad \text{(26)} $$

Using the calculated values from equations (22) and (24), the values of $\rho_A$ and $L_1$ can be found. If the rod is gripped so that an overhang $L_2$ exists on the left of the diagram, the diagram and equations are simply reflected about the vertical axis, and readings from the second pair of sensors are used.

Note that all of the above reasoning is invalidated if the fundamental assumption of known partial loading is violated. The validity of this assumption could be tested with redundant sensing of strains $\varepsilon_s(x, y)$ at other points, but was not done experimentally because of the great complexities involved.

4. EXPERIMENTAL RESULTS

The proposed thin-plate sensor design and inverse solution have been tested by implementation. The complete sensor consists of two pairs of opposing plates, manufactured of materials that can be expected to withstand the rough conditions of a working robot gripper. Due to the practical difficulties involved in implementing a technique for obtaining a full stress map, only the case where the loading is applied throughout the plate was experimentally evaluated.

Each plate was fabricated from low carbon stainless steel of thickness 0.46 mm. A strip of width 40 mm and length 120 mm was bent at right angles at a distance of 40.95 mm from both ends as shown in Figure 5. To obtain a firm fixed support for the two opposite sides, two aluminum blocks of size $39 \times 39 \text{ mm}$ and thickness 5 mm were rivetted to the sides. For the simple supports, an aluminum channel section of outer width and length 38.1 mm was used. The top of the channel was filed to obtain a sharp knife edge, acting as simple supports.

![Fig. 4. Free-body diagram of gripped rod.](image)

![Fig. 5. Design of a plate-sensor element.](image)
Both the channel section and the plate arrangement were then screwed to a wooden base block of size 120 × 38.1 × 24 mm, which was attached to a bench vice. The entire sensor consisted of four channel sections and four plate arrangements, with the distance between adjacent plates being 24 mm. Samples of the plate material were subjected to tensile tests, and the elastic modulus was found to be $233 \times 10^3$ MPa.

Four resistance strain gauges of resistance 350 ohms ($\pm 0.3\%$), gauge factor 2.13 ($\pm 0.5\%$), gauge length 1.57 mm and transverse sensitivity $+0.6 \pm 0.2\%$ were used for measuring the strain on each plate. These gauges were mounted on the bottom surface of the plate along the center line between the simple supports, and the locations (X and Y coordinates with respect to the origin) of these gauges were recorded.

In order to account for errors in strain gauge installation, such as misalignment, lack of bonding, etc., the gauge readings had to be calibrated against a known value of strain. For this purpose, the entire surface of the plate was subjected to uniform load conditions by placing parallel-opposed blocks, whose length and width matched those of the plate, on the top surface of the plate. Different uniform conditions were obtained by using blocks of varying height and material under simple gravitational load. The readings obtained from each strain gauge were then individually plotted against the theoretically calculated strains to obtain the calibration factor. The plots obtained for all the gauges revealed an almost linear relationship.

We note that the above procedure results only in the force calibration of the plate sensor, since the torque acting on the sensor is zero. Torque calibration, although essential and desirable, is hard to obtain. When the plate sensor is subjected to torque, as in the case when an object (whose length is greater than that of the plate) is gripped, it is very difficult to theoretically determine the strains from standard analytical techniques, because of the non-uniform nature of the loading pattern. However, methods that use piecewise-linear load approximation techniques, such as finite-element or finite-difference techniques, could likely be employed.

### 4.1 Experimental Procedure

Four different objects were used in the study. The width of all four objects was 38.1 mm, which is the same as that of the plate. Two of the objects were made of oak, one of pine and the other of aluminum. Table II shows the total length and total mass for each of the rod. Each of these objects was gripped at an intermediate location between its ends. The lengths extending on the two sides of the plate sensor are as shown in Table III.

When the object is gripped by both the left and right set of plate sensors (where the top plate and the bottom plate constitute a set), each set predicts the overhang plus the length of the gripped portion of the object. Thus, the partial length of the rod extending to one side of the plate sensor and the mass per unit length of the gripped object were determined using Equations 25 and 26. Four different sets of readings were obtained for each rod from different trials, and the mean was obtained for the length estimates. For the plate sensor, the total mass of the object given by the product of the mass per unit length and the total length of the object was determined using Equation 25.

### 4.2 RESULTS

Following calibration, the objects were grasped by the sensor and data were collected. The strain readings were then used to deduce the masses and overhang lengths of the test object. Four independent trials were conducted with each test object, to determine the sensor's repeatability.

The perceived mass of each rod “piece” (i.e. the left and right components) is presented in Table IV. The columns of perceived masses indicate the use of the raw sensor data in the closed-form solutions (found in Section 3.3). If these data are then fit linearly to the known masses, the corrective formula

$$M_c = 1.0012M_p - 1.1625$$

produces the corrected mass $M_c$ from the perceived mass $M_p$. The corrected values are accurate to $\pm 1.6\%$, and the standard deviations indicate the spread of the data for multiple readings. The measurement error was so great for the 60 mm object (with a mass of only 12 g) that it was discarded as a statistical outlier.

---

**Table II** Lengths and masses of test objects.

<table>
<thead>
<tr>
<th>Material</th>
<th>Length ($\pm 1$ mm)</th>
<th>Mass ($\pm 0.05$ g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>1138</td>
<td>229.2</td>
</tr>
<tr>
<td>Oak</td>
<td>878</td>
<td>447.1</td>
</tr>
<tr>
<td>Oak</td>
<td>1563</td>
<td>796.0</td>
</tr>
<tr>
<td>Aluminum</td>
<td>363</td>
<td>238.6</td>
</tr>
</tbody>
</table>

**Table III** Overhang length of test objects.

<table>
<thead>
<tr>
<th>Material</th>
<th>Left ($\pm 1$ mm)</th>
<th>Right ($\pm 1$ mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine</td>
<td>60</td>
<td>1060</td>
</tr>
<tr>
<td>Oak</td>
<td>510</td>
<td>350</td>
</tr>
<tr>
<td>Oak</td>
<td>895</td>
<td>650</td>
</tr>
<tr>
<td>Aluminum</td>
<td>225</td>
<td>120</td>
</tr>
</tbody>
</table>

**Table IV** Perception of object mass

<table>
<thead>
<tr>
<th>Perceived mass (g)</th>
<th>Corrected mass (g)</th>
<th>True mass (g)</th>
<th>Mean error (%)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>81.60</td>
<td>80.54</td>
<td>78.6</td>
<td>$+2.11$</td>
<td>0.17</td>
</tr>
<tr>
<td>173.00</td>
<td>172.35</td>
<td>178.0</td>
<td>$-3.18$</td>
<td>0.12</td>
</tr>
<tr>
<td>316.00</td>
<td>315.22</td>
<td>320.5</td>
<td>$-1.65$</td>
<td>0.13</td>
</tr>
<tr>
<td>428.00</td>
<td>427.35</td>
<td>424.0</td>
<td>$+0.79$</td>
<td>0.16</td>
</tr>
<tr>
<td>244.00</td>
<td>243.73</td>
<td>242.0</td>
<td>$+0.72$</td>
<td>0.12</td>
</tr>
<tr>
<td>214.00</td>
<td>213.09</td>
<td>213.0</td>
<td>$+0.10$</td>
<td>0.11</td>
</tr>
<tr>
<td>152.00</td>
<td>151.92</td>
<td>147.9</td>
<td>$+2.72$</td>
<td>0.14</td>
</tr>
</tbody>
</table>
The perceived length of each rod piece is presented in Table V. Again, the measurement error was so great for the object with a length of only 60 mm that it was discarded as a statistical outlier.

The raw perceived length produced an average error of 13.86%. In order to account for the torques subtended by the portion of the object between the two adjacent plate sensors, a correction was applied to the predicted length. However, the correction was very small, only improving the results of measured length to an average of 12.82%. This rather high error can be explained by the lack of torque calibration of the plate sensor.

We observed that the calculation of the length of an object is a function involving the squares of the distances of the resultant forces from the interior edge of the plate. Thus, an error in predicting the location of the resultant $x$ of the load on the plate will lead to an error that is a function of $x^2$. In order to account for this, a least-squares quadratic fit of the data was obtained and the new corrected lengths were computed with the empirically derived equation

$$L_c = 0.0001357L_x^2 + 0.6852L_x + 51.78$$

This produced the corrected length $L_c$ from the perceived length $L_x$ to an accuracy of $\pm 3.8\%$.

It is likely that the use of extremely small gauges (e.g. ones about 0.2 mm long) would improve these results further, as would more precise fabrication of the plates. Another improvement might be derived from using redundant sensors to better determine the actual deformation of the plate during the grasp.

5. CONCLUSIONS

There are many different ways of designing and constructing a tactile sensor. We have begun with a proposed task: determining the length and mass of an object that is gripped in a gravitational field. From this task, we performed a free-body equilibrium analysis that determined the physical components that must be determined in order to accomplish the task.

The design criteria of ruggedness and repeatability led us to consider a gripper in which thin plates were deformed by the object during the grasp. This unusual design required careful forward and inverse solutions, which we found in a closed form. The loading patterns we assumed can be generalized to other grasp situations, and additional transducers can be mounted on the thin plates in order to derive other sensory information.

Construction, calibration, and use of a prototype thin-plate tactile sensor validated our approach. In a laboratory situation, the sensor was capable of surprisingly high accuracy in measuring the mass and length of objects which had considerably different densities. The repeatability of the measurements was also high.

One limitation of our design is that we assumed that the loading pattern on the plate was due to an object that produced a contact region smaller than, or equal to, the size of the plate. If the object were larger than the sensor, further theoretical analysis and experimentation would be needed.

Improvements to our design include the introduction of load cells to better measure the support reactions, and perhaps the use of other boundary conditions. One intriguing alternative boundary condition is pinning only the corners of the plate, to permit greater flexure of the plate during grasping. Another intriguing issue is how to improve the gripping friction of the plates without compromising the loading pattern, perhaps by integrating our design with more traditional tactile "skins".

The principal lesson we learned from this research effort was the importance of designing the sensor for the task. Better task specification and consideration of many transduction methods present many opportunities for improvements in tactile sensing.

ACKNOWLEDGMENTS

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada, the Information Technology Research Centre, the Manufacturing Research Corporation of Ontario, and the federal government under the Institute for Robotics and Intelligent Systems (which is a National Centres of Excellence programme).

References


9. M.H. Raibert and J.E. Tanner, “Design and implementa-
47. S.R. Ganeshan, “Robotic and haptic perception of length of statically held objects”, Master’s Thesis (Queen’s University at Kingston, 1992).